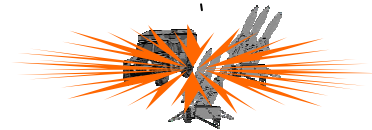


# Optimized Trajectory Shaping Guidance for an Air-to-Ground Missile Launched from a Gunship

Craig Phillips  
Ernie Ohlmeyer  
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DAHLGREN

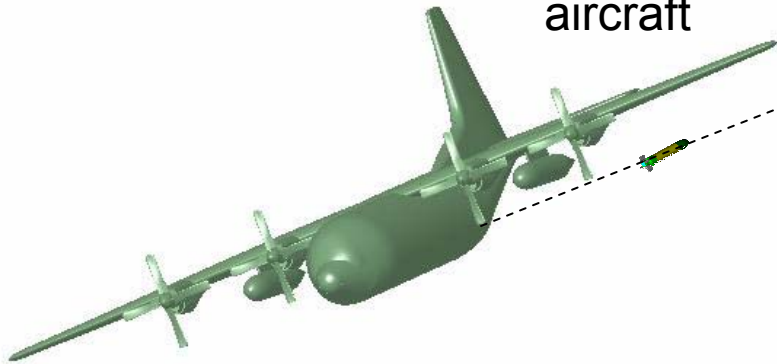


# Overview

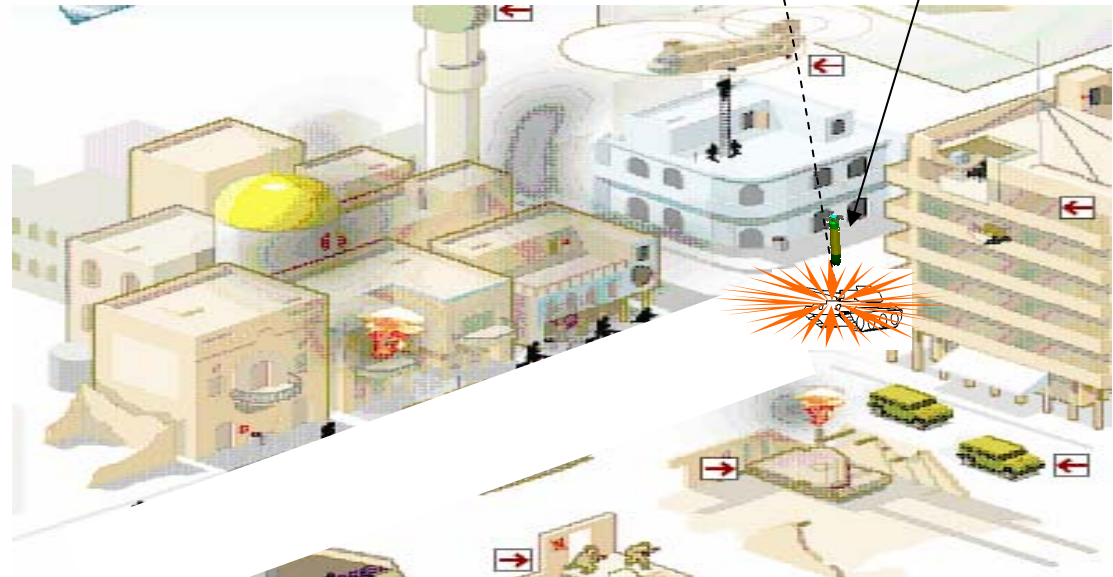
- Mission Scenario
- Notional Munition Concept
- Guidance Laws
  - Generalized Vector Explicit Guidance
  - Composite Guidance
- Footprints
- Trajectories
- Conclusions

# Next Generation Gunship Mission Scenario

Munition launched  
from port side of  
aircraft



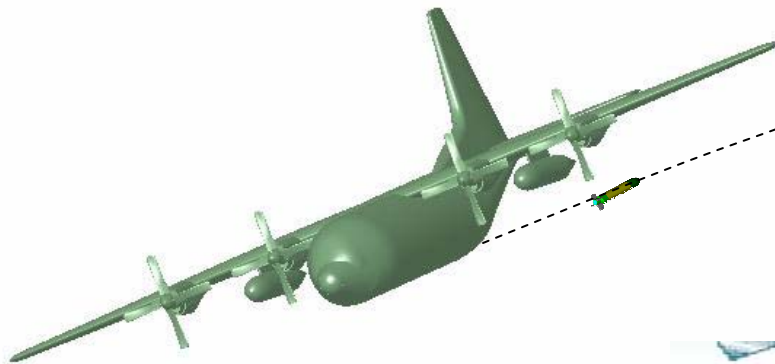
Munition will be able to reach extended ranges and have 360 degrees of coverage, which will help keep the gunship out of the range of an enemy's weapons.



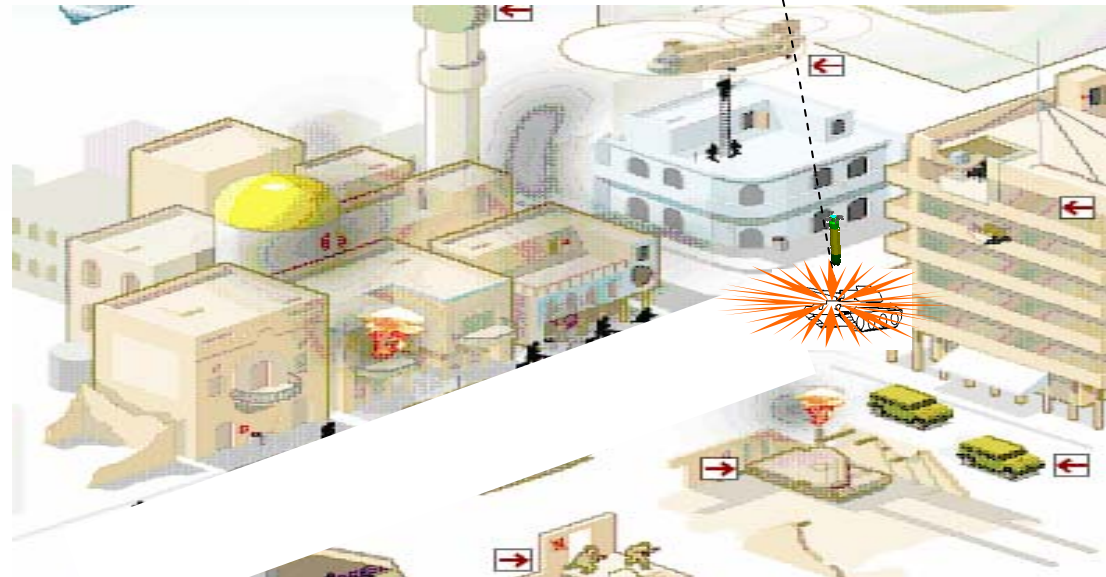
Vertical  
Impact  
Angle

# Next Generation Gunship Mission Scenario

- Munition must meet Military Operations on Urban Terrain (MOUT) requirements:
- Be able to attack in urban canyons
  - Have low collateral damage

