ATLAST Deployment & Push Pack Spares Optimization Module

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Abstract: *ATLAST (Aircraft Total Life-Cycle Assessment Software Tool) was developed to support life-cycle logistics impact forecasting for new and aging weapon-system fleets. ATLAST emulates airframe operations, using tail numbers and operation profiles, to predict unscheduled and scheduled removal events according to location and age of components over time. It then administers a capacity-constrained maintenance and logistic support process that return assets back to serviceable conditions. The latest version of ATLAST consists of a "Deployment & Push Pack Spares" optimization module. The module utilizes a hybrid analytical-simulation optimization approach to rank spare parts by their effectiveness in increasing availability. Decision makers can use ATLAST to determine the optimal level of spares for the deployment of aircraft at a specific location. Detailed mathematical formulation and three numerical examples are presented in this paper.*

I. Introduction

This paper describes the technical features of the *Aircraft Total Life-Cycle Assessment Software Tool* (ATLAST) developed by Clockwork Solutions for the military aviation communities and particularly focuses on forecasting life-cycle maintenance and logistics impacts associated with spare parts decisions and on Readiness-Based Sparing (RBS) optimization. The capabilities developed within ATLAST have been derived from analysis requirements gathered from recent initiatives in military transformation, recapitalization and Performance-Based Logistics (PBL). World-class business organizations have successfully used high-fidelity life-cycle SPAR™ models to reduce ownership cost and increase availability of capital assets such as fixed and rotary wing aircraft, tanks, radar, submarine combat systems, power plants, chemical plants, and gas exploration and production facilities. Knowing a system's life-cycle characteristics and future behavior in advance enables decision makers to assess the cost-effectiveness of utilization, logistic support and engineering improvements scenarios before they are implemented. However the ability to perform accurate what-if analysis to forecast costs and readiness in several instances is only a precursor to an effective decision support system. Initial provisioning of spares, spares replenishment and maintenance scheduling decisions are all characterized by an astronomically large set of possible scenarios, with each scenario yielding a different performance level. For these types of problems, quick and accurate decision support systems will include a systematic process of 'optimization' that reduces an astronomically large number of scenarios to a manageable set of the most-effective options to be considered for further analysis.

II. Life-cycle Analysis (LCA)

Life-cycle Analysis (LCA) is a formal process for establishing a quantitative basis in support of asset, or system management decisions [2-6]. LCA consists of: (i) building a model representation of a real world system or process, (ii) obtaining data to populate or instantiate the model, (iii) using the populated model for trade-off analysis by predicting future behavior - performance and costs - for a **set** of scenarios, (iv) validating the model predictions, and (v) presenting the analysis results to decision makers. LCA is used to support a range of system management decisions during all stages of a system life-cycle:

- During acquisition LCA is used in support of investment decisions. This includes identification of potential performance and cost weaknesses, assessment of alternative design options and evaluation of the cost and impact on system performance of alternative maintenance concepts.
- During deployment LCA is used in support of change management. This includes assessment of the effects of proposed engineering improvements on system performance and cost, changes in maintenance procedures and capacity and supply practices to reflect component and system aging and determination of spare pool implications for technology refresh.
- Finally, as an asset approaches its end of life LCA is used in support of transition management. This includes support investment allocation among the systems to be retired and their replacements, projected remaining life of end-of-life extensions and assessment of required support resources.

Knowing a system's life-cycle characteristics and future behavior in advance enables decision makers to assess the cost-effectiveness of utilization, logistic support and engineering improvements scenarios before they are implemented. With respect to weapon system inventories, maintaining affordable readiness while systems continue to age is becoming a growing problem. LCA provides program managers, item managers, and executive staff with rigorous quantitative support for strategic, tactical, and operational level decisions that previously had to be made based on crude approximations and intuition.

III. Readiness Based Sparing (RBS)

Readiness Based Sparing (RBS) optimization algorithm calculates the improvement in performance for each spare part being added to the field. Knowing the cost of each spare part type, the algorithm calculates the performance improvement per dollar spent, called the performance gradient. The performance gradient is computed for each additional spare part. The RBS algorithm ranks parts from the highest performance gradient to lowest. The RBS algorithm next adds the spare with the highest performance gradient and tallies up the expected performance improvement and budget spent, and then repeats the process until the desired performance level is achieved or the budget is exhausted. By selecting the most cost effective part in every iteration, the RBS algorithm ensures, in principle, that target performance levels are achieved with a minimal cost.

However, in practice traditional RBS optimizers produce grossly incorrect results. The problem lies in the use of simple analytic expressions or models to compute the performance gradient. These analytic models ignore key time-dependent factors that can

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have significant impact on spare part levels required to achieve a target performance level. Among the many important factors that analytic models ignore are the logical structure of the systems, components age and aging processes, removals stemming from life limits and maintenance protocols, parts replacement due to remaining life limits, build rules, *sunshine* repairs and variability in flow and lead times. In general, the analytic models assume systems with serial configuration, ageless systems with constant failure and removal rates, constant parts repair and condemnation rates in the depots and some of the real-life complexities e.g. redundancies, are accounted for by employing "correction" factors. These simplifications are made because the analytic models are unable to express these real life phenomena analytically.

IV. Hybrid Approach for Readiness Based Sparing Optimization

A hybrid approach, suggested by Dubi [11] is capable of providing quick and accurate decision support by combining the high fidelity of life-cycle (LC) simulation models together with the efficiency of analytic models. The purpose of the optimization modules is to eliminate the vast majority of possible scenarios, and to identify a few candidates for more rigorous evaluation by high-fidelity LC simulation models. The concept of Dubi's hybrid approach is illustrated in Figure 1.

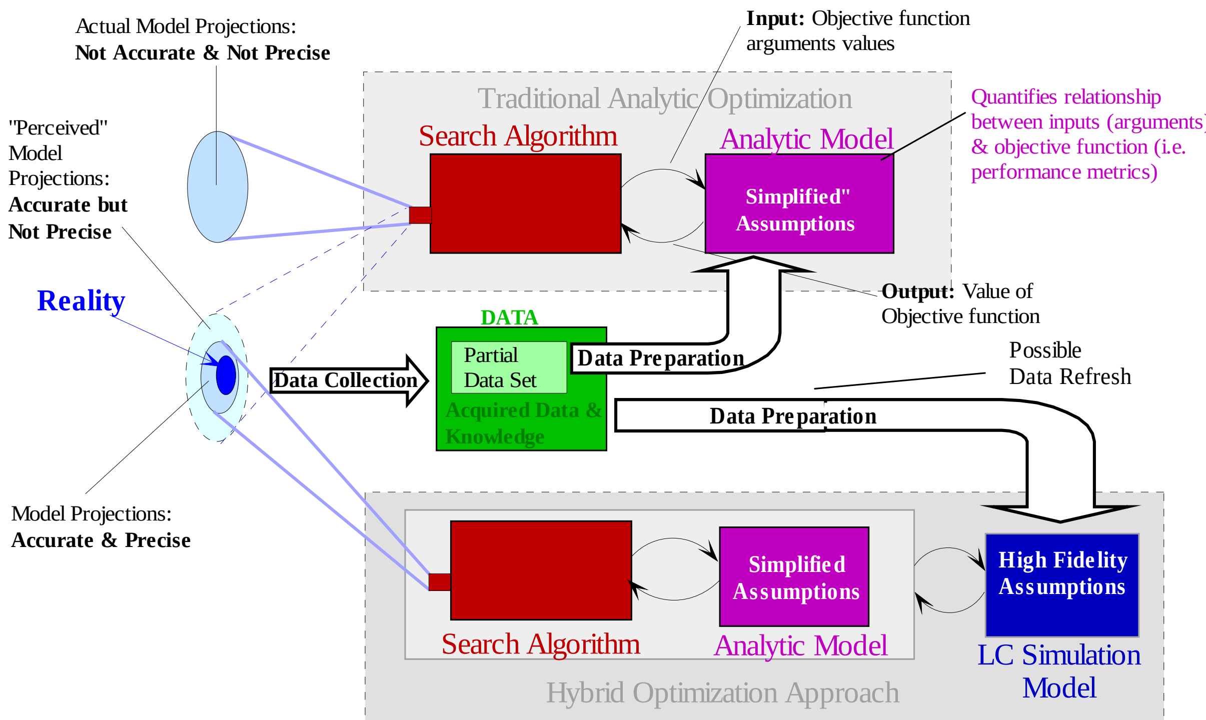

Figure 1: Traditional optimization versus the hybrid approach

In the hybrid approach for RBS optimization, the LC simulation models are used to generate accurate estimations of system performance, e.g. readiness and availability, to calculate various events-related metrics, e.g. time-dependent removal/condemnation rates and associated variance, and to identify and rank the culprit elements/components in the system that are responsible for the loss of performance for a given reference scenario in a 'criticality' table. The simulation model outputs are then fed as inputs into an analytic model incorporated into RBS optimization algorithm that produces a sparing policy in which target performance levels are achieved at minimal cost. Nonetheless, sparing policies optimization for complex systems in a single step may still produce erroneous results because simplified assumptions of the analytic model are too excessive compared to reality. Employing a stepwise optimization process will reduce this risk as shown in Figure 2. The analytic model will be used only to optimize limited performance targets. Once achieved, the LC simulation model will be used to generate new predictions of performance levels, the various event-related metrics and a new ranked criticality list that are then fed back into the analytic model. The 'adjusted' analytic model is then used until the next limited performance target is achieved. This process is repeated until either target performance levels are achieved or the given budget is exhausted.

