DEFENSE THREAT REDUCTION AGENCY

Making the World Safer

SENSOR PLACEMENT OPTIMIZATION

Science and Technology for Chem-Bio Information Systems

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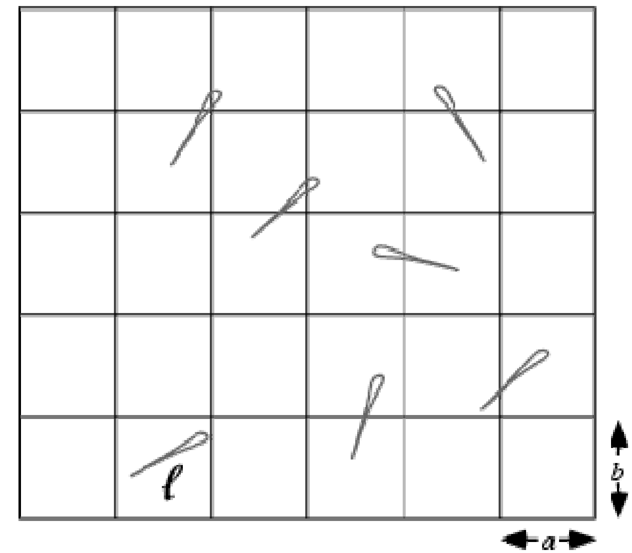
Problem of Interest

- Multiple Biological detectors to be placed around and within a fixed facility as passive defense measure
- Look at sensor placement options with fast running tool to generate statistical measures
- Definition of performance metric
  - Prior work accepted “at least one hit” on sensor as adequate
  - Relationship between metric and operational use of multiple sensors
  - Consider imperfect attacks
- Overall goal to create optimization tool to determine geometry, spacing and number of sensors

Theoretical approach

- Buffon's Needle: What is the probability that a needle hits crack in floor? It is a function of needle length and space between cracks.



$$P(\ell; a, b) = 1 - \frac{\int_{-\pi/2}^{\pi/2} F(\phi)\, d\phi}{\pi a b},$$

$$F(\phi) = a b - b \ell \cos\phi - \ell a |\sin\phi| + \frac{1}{2} \ell^2 |\sin(2\phi)|$$

$$P(\ell; a, b) = \frac{2 \ell (a + b) - \ell^2}{\pi a b}.$$

If the plane is instead tiled with congruent triangles with sides *a, b, c* and a needle with length *l*, less than the shortest altitude is thrown, the probability that the needle is contained entirely within one of the triangles is given by