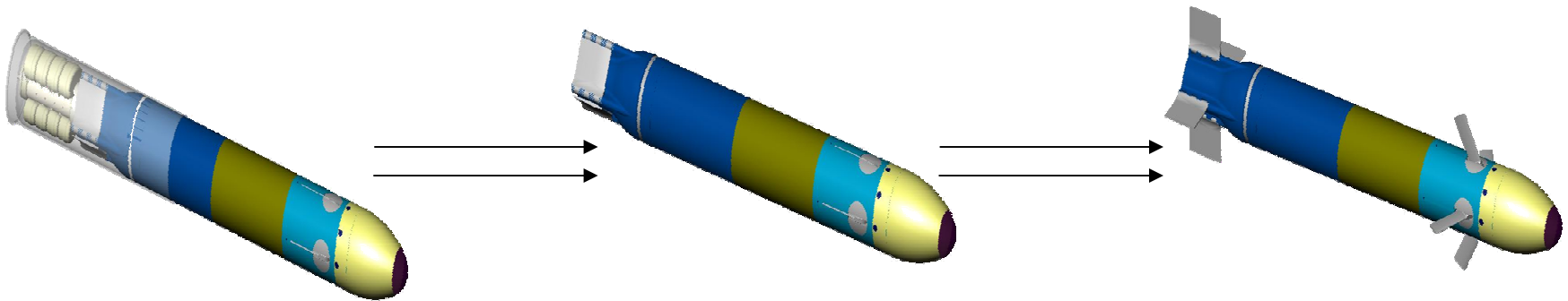


Trajectory shaping will help the munition reach these extended ranges, have 360 degrees of coverage, and meet MOUT requirements



# Notional Gun-Launched Munition Concept

- Weight: 47.5 lb  
15 lb (Warhead)  
20 lb (Propellant)
- Length: 42.5 inches
- Diameter: 4.7 inches (120 mm)
- Thrust: 1350 lb (Boost)  
163.9 lb (Sustain)
- Burn Time: 2 sec (Boost)  
15 sec (Sustain)
- Total Impulse: 5158.5 lb\*s



- Angle of Attack (Max): 20 deg
- Achievable Acceleration: 35 g's (Aerodynamic Limit)



# Generalized Vector Explicit (GENEX) Guidance

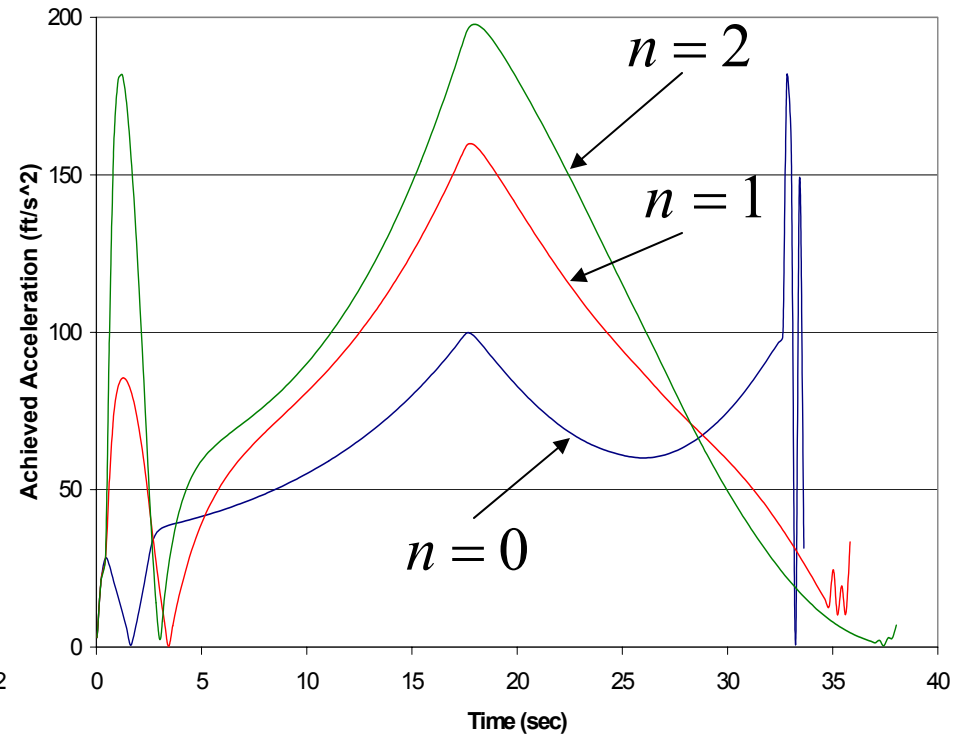
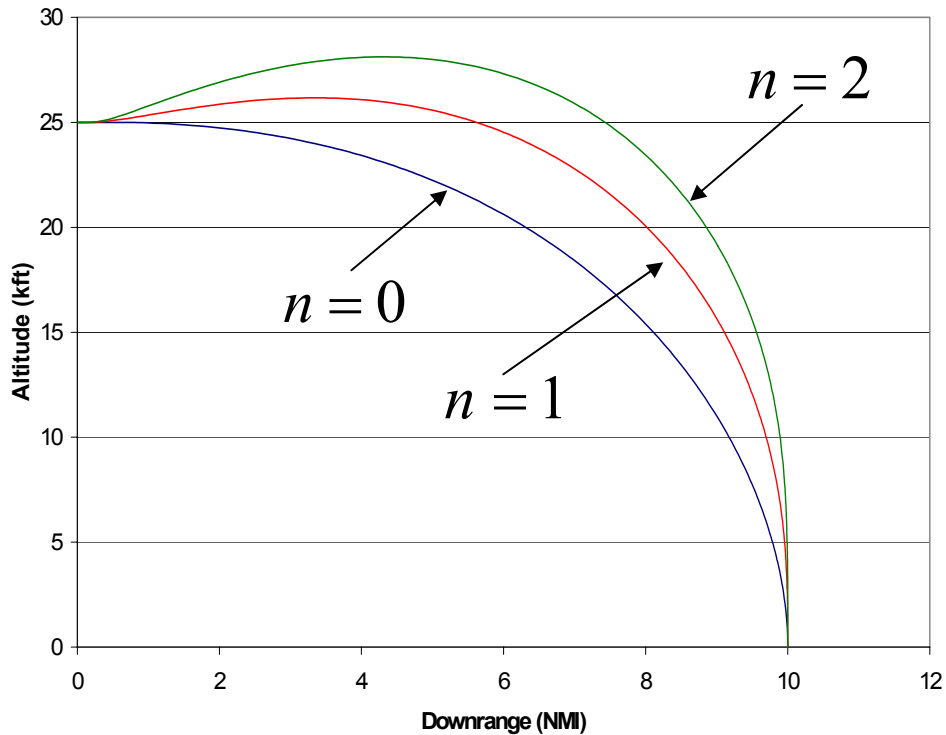
$$u^* = \frac{1}{T^2} \left[ \underbrace{K_1(y_f - y_M - \dot{y}_M T)}_{\text{ZEM}} + K_2(\dot{y}_f - \dot{y}_M)T \right]$$