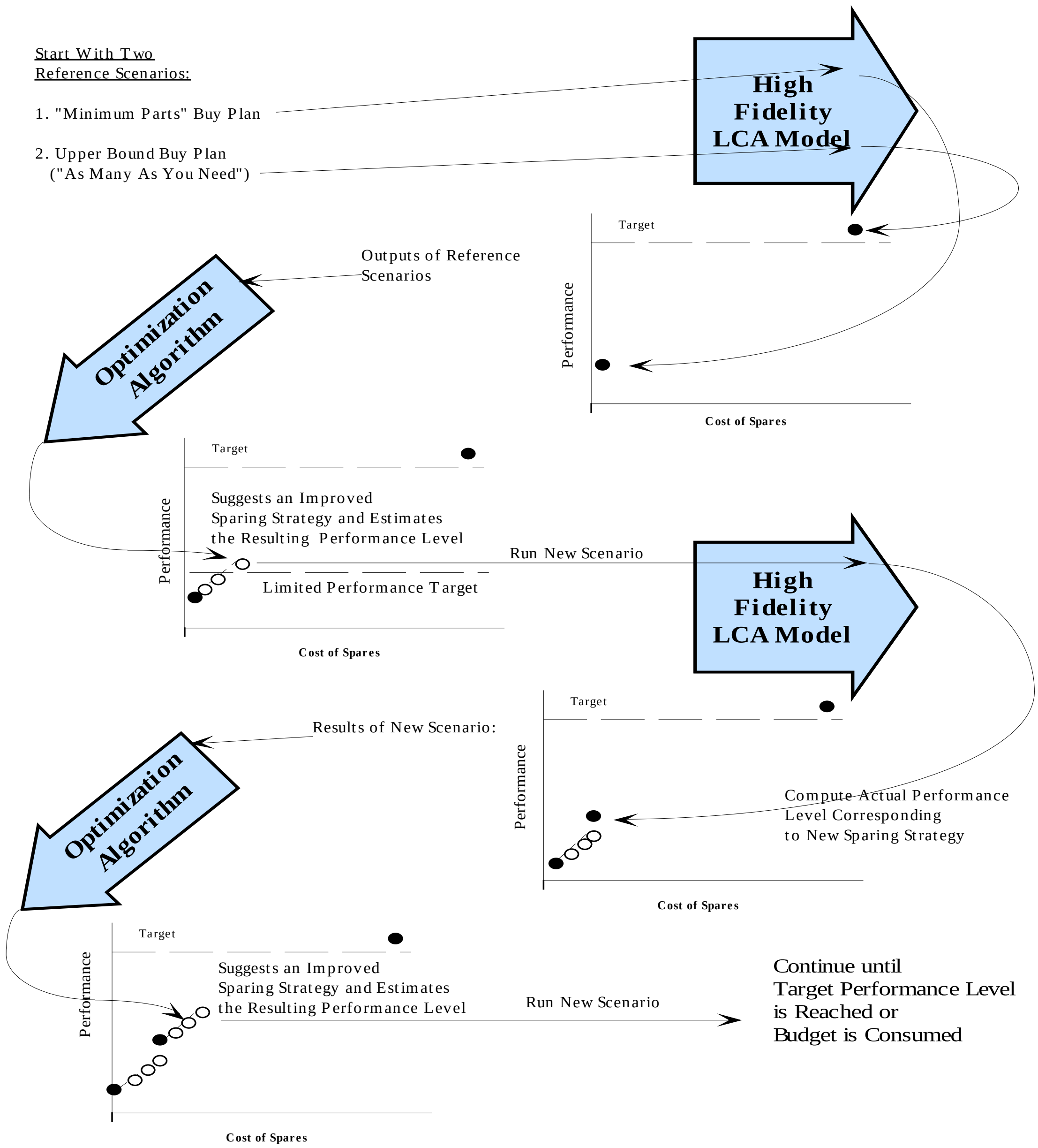

Figure 2: Stepwise optimization hybrid approach process

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V. ATLAST - Aircraft Total Life-cycle Assessment Software Tool

Clockwork Solutions Inc. has developed ATLAST *Aircraft Total Life-cycle Assessment Software Tool* [7]. ATLAST is specifically tailored for analyzing new and aging weapon system fleets. ATLAST is built on top of the SPAR simulation engine [8]. The SPAR simulation engine is a commercial grade software product that uses Monte Carlo techniques to simulate the life-cycle behavior of industrial systems. SPAR extends traditional Monte Carlo techniques [1-3] to handle such real world phenomena as uncertain and incomplete data, component aging and maintenance, spare parts, variable demands on

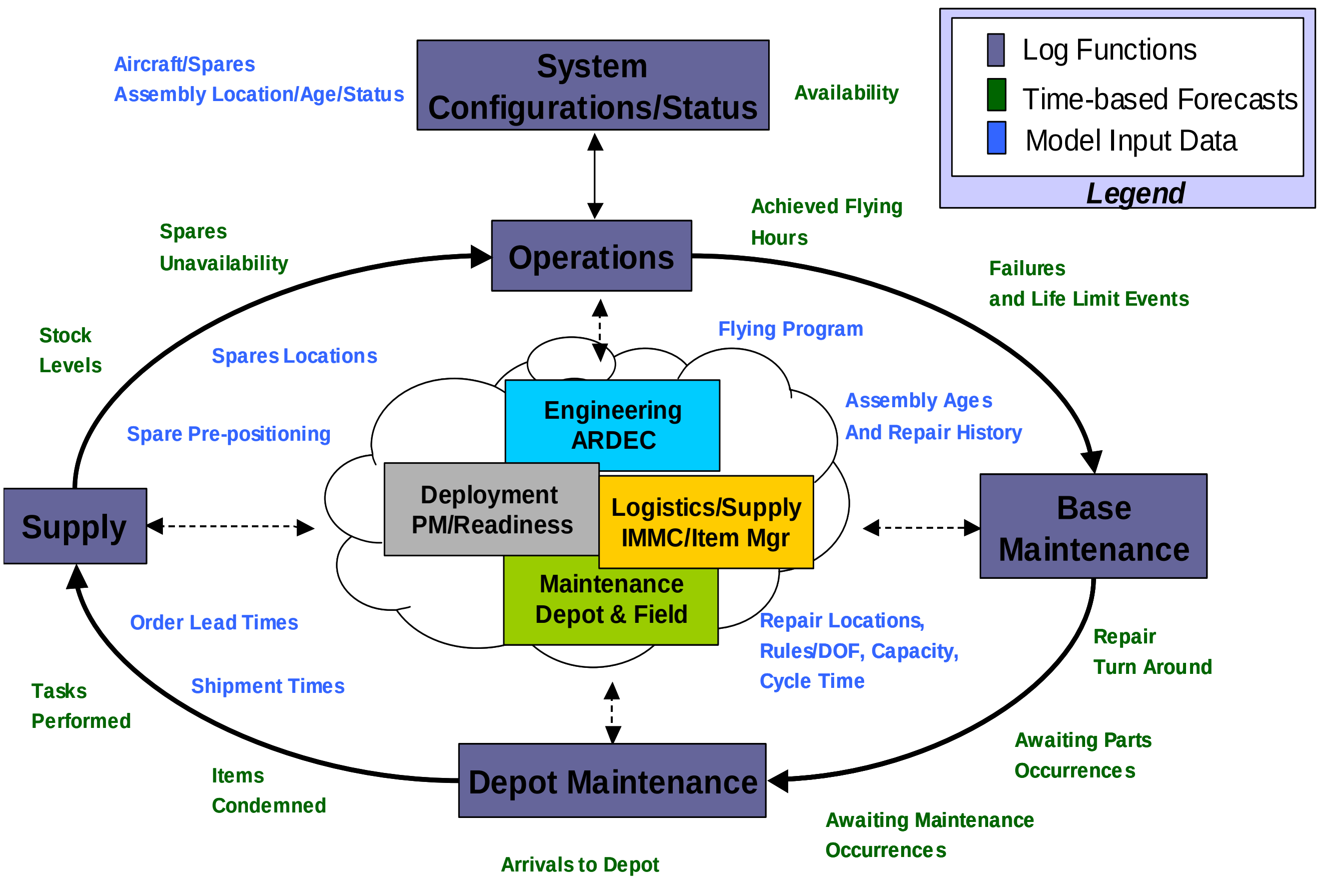

Figure 3: ATLAST high-level logic and functionality

the system, and component interactions. A high level illustration of ATLAST logic and functionality is shown in Figure 3. Much of the work involved in creating an aviation life-cycle model has already been designed into ATLAST (i.e. users must concern themselves only with changing inputs to the existing model, running life-cycle simulations, and evaluating life-cycle impacts). The life-cycle logistics impact forecasts of ATLAST are unique in that the model does the following:

- Captures the current state of a fleet of weapons systems as they are in the field today.
- Operates them according to operations profiles for the base of which they are located
- Forces failure and life-limited events according to location and age of components
- Administers a capacity constrained maintenance and logistics support process necessary to retrieve unserviceable assets back to operational conditions and states.

ATLAST is 'loaded' with an *initialized* fleet of equipment (weapons systems). Initialization includes the configuration of a serialized system from its main serialized Line Replaceable Units (LRU's), through it's modules, and down through its sub-parts and components. The age, location, and status (serviceable and installed, unserviceable, serviceable spare, in repair, etc) of every component is set to the most current representation possible. Each serialized system is then placed at a location where it is operated according to an operations profile dictated by that location. Once in operation, the weapon system accumulates hours against it parts and components, and stops operating upon a failure event to one of its systems or sub-systems, or due to a component life-limit that has been reached. When a system is down for repair, the model then reviews all other life-limited parts in the system to see if they are within remaining life-limits thresholds, and if they are, takes the opportunity to perform necessary repair tasks associated with that item. A typical ATLAST data collection process is illustrated in Figure 4 (for *T700 Family of Engines Logistics Requirements Forecasting Model* project).

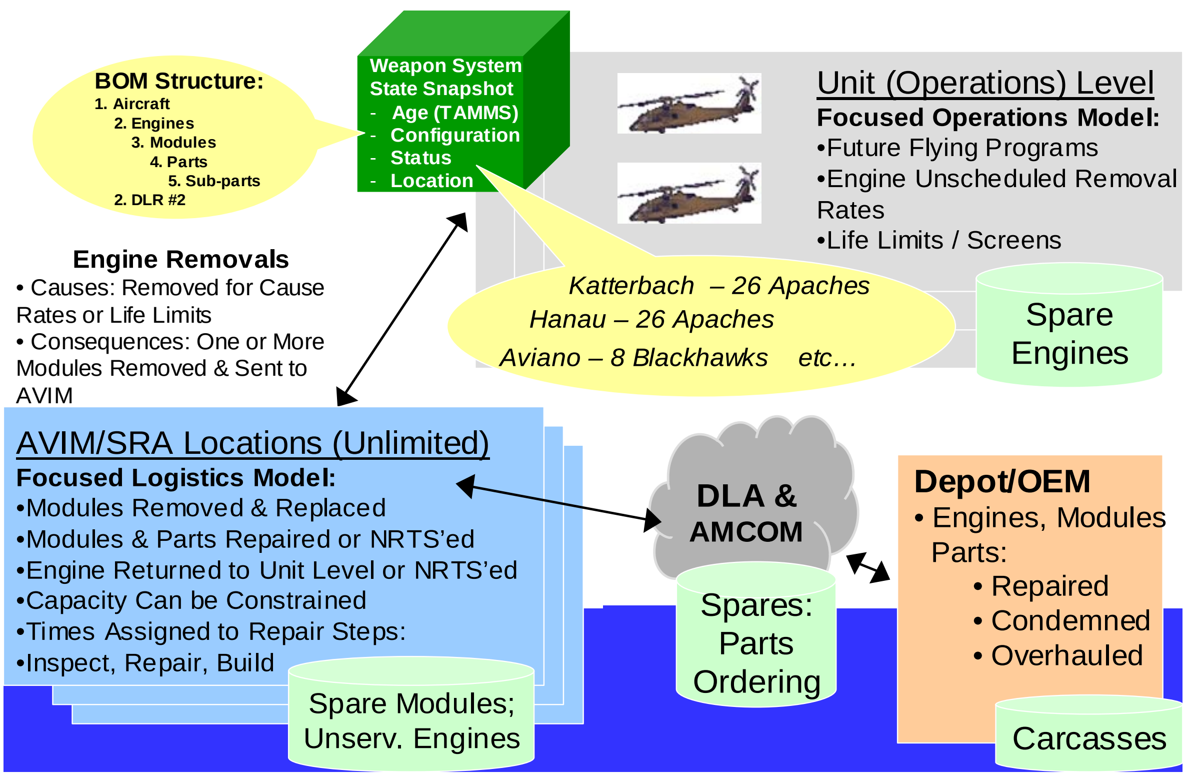

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Figure 4: High-level ATLAST data collection process