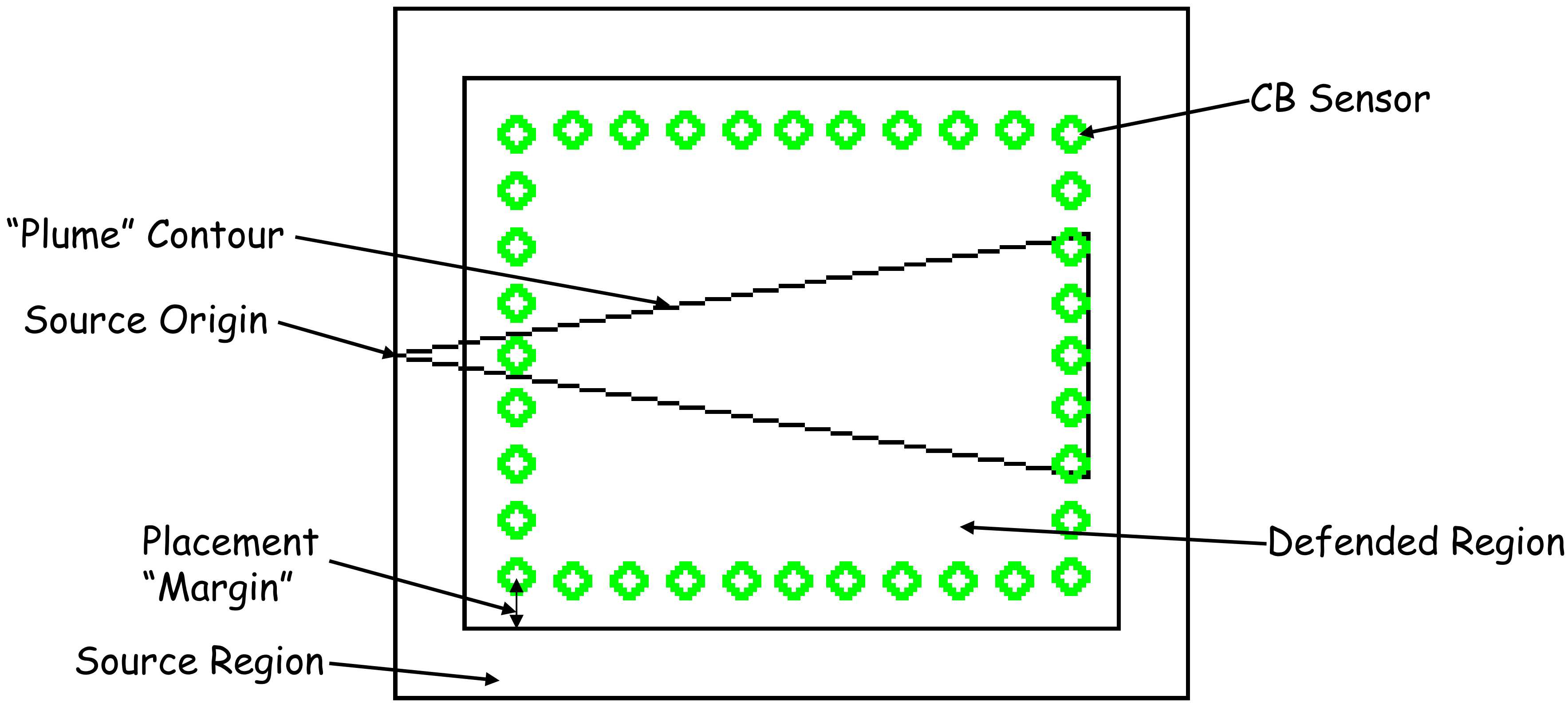
$$P = 1 + \frac{(A a^2 + B b^2 + C c^2)\ell^2}{8\pi K^2} - \frac{(4a + 4b + 4c - 3\ell)\ell}{4\pi K},$$

Where *A, B* and *C* are the angles opposite *a, b* and *c* respectively, and *K* is the area of the triangle.

What about dropping triangles on points, like a deadly plume on a sensor field? Too difficult – try a simulation.