$u^*$	: Missile acceleration	Define the gains as:
$T$	: Time-to-go	
$y_M, \dot{y}_M$	: Missile position and velocity	
$y_f, \dot{y}_f$	: Desired final missile position and velocity	
ZEM	: Zero Effort Miss wrt a specified final position	
		$K_1 = (n+2)(n+3)$
		$K_2 = -(n+1)(n+2)$
		$n = \text{design parameter}$

- The optimum guidance consists of a PRONAV-like term with gain  $K_1$  and a trajectory shaping term with gain  $K_2$ .
- The Generalized Explicit Guidance solution defines a family of guidance laws defined by the parameter  $n$ . The parameter  $n$  may be chosen to achieve specific performance objectives and to satisfy terminal constraints.
- In general,  $n$  may be used to modify the curvature of the trajectory, while enforcing a specified final position and final velocity orientation (impact angle).

**Reference: Ohlmeyer, E. J., "Control of Terminal Engagement Geometry Using Generalized Vector Explicit Guidance," Proceedings of American Control Conference, Vol. 1, 2003, pp. 396-401.**

# Effect of Varying Design Parameter, $n$ , in GENEX Guidance Law



Increased curvature as  $n$  is increased

For  $n > 0$ , large accelerations achieved earlier in flight to allow for smaller accelerations at terminal (e.g. smaller angle of attack)





# Composite Guidance Law

$$\underline{a}_N = C(t)\underline{B} + \underline{u}[1 - C(t)] + \underline{g}_{Bias}$$

- $\underline{a}_N$  : normal acceleration commanded by guidance
- $C(t)$  : time-dependent composite index
- $\underline{B}$  : composite bias acceleration vector
- $\underline{u}$  : GENEX control term
- $\underline{g}_{Bias}$  : one g up bias term

The bias acceleration vector B, when added to GENEX, provides higher gains early in flight, increases the curvature of the trajectory, and extends the kinematic reach of the munition.

Level flight commanded for  $t \leq 0.5$  sec

For  $0.5 < t \leq 2.5$  sec,  $C(t) = 1.0$

For  $2.5 < t \leq 7.5$  sec,  $C(t) = 1.5 - (0.2t)$

For  $t > 7.5$  sec,  $C(t) = 0.0$

For  $C(t) = 0.0$ , above equation becomes GENEX equation

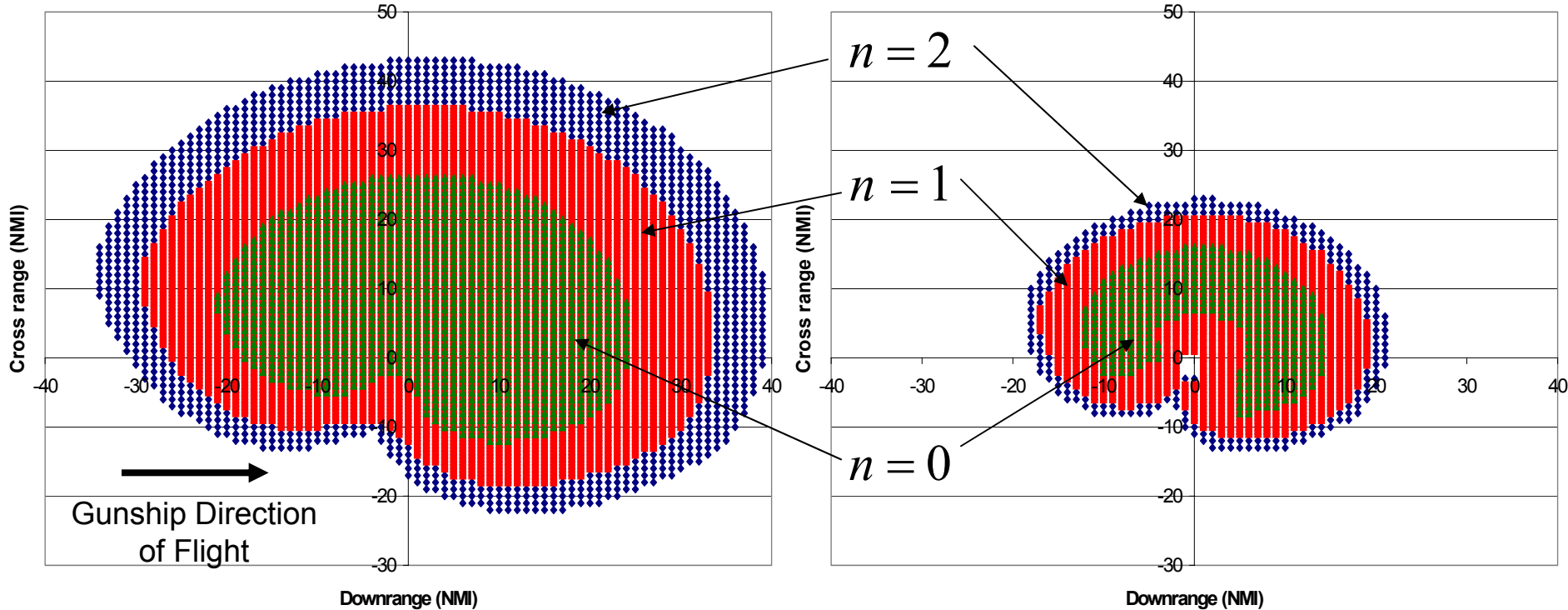
$C(t)$  picked to be linear for simplicity.  $C(t)$  should be optimized for best results.