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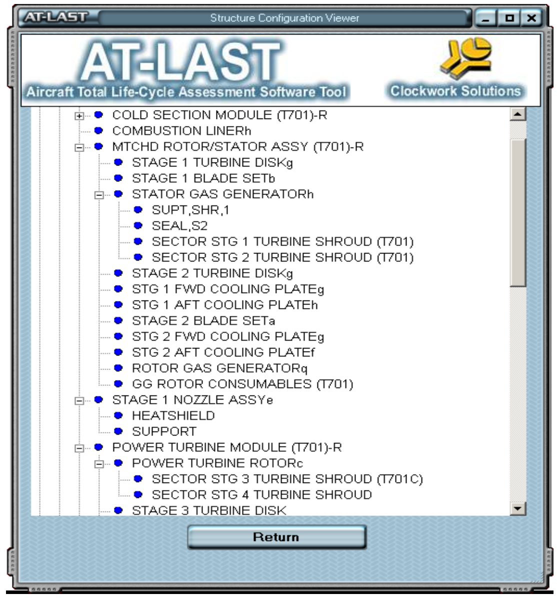
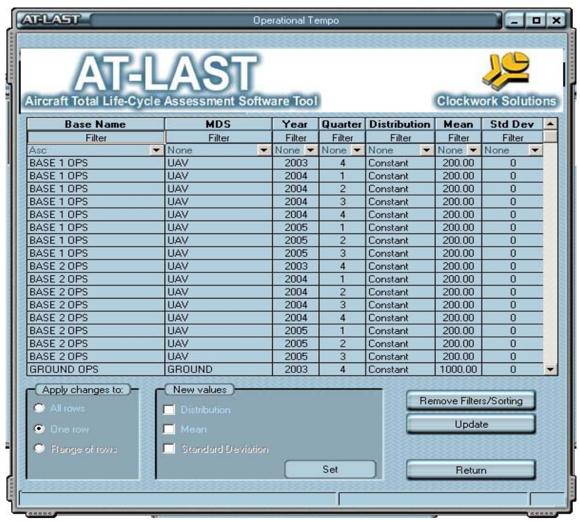
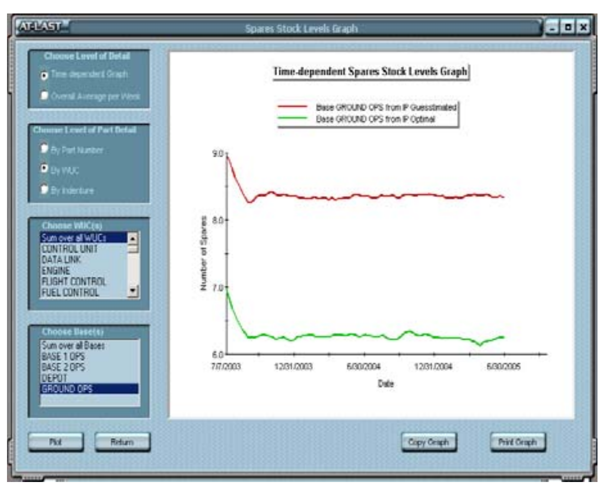
ATLAST uses the following types of input data:



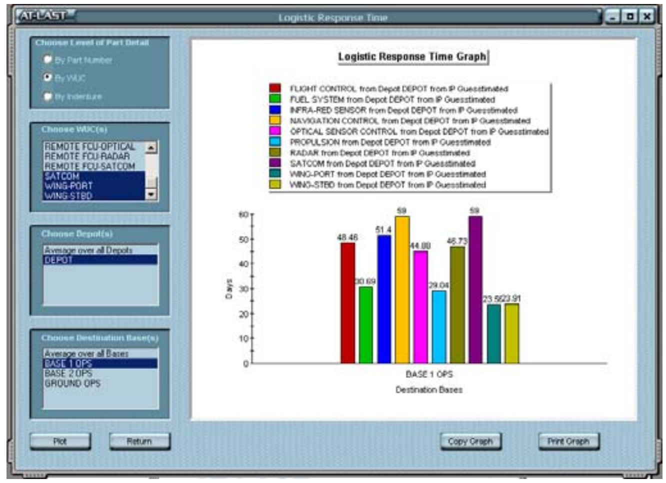
- System Structure Data – Breakdown structure (bill of materials), build rules of assemblies.
- Logistic Echelon Structure – Operation bases, intermediate support bases, depots, alternative support bases.
- Component Data – Part types, i.e. upgrades and versions, usage-tracking methods, life limits and overhaul intervals by part type and serial numbers.
- Program Data– Flying hour programs and stress profiles by base and tail numbers.
- Supply and Logistic Data – current inventories, supply lead times, shipment times, Not-Reparable at this Station (NRTS) rates/probabilities, part costs.



- Failure Data –failure distributions by part types, location and maintenance history.
- Maintenance Data – Maintenance tasks and time duration, resources and capacities, logistic consequences, i.e. condemnations, repairs with and without part replacements or no-fault-found rates/probabilities.
- Shop Rules – criteria for opportunistic replacements, repair policies, i.e. overhaul to like new, minimal, or upgrades/ recapitalization.
- Depot Induction Schedules



ATLAST uses the Monte Carlo simulation method to “fly” the aircraft according to the flying hour programs per base. The model creates LRU removal events, places demands for spares, implements appropriate procedures to repair the LRU and its constituent assemblies and parts, condemn parts, consume parts, and incur costs. The model repeats this process many times to create a statistically meaningful database of events, their consequent demands on the repair and supply systems, and their impact on aircraft availability. At the completion of its execution, the model uses the data collected in order to produce several outputs that include:



- Operational availability
- Planned and unplanned removals
- Achieved operating hours or Time On Wing (TOW)
- Repairs and condemnations at base and depot levels
- Spare parts unavailability, waiting times and stock levels
- Logistic delays – Awaiting resources, awaiting parts
- Responses periods - depot logistic response, depot flow times
- Requirements for spare parts
- Life-cycle Costs (LCC)

The questions that are supported by ATLAST outputs and capabilities include but are not limited to, the following:

- Will the fleet, or assets at some operating location, achieve required flying hour programs?
- Will the buy plan be suitable to maintain expected and target availability?
- What will the parts requirements be?
- How do improvements in repair capacity impact repair turn around time and time on wing?
- Where will the repair and supply bottlenecks be?
- What can I expect to have in the repair pipeline due to removals for cause and life-limited parts?
- What volume of part condemnations will occur and where?
- Will a repair location be able to keep up with the demands anticipated?
- What percent of time is repair held up due to awaiting parts or awaiting maintenance conditions?
- What performance gain (fleet availability, time on wing, repair turn around time) is obtained through selection of an alternate part type, with respect to part and vendor attributes such as order lead-time, ship time, purchase cost, and reliability?
- If fatigue-testing results in modified life limits on certain parts, how will that change affect maintenance and supply volume?
- I have a limited budget, how should/could I spend it across supply and maintenance functions to maximize fleet availability?

ATLAST has evolved over the past three years through development phases supporting both the US Air Force and US Army. The extent of the logic included in the simulation capability and the weapon systems that have been specifically included has increased with each project supported by Clockwork Solutions. The projects supported to date that have contributed to the advancement of ATLAST technology include the following:

RAMSS (Requirements, Analysis, and Management Support System: A production requirements forecast system for the GE F100 family of engines for use at the Oklahoma City Air Logistics Center (OC-ALC). This system addresses a critical United States Air Force (USAF) need — accurate production requirement forecasts. D041, the USAF's current system, uses historical supply system data to project a future profile of demands, repairs, condemnations, and buys per National Stock Number (NSN). Because D041 was designed to handle all of the USAF's weapon systems—one size fits all—it is forced to use generic, system-independent forecasting methods that ignore the singular aspects of individual weapon systems. Initial airframe platforms supported in the model include the F16, B1B, B2, KC135, and five configurations of the F100 family. Historical maintenance and airframe performance data was accessed and collected from the Comprehensive Engine Management System (CEMS).

T55 Turbine Engine Maintenance Workload Simulation Model: The T55 turbine engine is used to power the Army's twin engine CH-47 Chinook helicopter. The US Army Logistics Integration Agency (LIA) contracted with Clockwork Solutions to create and utilize predictive models for analyses of life-cycle cost issues related to T55 engine reliability and logistics. A second contract involved a data gathering and assessment project to provide the background data necessary to build and sustain predictive simulation models. The work was conducted in coordination with the US Army Aviation and Missiles Command (AMCOM), PM Cargo Helicopters, and the Integrated Material Management Center (IMMC).

T700 Family of Engines Logistics Requirements Forecasting Model The T700 family of turbine engines is used to power the US Army's fleet of AH-64 Apache and UH-60 Blackhawk helicopters. The US Army LIA contracted with Clockwork to create and utilize predictive models for analyses of life-cycle costs related to T700 engine reliability and logistics. The work was conducted in coordination with AMCOM, Utility Helicopter Program Management Office (UHPMO), and IMMC. The primary army data sources used to construct and refresh the living aviation model included the DA-Form 2410 Maintenance Management system, OSMIS, NSN MDR, and HAS. The model includes over 1400 airframe platforms, UH60 and AH64, and a total of over 5500 total engines. They system manages the flight time, repair activities, shipping, sparing etc. for nearly 300,000 serialized parts/components in each life-cycle simulation.

UH60 A to A Recapitalization Impact to Sustainment Simulation Model: As a extension to the T700 model, the Utility Helicopter Program Management Office at AMCOM contracted Clockwork to develop an airframe model, consisting of 31 total LRUs and their modules and sub-parts. These LRUs are primarily those that are tracked in the Army's Maintenance Management System (TAMMS). The model has been developed to support its expansion across additional LRU's on the aircraft, and across greater number of aircraft platforms and configurations.