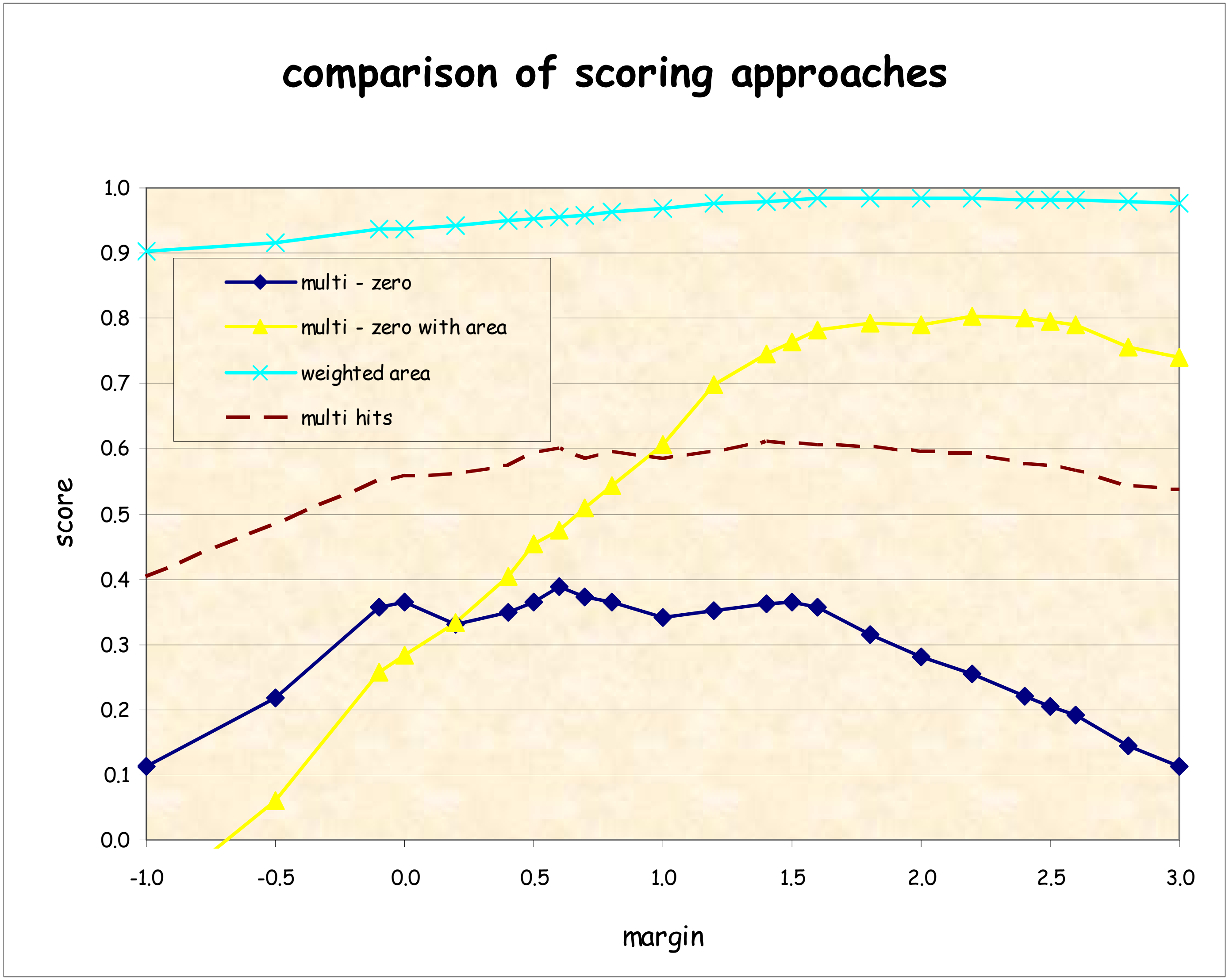
Mathworld.wolfram.com

Example Configuration

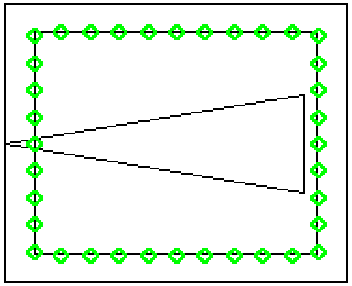


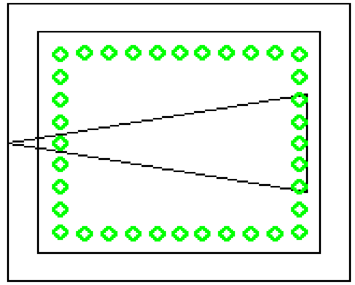
Basic Scoring Approaches

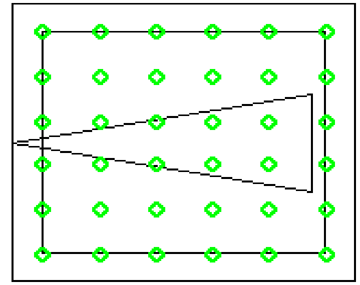
- Count number of detections
  - **Score =** number of detections
  - Problems: unbounded, had to compare different size arrays; sensitivity
- One or more hits is good (war posture, false alarms not considered)
  - **Score =** number of runs with one or more hits / total number of runs
- More than one is better (homeland posture, avoid false alarms)
  - **Score =** number of runs with two or more – number of runs with zero hits / total runs
- Areas weights =>> score * plume area / base area
  - Values cases where plume covers center of defended region
- Power law weights (optimization routine, declining return)
  - **Score** = $(2^i-1)/2^{(i-1)}$ or {0, 1, 1.5, 1.75, .. => 2.0}
  - Allows additional weight (discrimination) for more hits