# GENEX Footprints

25,000 ft Launch

10,000 ft Launch



GENEX less effective with drop in launch altitude

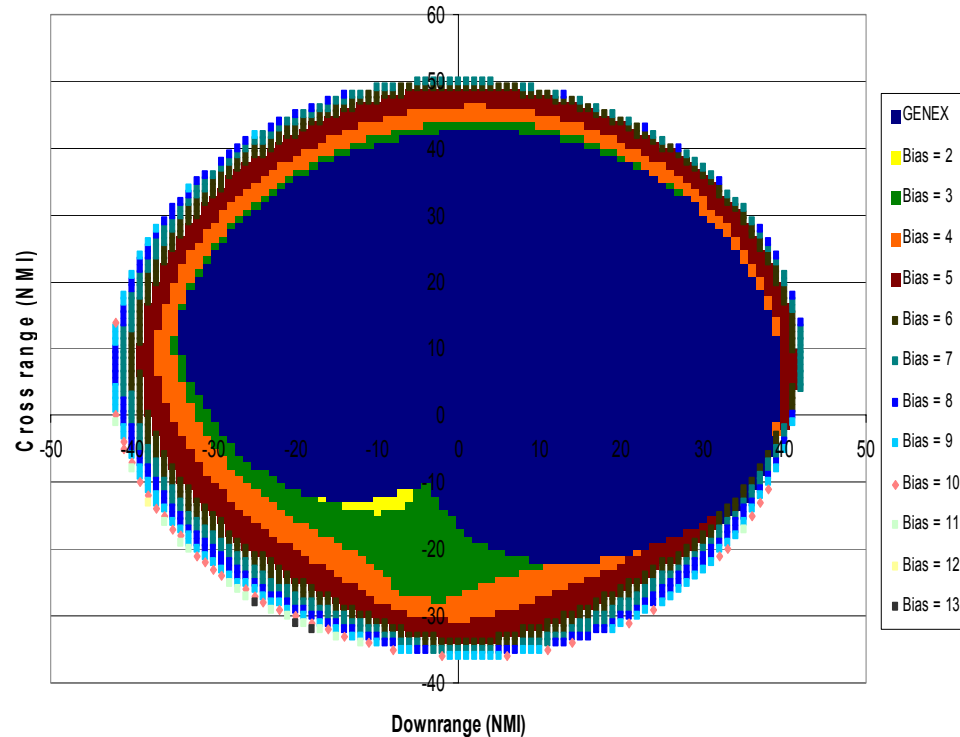
Muzzle Velocity = 1100 fps  
Gunship Velocity = 405 fps  
Launch Angle = 0 deg

Hit Criteria:

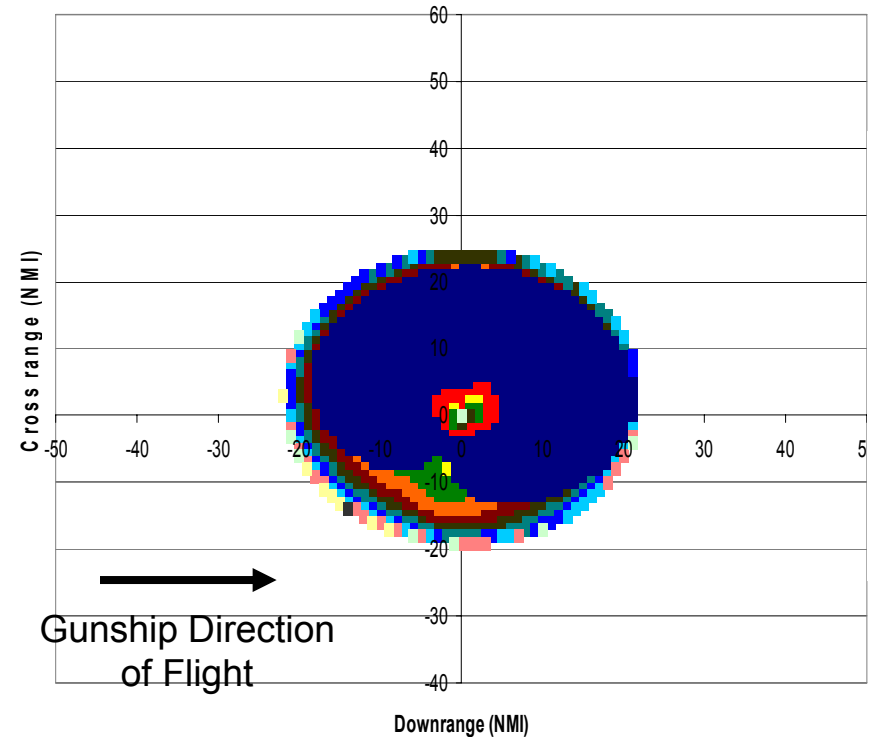
- less than 5 ft Miss Distance
- within 10 deg of vertical impact angle