VI. ATLAST Deployment Push Pack Spares Optimization Module

ATLAST version 4.01 consists of the *Deployment Push Pack Spares Optimization Module*'. The module utilizes Dubi's hybrid approach to optimize 'push pack' spares that accompany aircraft when deployed to new bases or locations. The module was designed to use the most up-to-date information on the state of the aircraft that are to be deployed in a new location. For this purpose in the first step, the module generates a *deployment scenario* from ATLAST *master data model* file. Prior to the use of the optimization module a *master data model* that represents the state of the system must be already established. Clockwork's services include the establishment of master data models, including obtaining, cleaning, and preparing the data in appropriate formats. It is expected that ATLAST users will concern themselves only with changing inputs to the existing master data model, running life-cycle simulations, and evaluating life-cycle impacts. A myriad of *Scenario Editors* aimed to assist users in preparation of necessary scenarios for 'What-if' analysis plans.

Deployment Wizard: A wizard is used to create a deployment scenario from a master data model. The user:

- Selects aircraft tail numbers to be deployed
- Selects deployment base or location
- Assigns deployment duration
- Establishes profiles of operation of deployed aircraft. A profile of operation is defined by amounts of scheduled flying hours aircraft are planned to fly in each quarter during the deployment duration.
- Defines logistic delay period by Work Unit Code (WUC) of Line Replaceable Units (LRU). The logistic delay period is defined as expected time elapsed from a request to ship a spare part until part actually arrives to the deployment base or location.
- Defines number of histories i.e. repetitions in a simulation run.

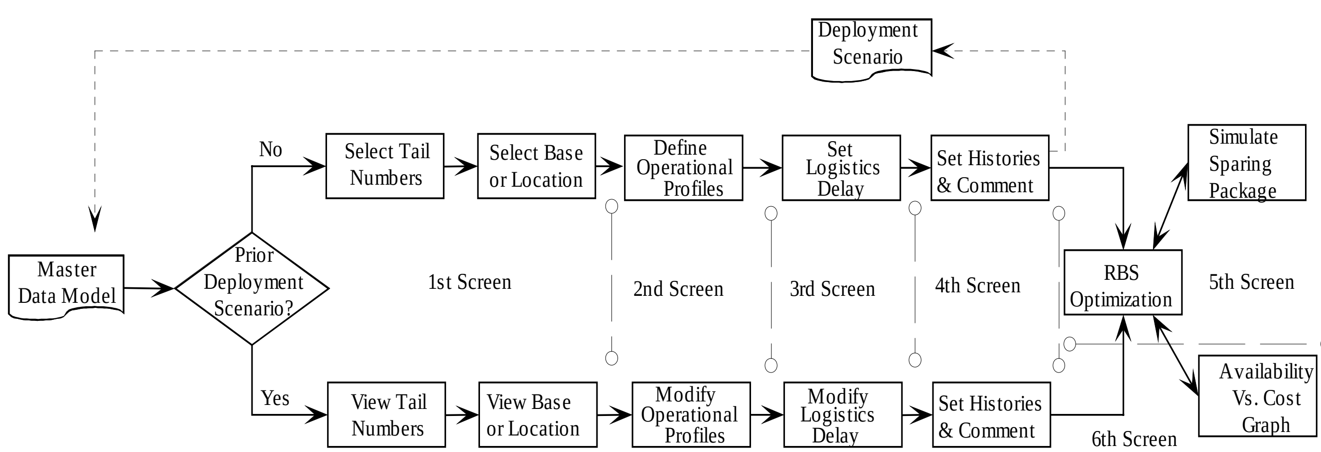


Figure 5: Deployment scenario optimization flow process

Users are capable of modifying additional elements in deployment scenarios, e.g. failure distributions or costs, by using the standard ATLAST scenario editors.

The optimization module also allows modifications of deployment scenarios so users may analyze push pack spares policies in various conditions. When the optimization module recognizes a master data model as a prior deployment scenario, it will allow users

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either to generate from it a 'sub' deployment scenario (deployment scenario is now defined as a master data model) or only to modify deployment duration, operation profiles, logistics delay period and number of histories. The creation process of a deployment scenario is illustrated in Figure 5. Associated snapshots of the deployment wizard screens are illustrated in Figures 6-9.

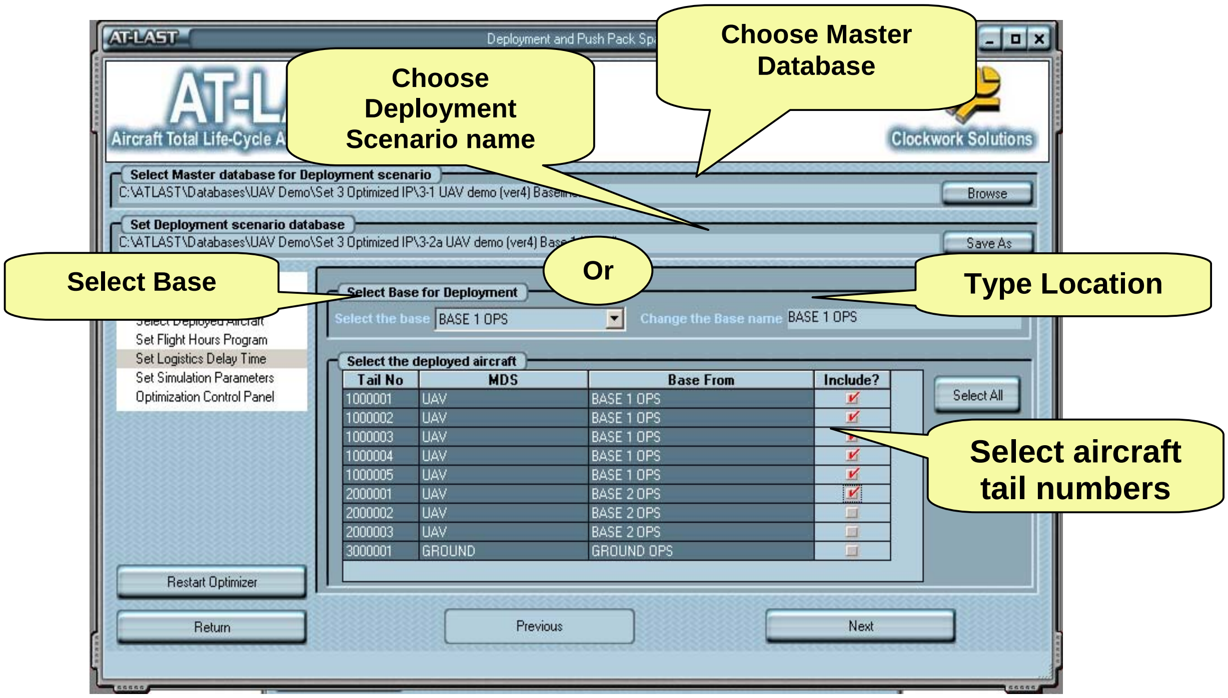

Figure 6: Open/Save scenario files & Select aircraft tail numbers screen

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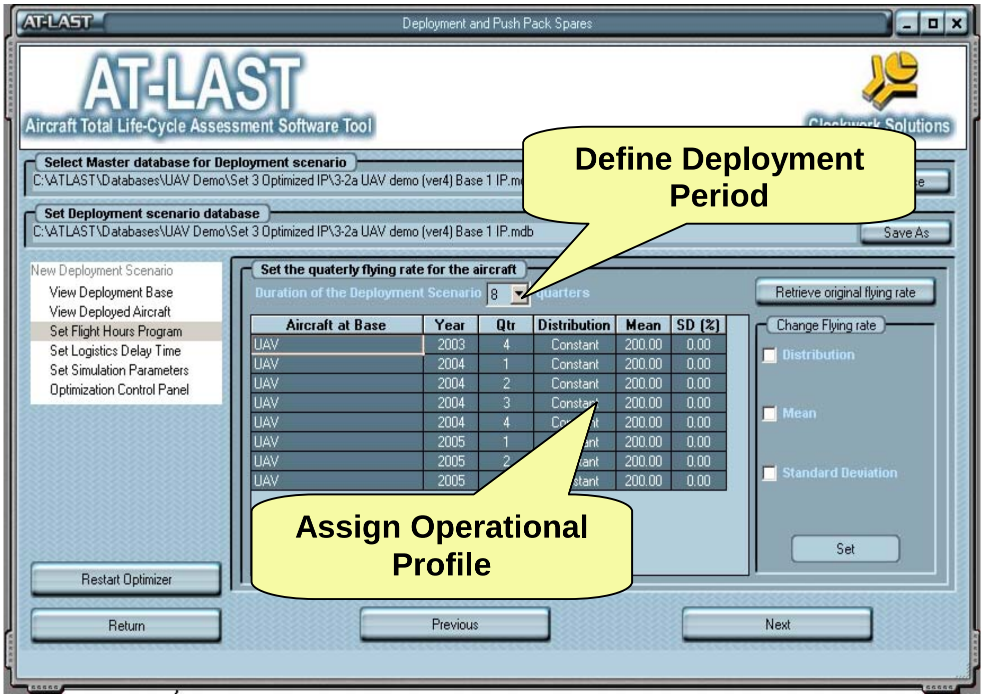


Figure 7: Define deployment period & Assign operational profile screen

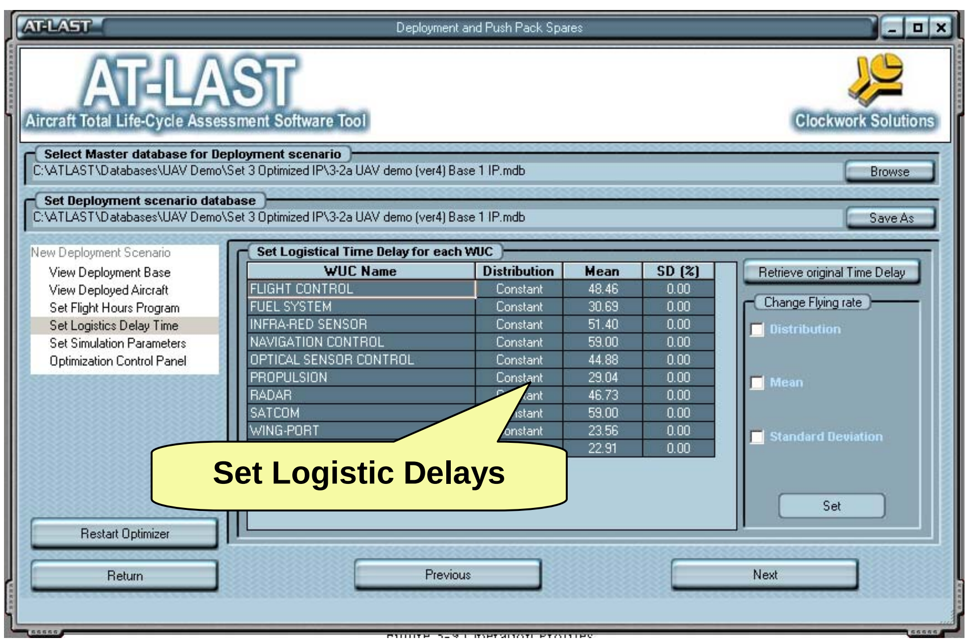
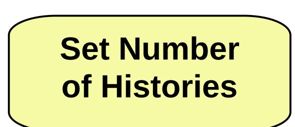