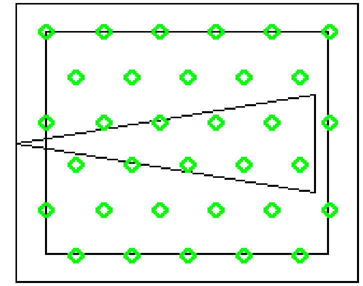

comparison of scoring approaches
score
margin
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-1.0
-0.5
0.0
0.5
1.0
1.5
2.0
2.5
3.0
multi - zero
multi - zero with area
weighted area
multi hits

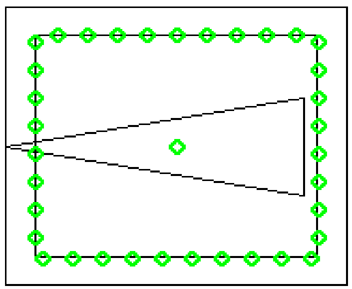
Grid Configurations

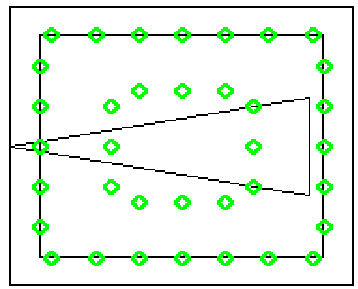

Perimeter