# Composite Guidance Footprints

25,000 ft Launch  
 $n=2$



10,000 ft Launch  
 $n=2$



Composite Guidance increases the footprint significantly compared to GENEX only

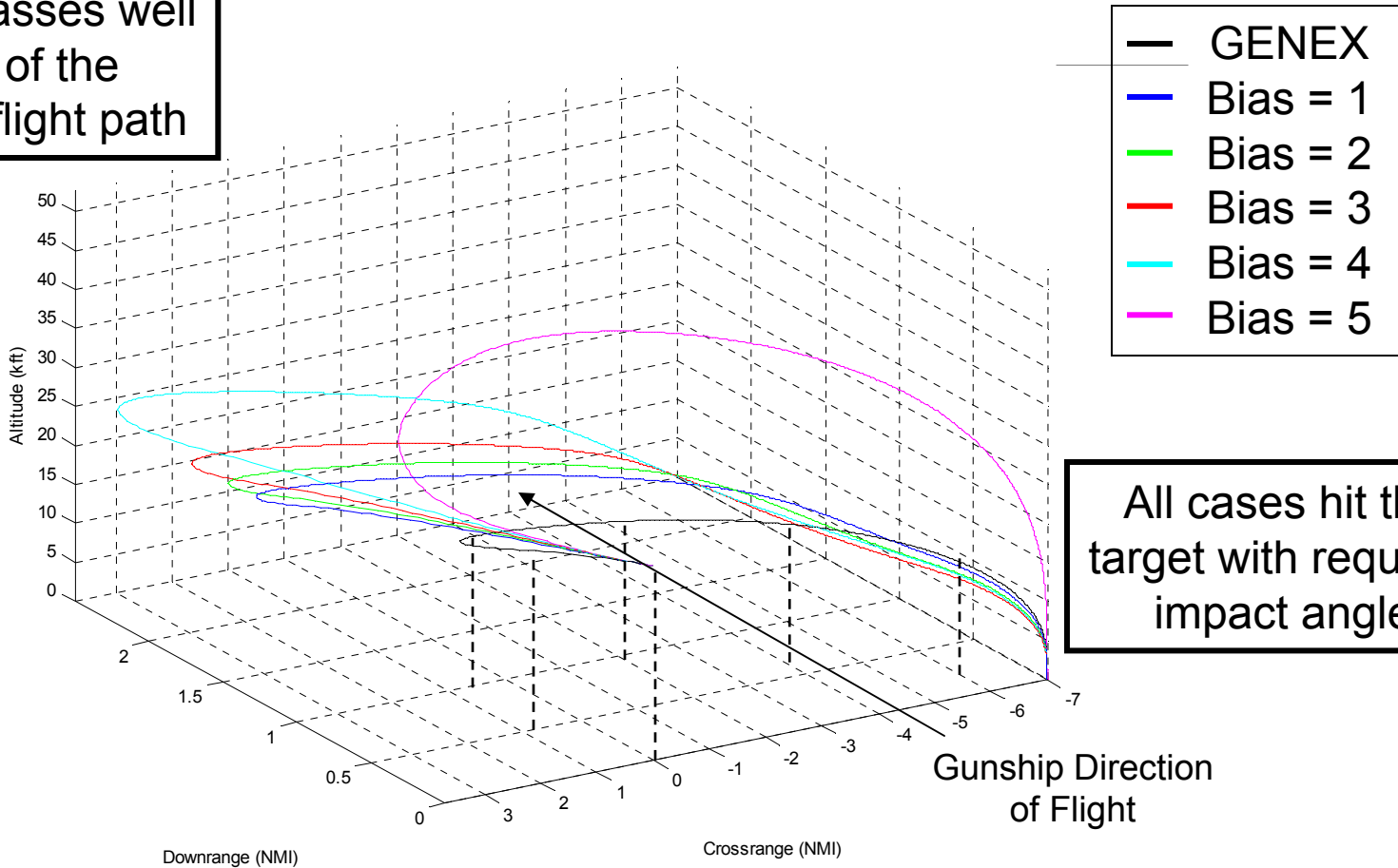
- 10-25 NMI farther for 25 kft launch
- 5-10 NMI farther for 10 kft launch

Like GENEX, Composite Guidance becomes less effective with drop in launch altitude

# Trajectories

Target: Cross range = -7 NMI  
25,000 ft Launch  
 $n=2$

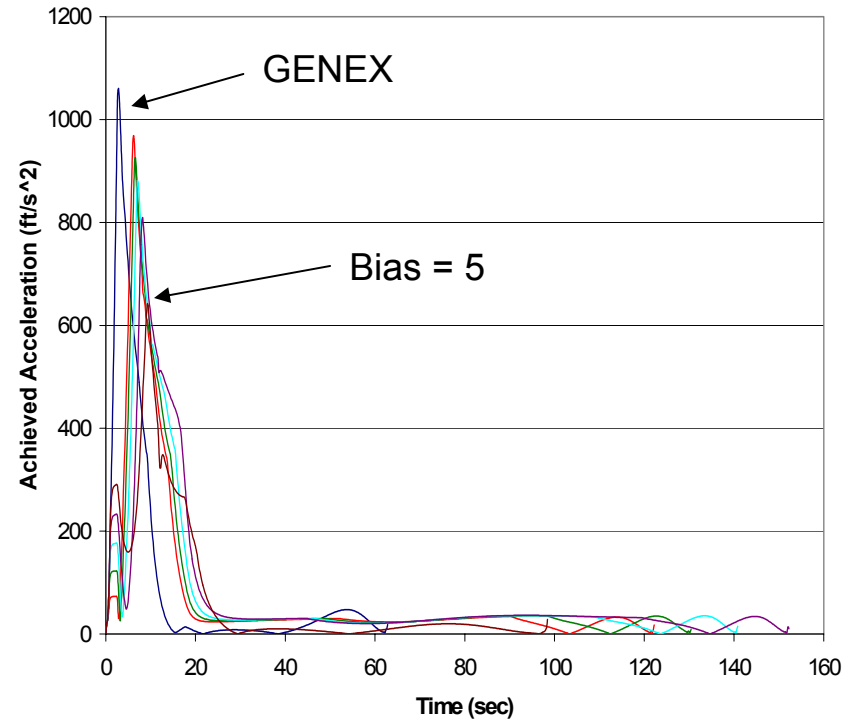
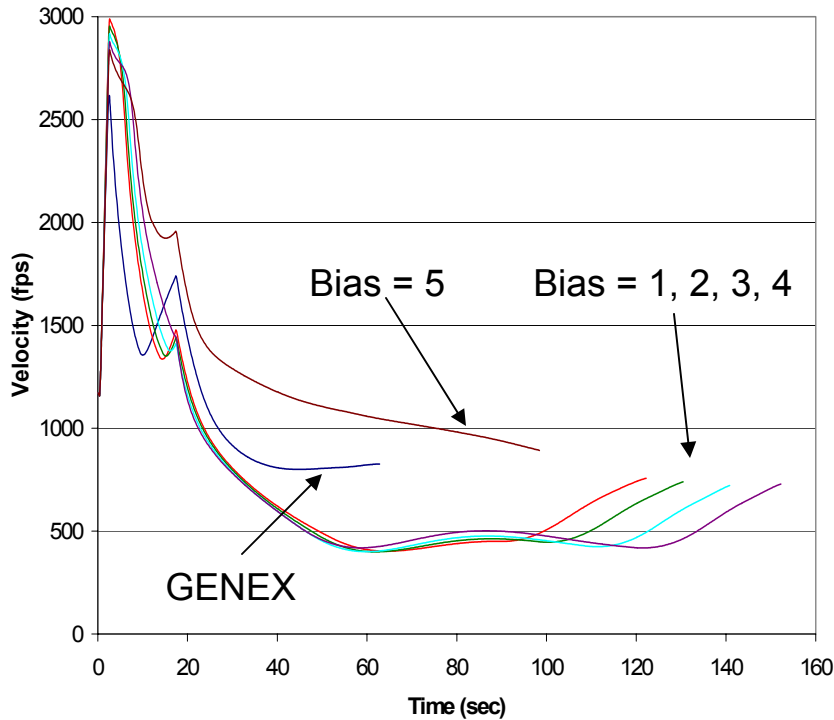
In all cases, the munition passes well in front of the gunship's flight path