Figure 8: Set logistic delays screen



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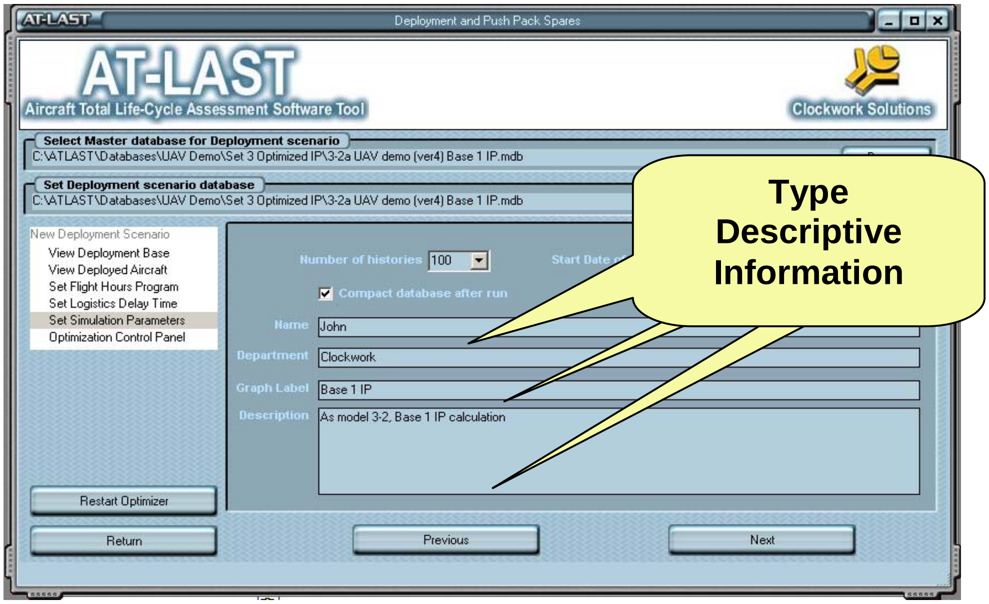

Figure 9: Set number of histories & Type descriptive information screen

Once the user enters the data, ‘reference scenario’ is simulated (details on ‘reference scenario’ are provided in following section). The outputs from the ‘reference scenario’ are fed to an RBS optimization algorithm to serve as inputs for an analytical model (analytic model details described in next section). The RBS algorithm generates two tables of spare parts ranked by either by ‘effectiveness’ or by ‘cost-effectiveness’ along with unavailability estimates which can be represented in graphical form, i.e. ‘availability vs. cost graphs’ as shown in Figures 10-11. The module also allows users to verify availability estimates by simulating models with corresponding push-pack spares packages. Verification of expected availability is a recommended practice because availability estimates with specific sparing policies in complex systems based on single ‘reference’ simulation scenario may not be sufficiently accurate.

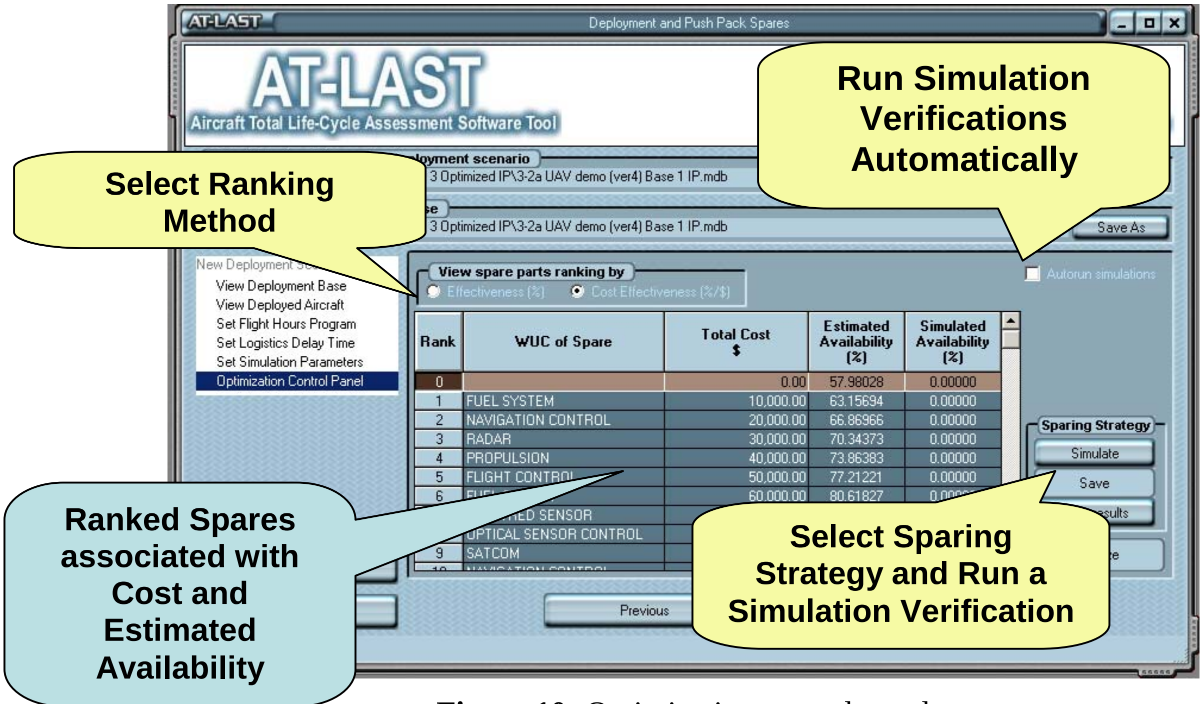

Figure 10: Optimization control panel

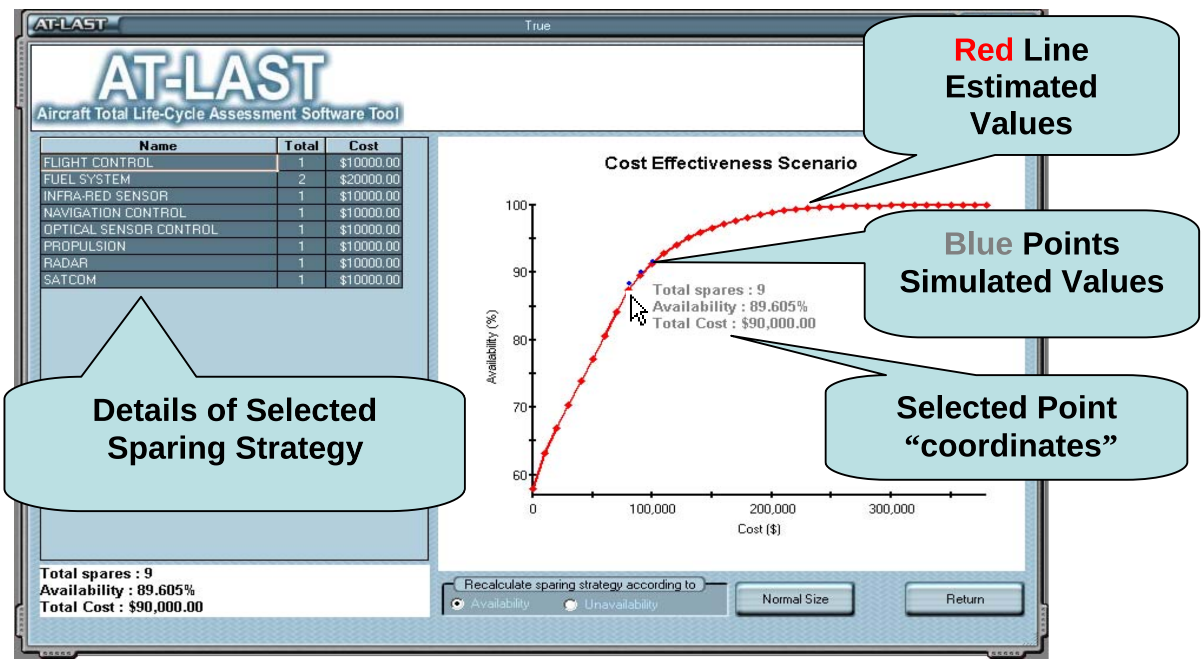

Figure 11: Availability versus Cost graphical display screen

Analytic Model: The analytic model is divided into two steps. In the first step, the model calculates the expected waiting time for spares in a given sparing policy, and in the second step, these waiting times are used to estimate aircraft availability. The model assumes that each deployed aircraft is independent of the others, i.e. lost flight hours by grounded aircraft are not being compensated by increasing the flight load on other

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aircraft. The model also assumes that LRUs are operationally independent, i.e. failure in one LRU does not induce a failure in any other LRU). Consequently LRU's of the same type are modeled independently in the first step, i.e. in derivation of the mathematical expressions for expected waiting times.

Let us assume that the model consists of n deployed aircraft and each aircraft consists of m LRU's of the same type. We shall assume that the LRU's fail at a constant effective failure rate, λ, and a constant logistic delay rate μ. The system can be modeled as a Markov chain with a finite set of states as shown in Figure 12.