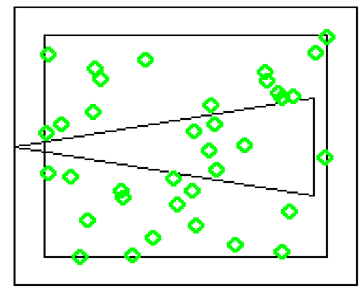

Perimeter with Margin

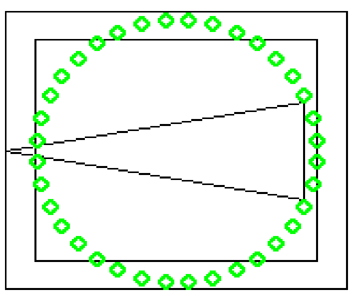

Uniform Array

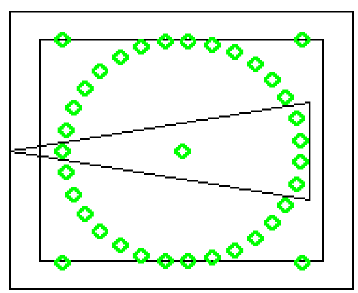

Dice 5

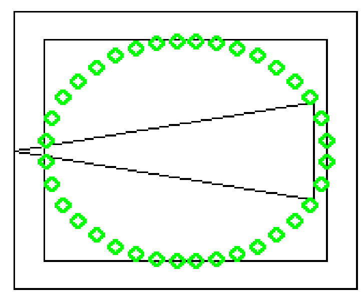

Perimeter with Center


Perimeter - 2 Tiers


Random

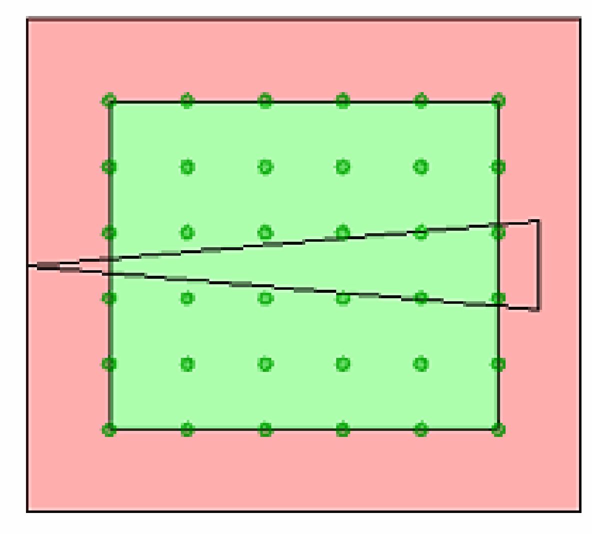

Circle

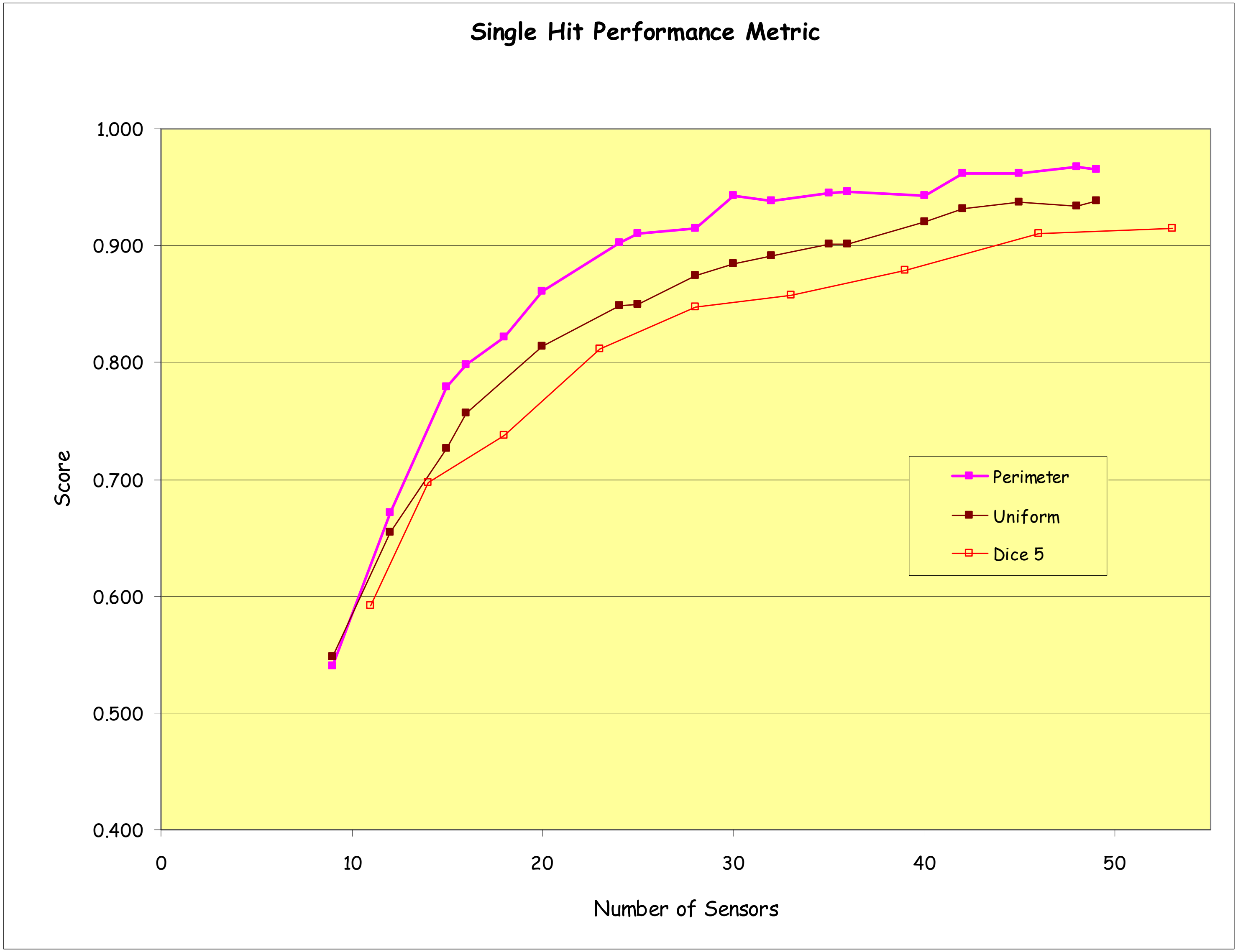