All cases hit the target with required impact angle

# Velocities and Accelerations

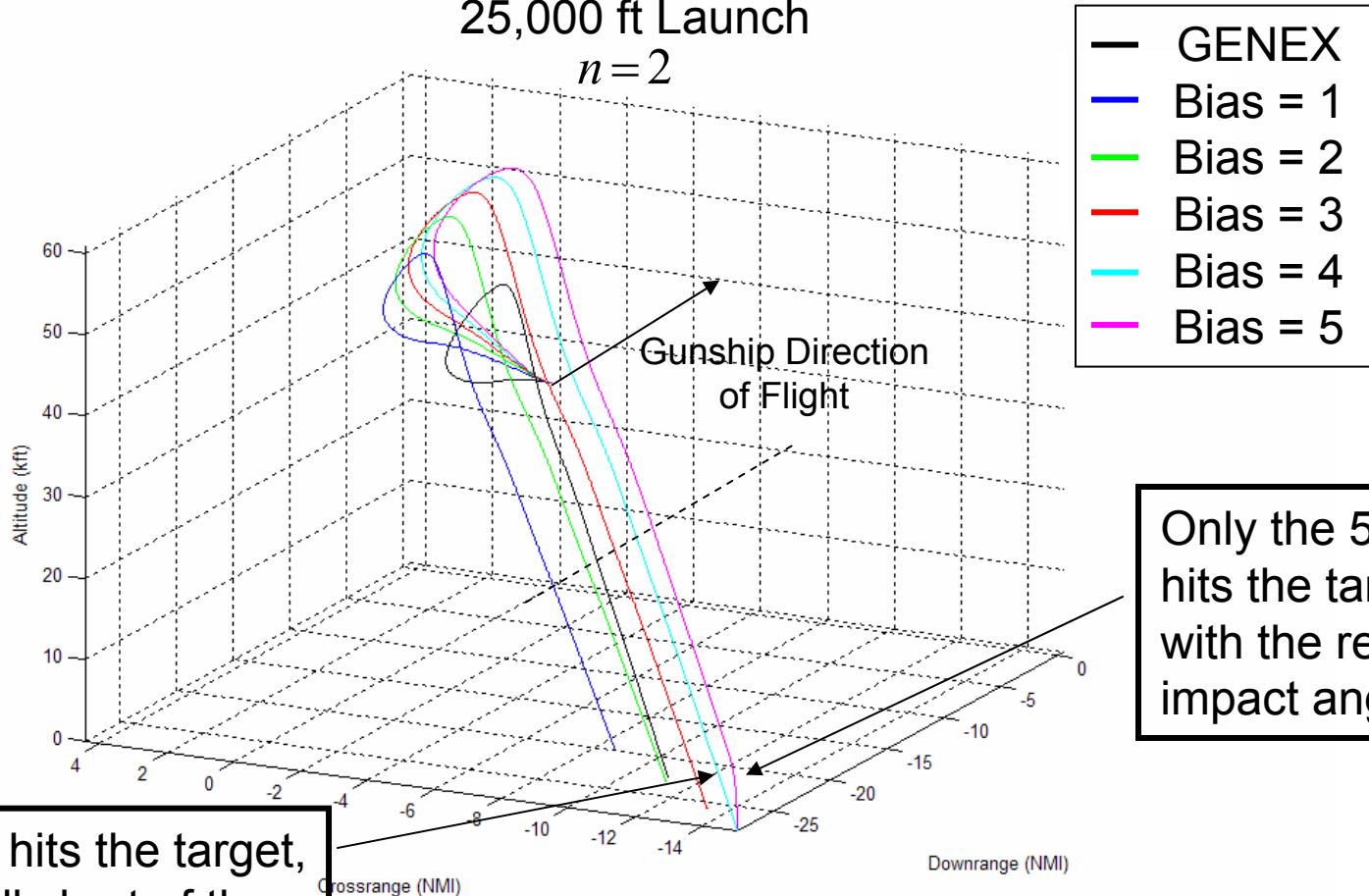
Target: Cross range = -7 NMI  
25,000 ft Launch  
 $n=2$



The 5 g Bias case gives a 40% reduction in max acceleration and still achieves approximately the same impact velocity

# Trajectories

Target: Downrange = -28 NMI, Cross range = -15 NMI  
25,000 ft Launch  
 $n = 2$

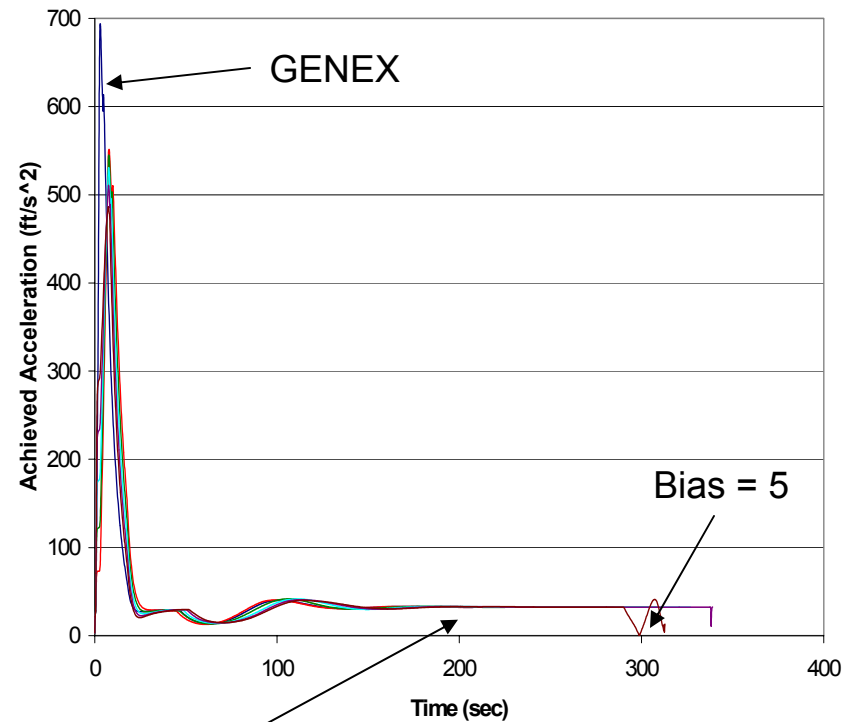
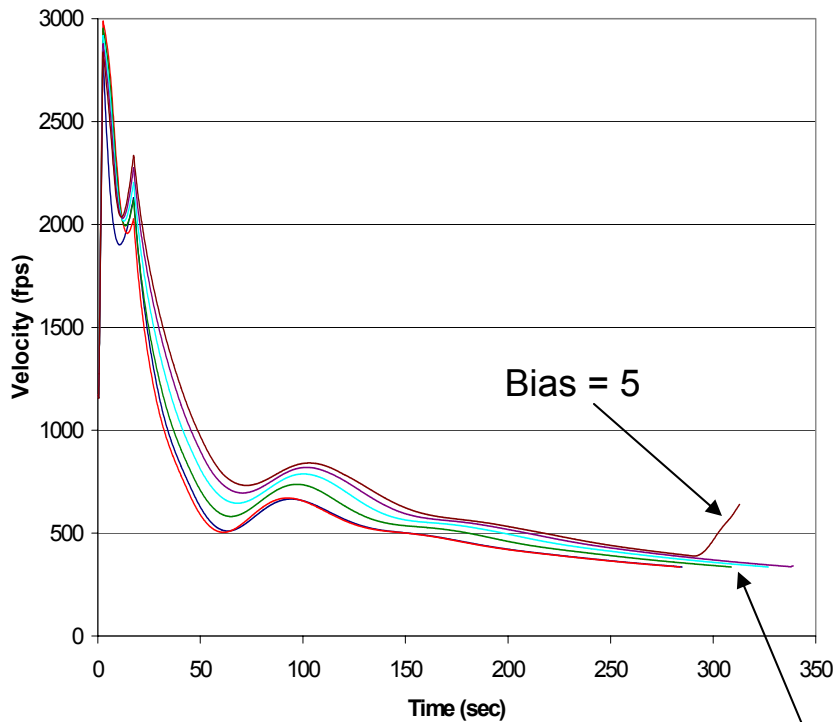