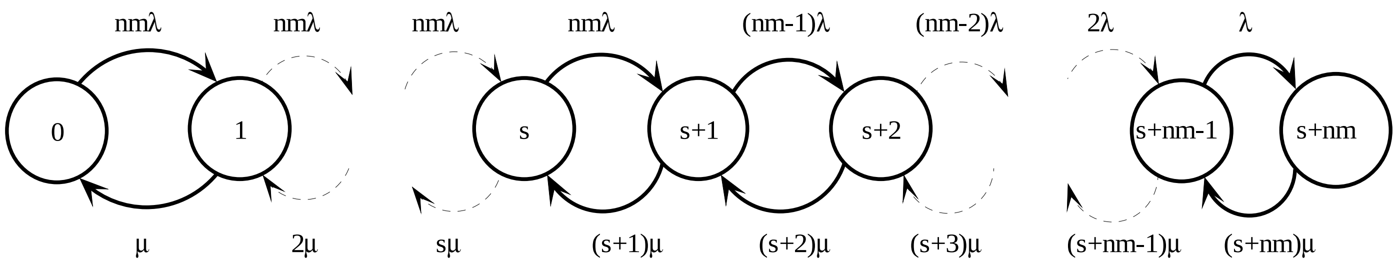

Figure 12: Finite Markov Chain

Where:
n = number of deployed aircraft
m = number of installed LRUs of the same type in each aircraft
s = number of spares in stock
λ = effective failure rate
μ = logistics delay rate

The set of steady state linear equations for a Markov chain depicted in Figure 12 takes the form:

$$
\begin{aligned}
&0 = \mu P_1 - nm\lambda P_0 \\
&\vdots \\
&0 = nm\lambda P_s + (s+2)\mu P_{s+2} - (nm-1)\lambda P_{s+1} - (s+1)\mu P_{s+1} \\
&0 = nm\lambda P_{s-1} + (s+1)\mu P_{s+1} - nm\lambda P_s - s\mu P_s \\
&\vdots \\
&0 = \lambda P_{s+nm-1} - (s+nm)\mu P_{s+nm}
\end{aligned}
$$

Equation 1: Steady State Markov Linear Equations

With the boundary condition: $\sum_{i=0}^{s+nm} P_i = 1$, the set solution takes the form:

$$
P_i = \begin{cases} \dfrac{nm^i}{i!}\left(\dfrac{\lambda}{\mu}\right)^i P_0 & \text{if } 1 \le i \le s \\[2ex] \dfrac{nm!s!}{(nm+s-i)!i!}\left(\dfrac{\lambda}{\mu}\right)^{i-s}\dfrac{nm^s}{s!}\left(\dfrac{\lambda}{\mu}\right)^s P_0 & \text{if } s \le i \le s+nm \end{cases}
$$

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$$P_o = \left[1 + \sum_{i=1}^{s} \frac{nm^i}{i!}\left(\frac{\lambda}{\mu}\right)^i + \frac{nm^s}{s!}\left(\frac{\lambda}{\mu}\right)^s \sum_{i=s+1}^{s+nm} \frac{nm!s!}{(nm+s-i)!i!}\left(\frac{\lambda}{\mu}\right)^{i-s}\right]^{-1}$$

Equation 2: Steady State Probabilities

The expected waiting time as a function of the number of spares, W(s), can be derived using Little's theorem:

$$< \lambda(s) > \cdot W(s) = H(s) \quad \Rightarrow \quad W(s) = \frac{H(s)}{< \lambda(s) >}$$

Equation 3: Little's Theorem

For the Markov chain depicted in Figure 12, W(s) takes the form:

$$W(s) = \frac{H(s)}{< \lambda(s) >} = \frac{\frac{nm^s}{s!}(\lambda T)^s \cdot \sum_{i=s+1}^{s+nm}(i-s) \cdot \frac{nm!s!}{(nm+s-i)!i!}(\lambda T)^{i-s}}{\lambda\left[\sum_{i=0}^{s} \frac{nm^{i+1}}{i!}(\lambda T)^i + \frac{nm^s}{s!}(\lambda T)^s \cdot \sum_{i=s+1}^{s+nm-1} \frac{nm!s!}{(nm+s-i-1)!i!}(\lambda T)^{i-s}\right]}$$

$W(0) = T$

where:

$T = \frac{1}{\mu}$ is the expected logistic delay period.

Equation 4: Expected waiting time as a function of number of spares

It is interesting to note that analytic RBS software tools that are based on the Poisson distribution in fact assume an infinite Markov chain as shown in Figure 13.

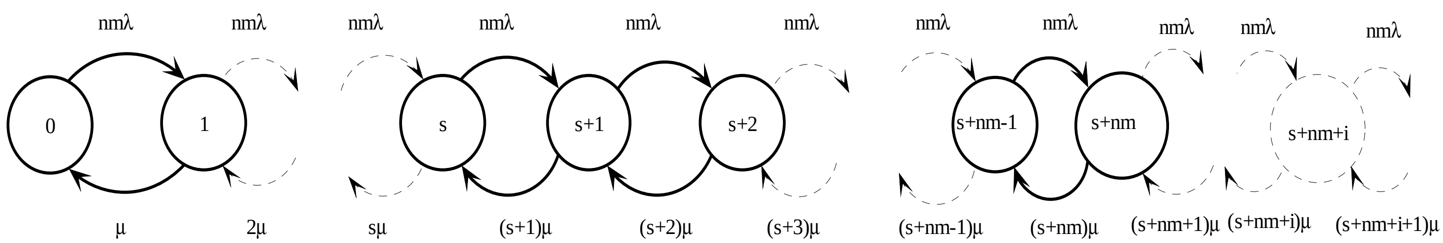

Figure 13: Infinite Markov chain

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with a infinite set of steady state linear equations:

$$
\begin{aligned}
&0 = \mu P_1 - nm\lambda P_0 \\
&\vdots \\
&0 = nm\lambda P_{s-1} + (s+1)\mu P_{s+1} - nm\lambda P_s - s\mu P_s \\
&\vdots \\
&0 = nm\lambda P_{s+nm-1} + (s+nm+1)\mu P_{s+nm+1} - nm\lambda P_{s+nm} - (s+nm)\mu P_{s+nm} \\
&\vdots
\end{aligned}
$$

Equation 5: Steady state equations of infinite state Markov chain

With the boundary condition of $\sum_{i=0}^{\infty} P_i = 1$ the state probabilities of takes form:

$$P_i = \left(\frac{nm\lambda}{\mu}\right)^i \frac{1}{i!} P_0 \quad ; \; P_0 = e^{-\frac{\lambda}{\mu}}$$

Equation 6: Steady state equations of infinite Markov chain

The state probabilities (of Eq. 6) corresponds to a Poisson distribution and the expected waiting time in this case is similarly obtained by using Little's Theorem:

$$W(s) = \frac{H(s)}{<\lambda(s)>} = \frac{\sum_{i=s+1}^{\infty} (i-s) \cdot \frac{(nm\lambda T)^i e^{-nm\lambda T}}{i!}}{nm\lambda}; \quad W(0) = T \quad \left[= \frac{1}{\mu}\right]$$

Equation 7: Expected waiting times of an infinite Markov chain

The fact that there are a finite number of LRU's in the field (installed or spares) suggests that expressions derived for a finite Markov chain (Eq. 4) are more appropriate than the expressions of a infinite Markov chain (Eq. 7) especially in cases in which the analysis involved a very small fleet of deployed aircraft.

The expected waiting time expressions (Eq. 4) are used to estimate the steady-state system availability as a function of sparing policies, $A_{sys}(s_1,...,s_L)$. In developing an availability expression, a serial configuration of the system is assumed, i.e. aircraft fails upon failure of any of its LRUs, and that operational LRU's do not fail as long as the aircraft is grounded or in a "down" state i.e. LRUs are in passive state with zero failure rates. With these two assumptions, the steady state availability can be derived as follows:

$$A_{sys}(s_1,...,s_L) = \frac{MTTF_{sys}}{MTTF_{sys} + MDT_{sys}} = \frac{\dfrac{1}{\sum_{j=1}^{L} \dfrac{m_j}{MTTF_j}}}{\dfrac{1}{\sum_{j=1}^{L} \dfrac{m_j}{MTTF_j}} + \sum_{j=1}^{L} P_j \cdot [MTTR_j + W(s_j)]}$$

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$$= \frac{1}{1+\left[\sum_{j=1}^{L} m_j \cdot \lambda_j\right] \cdot \left[\sum_{j=1}^{L} P_j \cdot [MTTR_j + W(s_j)]\right]} =$$

$$= \frac{1}{1+\left[\sum_{j=1}^{L} m_j \cdot \lambda_j\right] \cdot \left[\sum_{j=1}^{L} \left[\frac{m_j \cdot \lambda_j}{\left[\sum_{j=1}^{L} m_j \cdot \lambda_j\right]}\right] \cdot [MTTR_j + W(s_j)]\right]}$$

and thus takes the form:

$$A_{sys}(s_1,...,s_L) = \frac{1}{1+\cdot\left[\sum_{j=1}^{L} m_j \cdot \lambda_j \cdot [MTTR_j + W(s_j)]\right]}$$

Equation 8: Steady state availability as a function of sparing policies

Where:

m_j = number of LRU of type j in a single aircraft

L= number of types of LRU's composing an aircraft

$MTTF_j$ = Mean time to failure of an LRU of type j.

$MTTR_j$ = Mean time to replace an LRU of type j in an aircraft.

s_j = Number of provisioned LRU spares of type j.

$MTTF_{sys}$ = Mean time to aircraft failure

MDT_{sys} = Mean down time of an aircraft (when aircraft fails).

$\lambda_j \left[= \frac{1}{MTTF_j} \right]$ = Internal failure rate of an LRU of type j.

$P_j \left[= \frac{m_j \cdot \lambda_j}{\sum_{j=1}^{L} m_j \cdot \lambda_j} \right]$ = Probability that an aircraft fails because of a failure of an LRU of type j.

$W(s_j)$ = Expected waiting time for spare LRU of type j when s_j spares are provisioned to the deployment base.

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By substituting Eq. 4 for every LRU type in Eq. 8, an expression is obtained that quantifies aircraft availability as a function of a sparing strategy. [It should be noted that the effective failure rate (denoted by λ) in Eq. 4 is equivalent to $(MTTF + MTTR)^{-1}$ while the internal failure rate (also denoted by λ) equals to $MTTF^{-1}$]. This expression is essentially the analytic model that is incorporated in the ATLAST deployment push pack spares optimization module. The coefficients of the expressions, e.g. internal failures are calculated by post-processing the outputs of a reference scenario simulation run as explained in the following section.

Reference Scenario: The reference scenario is initialized with the 'current' state of the deployed aircraft as declared in the master data model, i.e. with installed parts that have accumulating age and have a maintenance history. These attributes affect:

- Time to next scheduled removal either because of reaching a life limit or due to limited allowed periods between overhaul or inspections,
- The unscheduled removal distributions of LRUs because these distributions are a function maintenance history (i.e. number of overhauls) of installed LRU's.
- The instantaneous failure rates or hazard functions of the installed LRU's that are affected by the 'cumulative damage' that installed LRU's have incurred due to prior usage [9].

Table 1: Coefficients used in analytic model

Item No.	Symbol	Description	Input/Output in Reference Scenario	Formula/Explanation
1	N	Number of deployed aircraft	Input	
2	m_j	Number of installed LRU's of type j per aircraft	Input	
3	$MTTR_j$	Mean time to replace LRU type j in a failed aircraft	Input with processing	Mean (first moment) calculated from distribution parameters according to known formulas
4	T_j	Mean time	Input with processing	Mean (first moment) calculated from distribution parameters according to known formulas
5	λ_j^e	Effective failure rate of LRU of type j. [denoted by λ in Eq. 2]	Output with processing	$\lambda_j^e = \left(\frac{1}{\lambda_j} + MTTR_j\right)^{-1}$
6	λ_j	Internal failure rate of LRU of type j [Eq. 4]	Output with processing	$\lambda_j = \frac{F_j}{A_\infty \cdot T_{max}}$
7	T_{max}	Deployment period	Output	Used for calculating λ_j
8	A_∞	Average (upper bound) availability during deployment period	Output	Used for calculating λ_j
9	F_j	Expected number of LRU removals of type j during deployment period	Output	Used for calculating λ_j. **Note:** F_j include all removals

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				regardless of cause.