Circle, Margin, Center, Corners


Ellipse

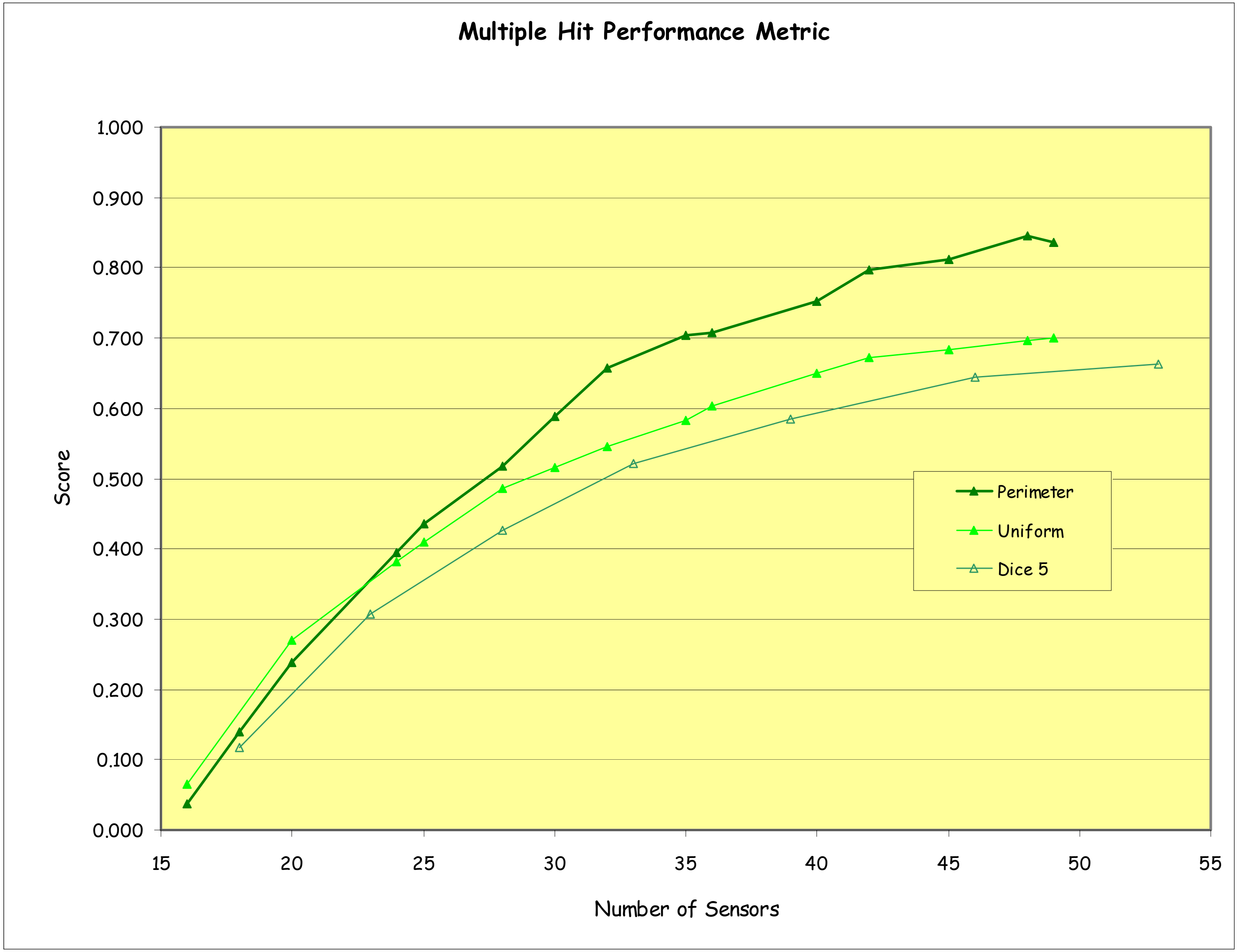
Scenario Parameters

- Defended Region: 16 km x 19 km
- Plume Source Region: 24 km x 27 km, centered on Defended Region
- Plume: 25 km length, 10 degree arc width
- Scenario Control: 2500 trials per run, fixed seed
- Sensor Configuration: Margin = 0.0