Only the 5 g Bias hits the target with the required impact angle

The 4 g Bias hits the target, but falls well short of the required impact angle

# Velocities and Accelerations

Target: Downrange = -28 NMI, Cross range = -15 NMI  
25,000 ft Launch  
 $n=2$



The munition expends too much energy to reach the target. Only the 5 g Bias case, with its increased curvature, is able to reach the target with the required impact angle.



# Conclusions

- Composite Guidance, a blending of GENEX guidance with a bias acceleration vector, provides a larger footprint than GENEX alone
- Composite Guidance extends the effectiveness of GENEX guidance
  - Allows the munition to reach 15-25 NMI farther on the starboard side and 10 NMI farther behind the gunship for a 25 kft altitude launch
  - Allows the munition to reach 5-10 NMI farther on the starboard side and 5 NMI farther behind the gunship for a 10 kft altitude launch
- Able to meet strict Military Operations on Urban Terrain (MOUT) requirements
  - Munition comes in vertically to allow an attack in urban canyons, simplifying mission planning
  - Small miss distance for low collateral damage



# Backup Slides



# Generalized Vector Explicit Guidance (GENEX)

Define a set of linear state equations and boundary conditions as:

$$\underline{\dot{X}} = A\underline{X} + \underline{b}u \quad \underline{X}(t_0) = \underline{X}_0 \quad \underline{X}(t_f) = \underline{X}_f = \underline{0}$$

where  $\underline{X}$  is the state and  $u$  is the control (missile acceleration).

Select a cost function of the form:

$$J = \int_0^{T_0} \frac{u^2}{2T^n} dT = \int_0^{T_0} L(u, T) dT$$

where  $T = t_f - t$  is the time-to-go, and  $n$  is an integer  $\geq 0$ .  
*The above equation comprises a family of cost functions parameterized by the index  $n$ .*

**Reference: Ohlmeyer, E. J., "Control of Terminal Engagement Geometry Using Generalized Vector Explicit Guidance," Proceedings of American Control Conference, Vol. 1, 2003, pp. 396-401.**



# Generalized Vector Explicit Guidance (GENEX)

Define the states as:

$$X_1 = z = y_f - y_M - \dot{y}_M T$$
$$X_2 = v = \dot{y}_f - \dot{y}_M$$

subject to the terminal conditions  $z = 0$  and  $v = 0$  at  $T = 0$  .

$z$  = zero effort miss with respect to a specified final position  
 $v$  = difference between current velocity and desired final velocity  
 $y_f, \dot{y}_f$  = desired final missile position and velocity

Define the gains as:

$$K_1 = (n+2)(n+3)$$

$$K_2 = -(n+1)(n+2)$$

**Reference: Ohlmeyer, E. J., "Control of Terminal Engagement Geometry Using Generalized Vector Explicit Guidance," Proceedings of American Control Conference, Vol. 1, 2003, pp. 396-401.**



# Composite Guidance Rationale

Treat Guidance Command as composed of a set of basis functions

Increasing the number of degrees of freedom in the Basis Set increases flexibility and thus allows performance to equal or surpass lower order forms

$$\underline{a}_N(t) = a_1(t)\phi_1(t) + a_2(t)\phi_2(t) + a_3(t)\phi_3(t)$$

In comparison with Open-Loop Trajectory Optimization, GENEX forms have shown excellent match with the mid-trajectory regions

GENEX matching in regions of transition at initiation and termination of trajectories may be improved