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The deployed aircraft are flown in the deployment base according to operational profiles defined by the user. Aircraft fail or being grounded because of unscheduled events and because of a need to remove parts because they have reached a limit – end of life limit or overhaul/inspection limit. When an LRU is removed it is shipped to higher echelon and new LRU is immediately send back to deployment base. The reference scenario represents the upper bound of aircraft availability as no waiting time is ever encountered.

A list of coefficients that are obtained from the reference scenario file from either model inputs or outputs and used in Eq. 4 is provided in table 1.

VII. Numerical Examples

Analytic Model Validation: For the purpose of validating availability estimates generated by the analytic model, a simplified test case was developed with identical assumptions to the analytic model. The deployment scenario test case consists of 5 deployed aircraft for a period of 2 years. Each aircraft consists of 146 LRUs and all together 92 LRU types. All LRUs have the same exponential failure distribution regardless of maintenance history. LRU usage in not limited (no end-of-life limits) and the LRU are not preventively maintained or proactively inspected. All aircrafts fly with the same operational profile – 2000 flying hours per year. Logistic delay is set 4 days exactly. Aircraft are of serial configuration and LRUs do not fail when system is down.

The availability vs. cost graph generated by the deployment push pack optimization module is shown Figure 14 as connected red points. Some of the predicted values were verified through simulation runs (for every 5 additional spares). These simulated values are shown in Figure 14 as green (unconnected) points. As seen the match between predicted and simulated values for the test case is excellent. The level of match can be quantified through two parameters. The first is “average relative difference” defined as:

$$\begin{bmatrix} \text{Average} \\ \text{Relative} \\ \text{Difference} \end{bmatrix} = \frac{1}{N} \cdot \frac{1}{\left[A_{UB} - A_{LB} \right]} \cdot \sum_{i=1}^{N} \left| A_{\text{simulated},i} - A_{\text{predicted},i} \right|$$

Equation 9: Average relative difference definition

where:
N = Number of comparison runs
$A_{simulated,i}$ = Average availability in simulated run i.
$A_{predicted,i}$ = Predicted average availability in case I by analytic model.
A_{UB} = Upper bound availability (in reference scenario)
A_{LB} = Lower bound availability in a simulated run with no spares.

The second is the maximum relative difference encountered in the comparison study, i.e.:

$$\begin{bmatrix} \text{Maximum} \\ \text{Relative} \\ \text{Difference} \end{bmatrix} = \max \left\{ \frac{\left| A_{\text{simulated},i} - A_{\text{predicted},i} \right|}{\left[A_{UB} - A_{LB} \right]} \,\middle|\, i = 1,2,\ldots N \right\}$$

Equation 10: Maximum relative difference definition

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In the validation case the average relative difference amounted to 1.05% and the maximum relative difference to only 3.34%.

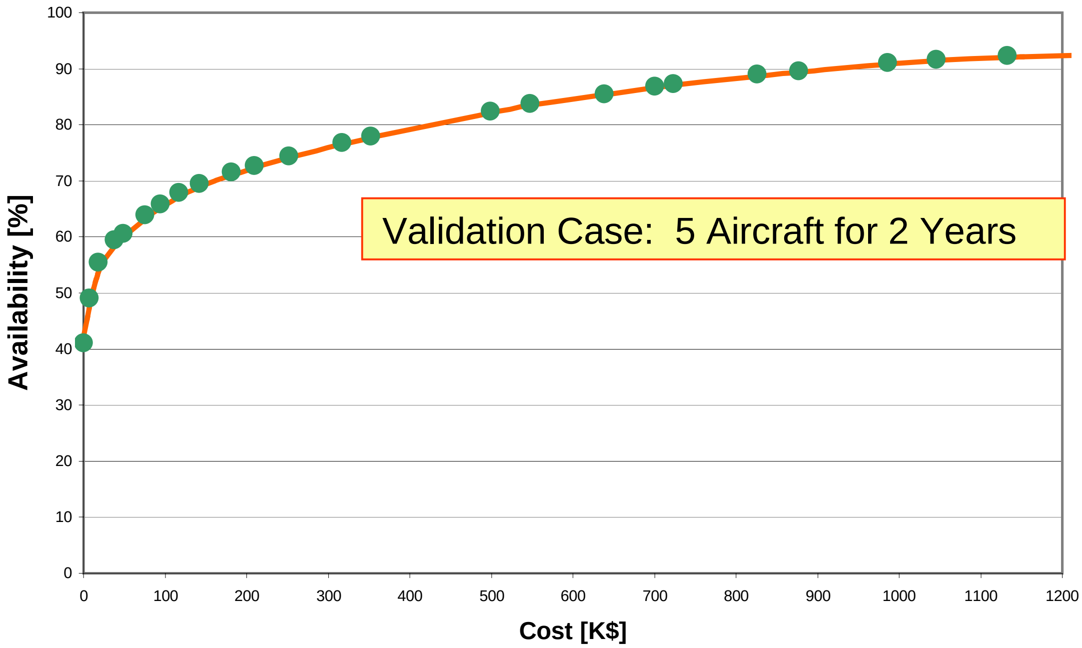

Figure 14: Availability vs. cost of 'Validation' scenario

Deployment Scenarios with Real Data: In the following three numerical examples, the number of deployed aircraft and the length of deployment period are consistently being reduced. The objective is to test the hybrid method with increasing difficulty. As the number of deployed aircraft is decreasing the deterministic processes, e.g. scheduled removals, become predominant. As the deployment period is shortened the time dependent probabilistic processes become predominant and the system is still in transient and not in steady state. The number of aircraft and the deployment duration in the three cases are set to:

1. Case A: 10 aircraft for 2 years
2. Case B: 5 aircraft for 1 year
3. Case C: 1 aircraft for ¼ year

In all three cases real aircraft were used i.e. data obtained from the "UH60 A to A recapitalization impact to sustainment" data model. In these three examples, aircraft and all installed parts are initialized (with accumulated age and maintenance history) and the LRUs fail according to Weibull distributions [10] that also depend on the number of depot overhauls. A full account of the data details on "UH60 A to A recapitalization impact to sustainment" data model will not be presented in this paper, but it shall be provided upon request. Availability vs. cost graphs that were generated in each of three cases, are presented in Figures 15-17.

By comparing the three graphs, we can come to several conclusions. As suspected, with smaller fleets and shorter deployment periods the analytic model prediction worsens and it is recommended to verify the expected availability through simulation.