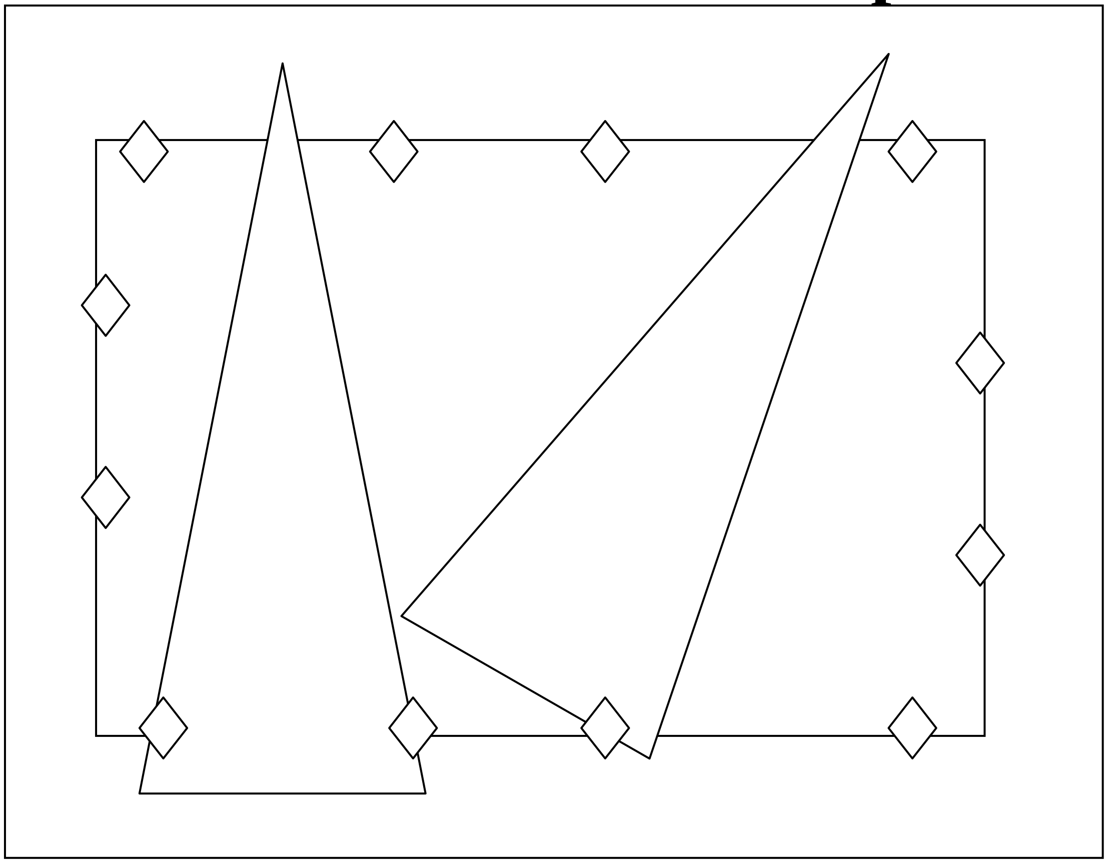




Single Hit Performance Metric
Score
1.000
0.900
0.800
0.700
0.600
0.500
0.400
0
10
20
30
40
50
Number of Sensors
Perimeter
Uniform
Dice 5