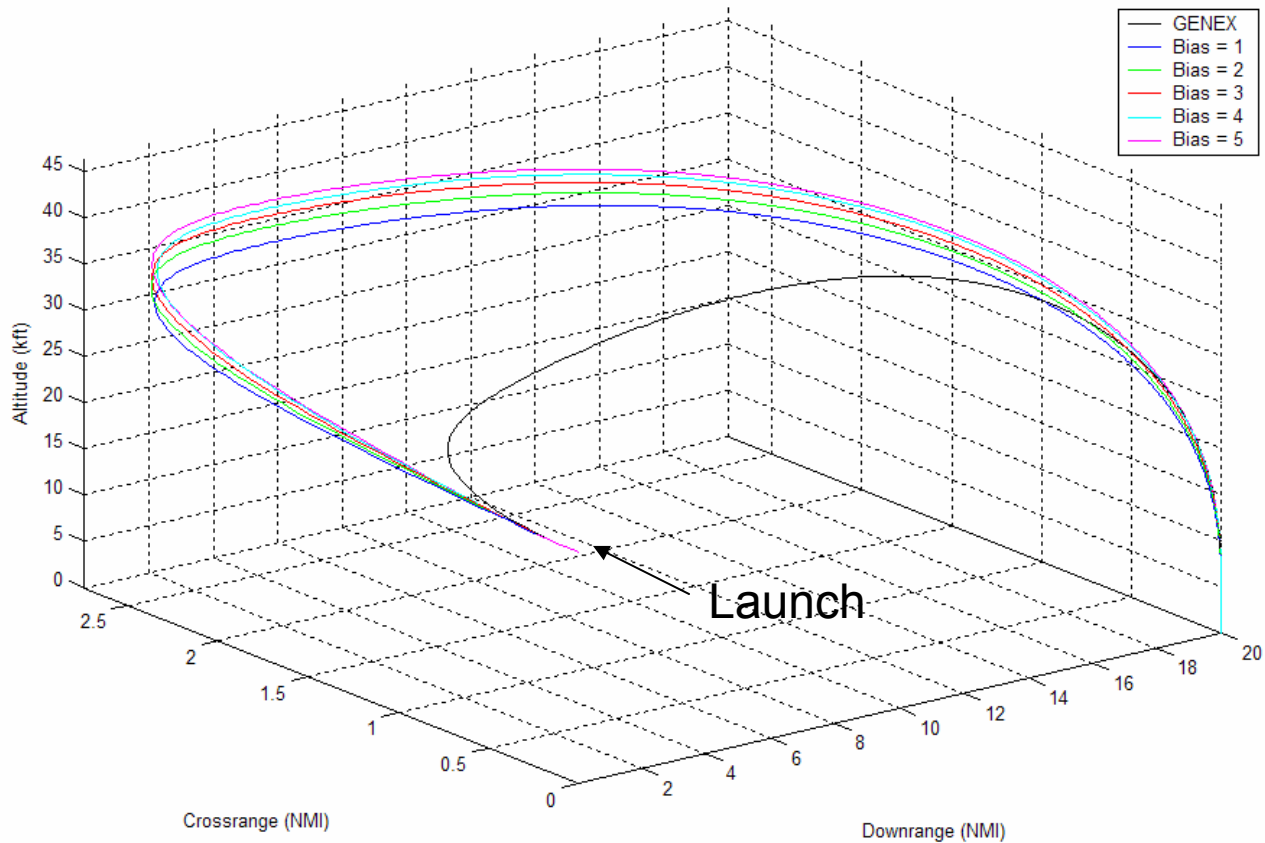
Thus, use GENEX as one of the basis functions

Add another function that is active only for initial phase (ramp)

Select Bias as simplest form for initial investigations

# Trajectories

Target: Downrange = 20 NMI  
25,000 ft Launch  
 $n = 2$

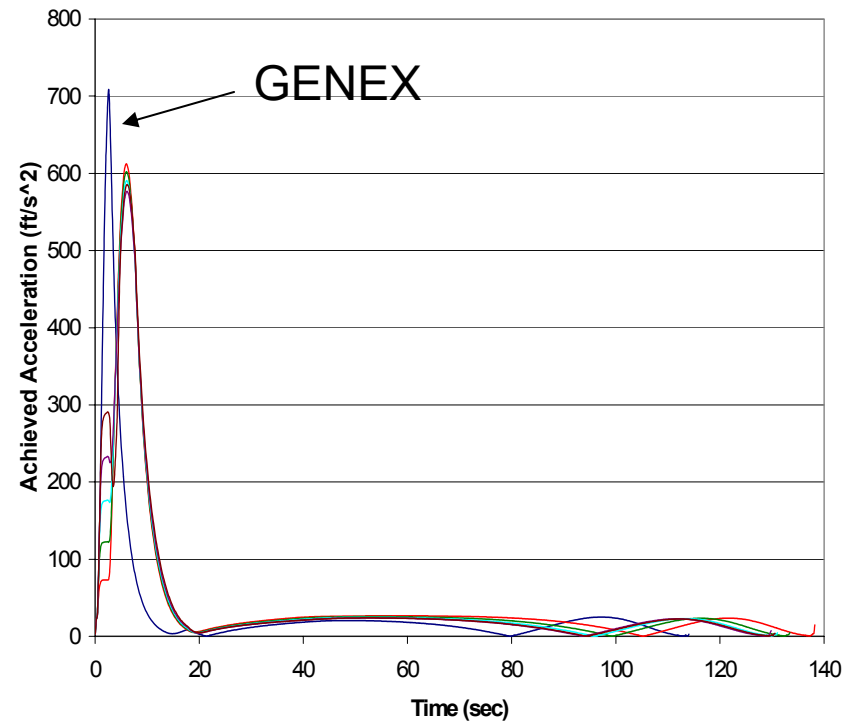
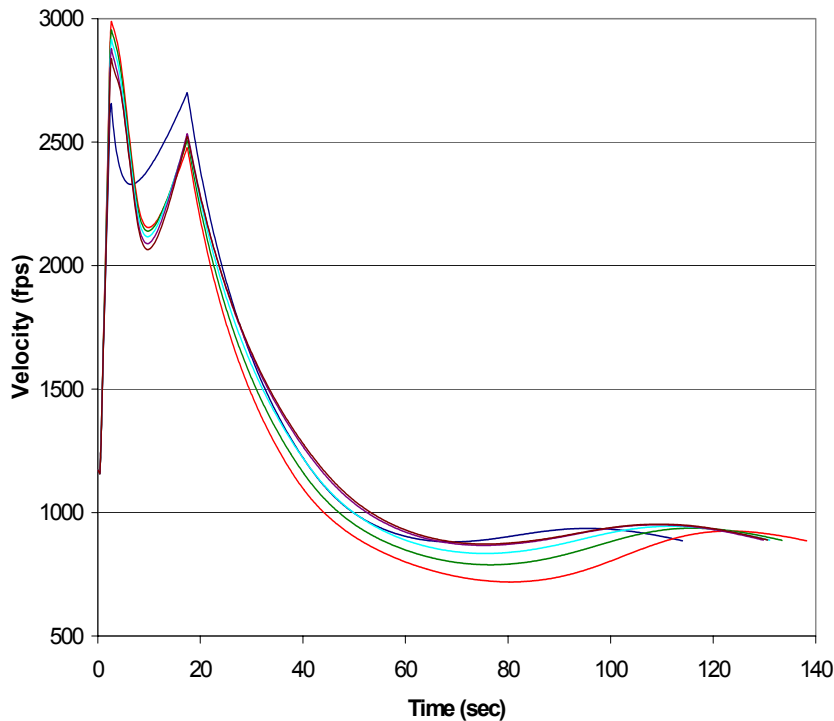


# Velocities and Accelerations

Target: Downrange = 20 NMI

25,000 ft Launch

$$n = 2$$



Composite Guidance allows for smaller accelerations to hit target with approximately the same impact velocity