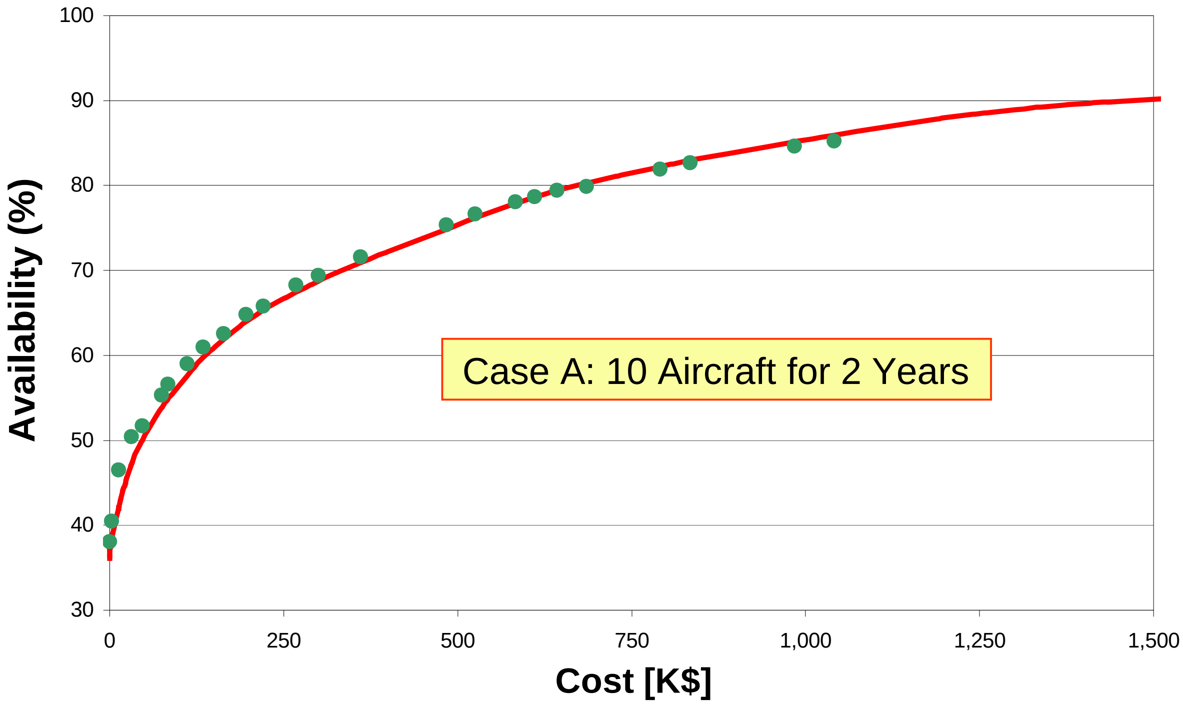

Figure 15: Availability vs. cost of 'Case A' scenario

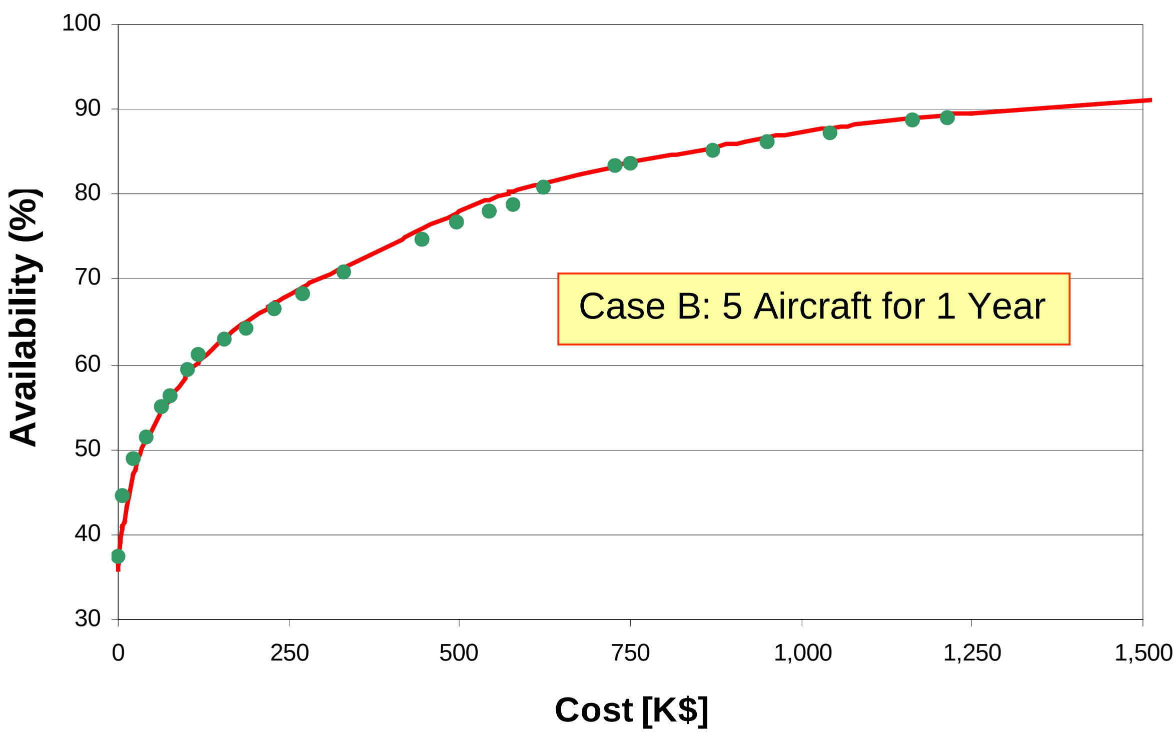

Figure 16: Availability vs. cost of 'Case B' scenario

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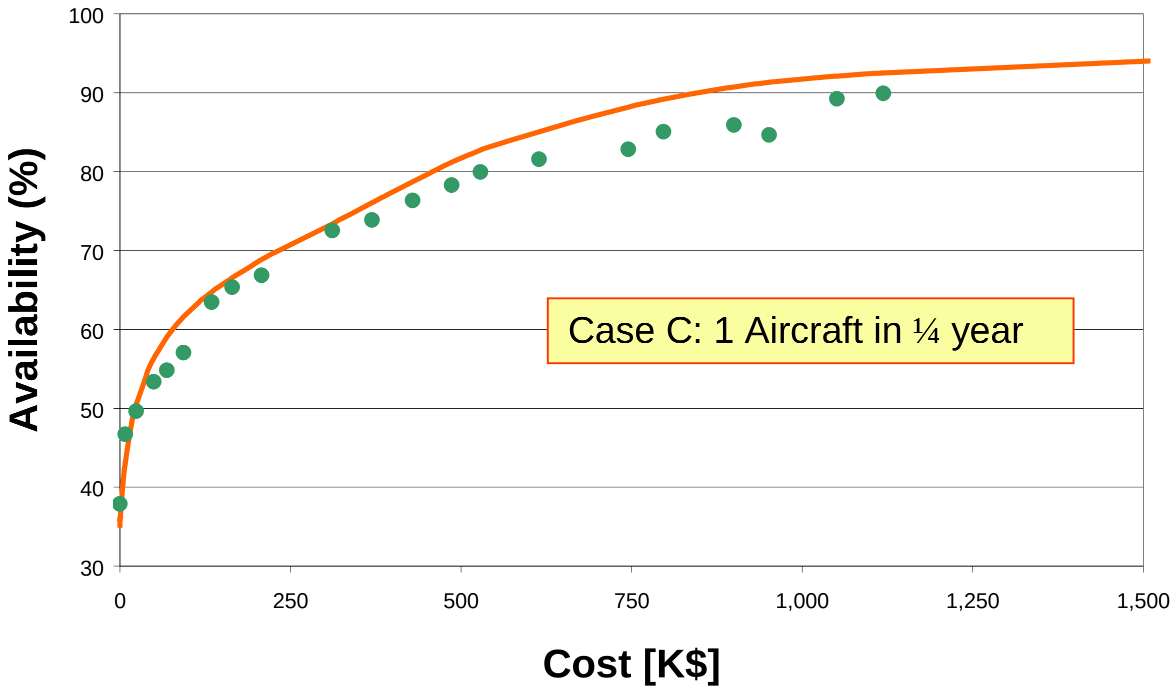

Figure 17: Availability vs. cost of 'Case C' scenario

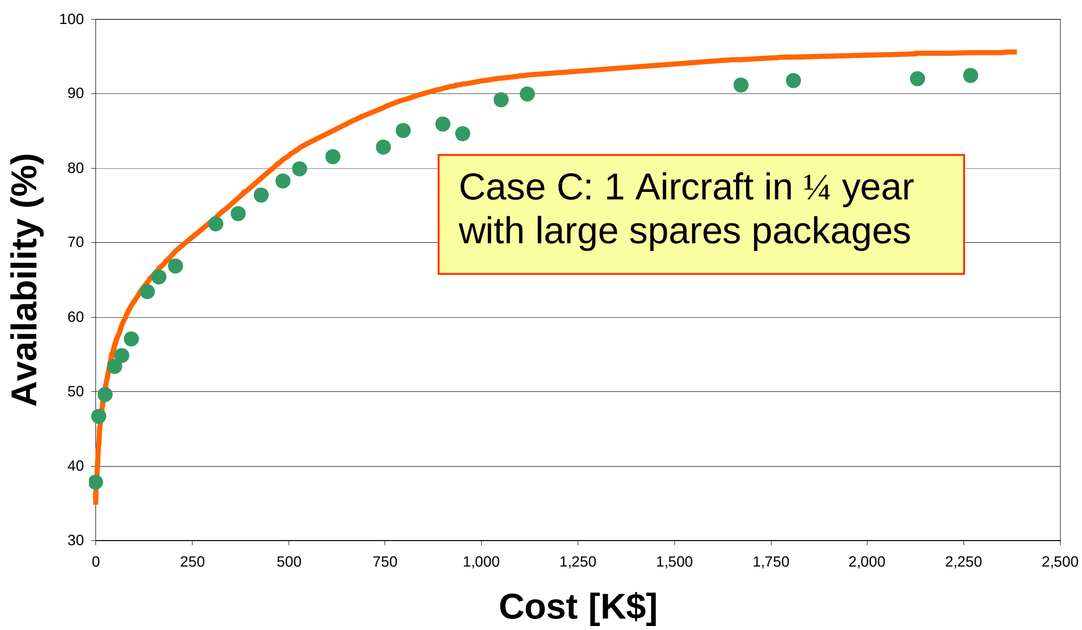

Figure 18: Extended Availability vs. cost of 'Case C' scenario

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One phenomenon that is not revealed in these three graphs, but is apparent in a availability vs. cost graph shown Figure 18. This is a variation of case C but for which the spares packages grow very large in number. It is evident that the predicted availabilities are underestimated i.e. consistently higher average availabilities are obtained in the simulation runs compared to the analytic model. This is because when availability is high aircraft fly more. When an aircraft is flying more there are more failures. The failed LRU's are replaced with new spares. Initially the instantaneous failure rates (hazard) of new spares, as in a case with Weibull distributions with shape parameters larger than one, are low. Therefore, the aircraft is being renewed or rejuvenated during the deployment period, but it is too short for the effects of aging to become significant. Once again it is recommended to verify availability estimates through simulation. In most cases the 'actual' availability will be higher than the predicted.

IX. Summary

The growing demand to reduce cost without adversely impacting system performance has produced requirements to develop accurate, efficient and user-friendly decision support applications. The hybrid approach is found to be a useful method for optimizing logistic and maintenance resources in general, and spare parts strategies in particular. The deployment and push pack spares optimization module within the latest version of ATLAST, combines a high fidelity life-cycle simulation model together with an efficient analytic technique for push-pack spares. The hybrid approach forms the basis for the development of additional optimization applications in ATLAST and general SPAR-based models.

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Biographies

Naaman Gurvitz (Ph.D.)- VP Technology & Products **-** 1996 to present – at Clockwork Solution Inc. (a privately held engineering firm specializing in computer modeling of operation systems, ERP, and manufacturing processes in the military, electrical, chemical, and utility industries). Manages large, complex model development projects for key accounts including the United States Air Force, Israel Defense Forces, Japan Energy Corporation, Dow Chemicals, plus other government and Fortune 500 organizations. Developed simulation algorithms of ATLAST (Aircraft Total Life-cycle Assessment Tool) and RAMSS (Requirements, Analysis, and Management Support System) integrating all aspects of military aviation systems involving operation profiles, engineering, maintenance, and supply of parts in multiple sites with various constraints and with flexibility to deal with unforeseen situations. Professional interests: predictive simulation modeling, optimization techniques and statistical analysis. **Education**: Ph.D. in Nuclear Engineering, University of Ben Gurion, Beer-Sheva, Israel (1996). M.S. in Nuclear Engineering, University of Ben Gurion, Beer-Sheva, Israel (1991)

Sergey Borodetsky (Ph.D.) – Chief Programmer – 1991 to present**-** at Clockwork Solution LTD. Oversees application of modern software technologies and software development policies. Designed and developed Clockwork's proprietary SPAR model development platform and its' integration with automated tools and component activation mechanisms for interpretation of logical assertions and accessing standard COM interfaces. Designed SPAR object model used in STORM (Material Flow Modeling Tool) and in PS+(Power System Modeling Tool). Developed sub-graph analysis algorithms used in ENRICO (electrical networks modeling Tool) and in Power Plant Workbench. Programmed very large-scale simulation tools – ATLAST and RAMSS for AMCON and USAF. Dr. Borodetsky gives lectures in Ben Gurion University and teaches wide series of computer related courses such as "Operating Systems", "Algorithms", "Data Structures", "Programming: Advanced Topics", "Windows 95/NT C++ Programming", "Object-Oriented Design and Programming with C++", "Foundation of Programming". **Education**: Ph.D. in Computer Science, State Technical Academy, St. Petersburg (1988). M.S. in Computer Science, Electro-technical University, St. Petersburg(1980).

Pierre van Eck is uniquely qualified to work in the area of RAM (reliability, availability, maintainability) with a significant breadth of experience in systems analysis, design, and programming as well as extensive knowledge of the logistics, maintenance, and operations domains. Mr. van Eck has developed many key components of the Aviation Total Life-cycle Assessment Software Tool (ATLAST), now in use by the United States Army and Original Equipment Manufacturers (OEMs) supporting Rotary Wing Aircraft systems. He graduated from the University of Stellenbosch, Stellenbosch, South Africa with a degree in Electrical and Electronic Engineering and was involved for 4 years on major South African Military projects doing reliability prediction and logistical analysis