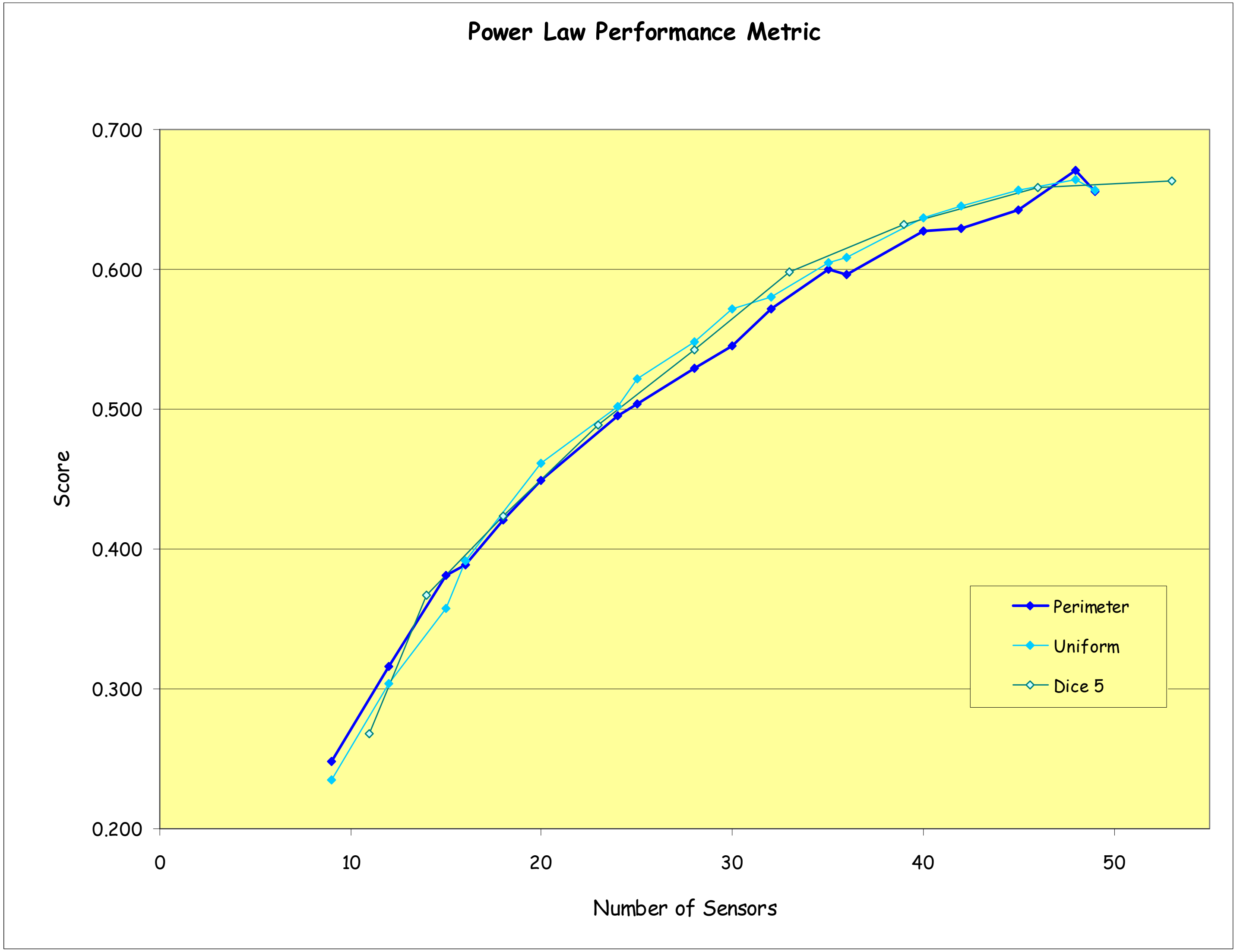

Multiple Hit Performance Metric
Score
Number of Sensors
1.000
0.900
0.800
0.700
0.600
0.500
0.400
0.300
0.200
0.100
0.000
15
20
25
30
35
40
45
50
55
Perimeter
Uniform
Dice 5

Perimeter vs Uniform for multiple hits



If the sensors are far apart, it is difficult to hit two or more with Perimeter.

Uniform is preferred with limited sensors.


Power Law Performance Metric
Score
0.700
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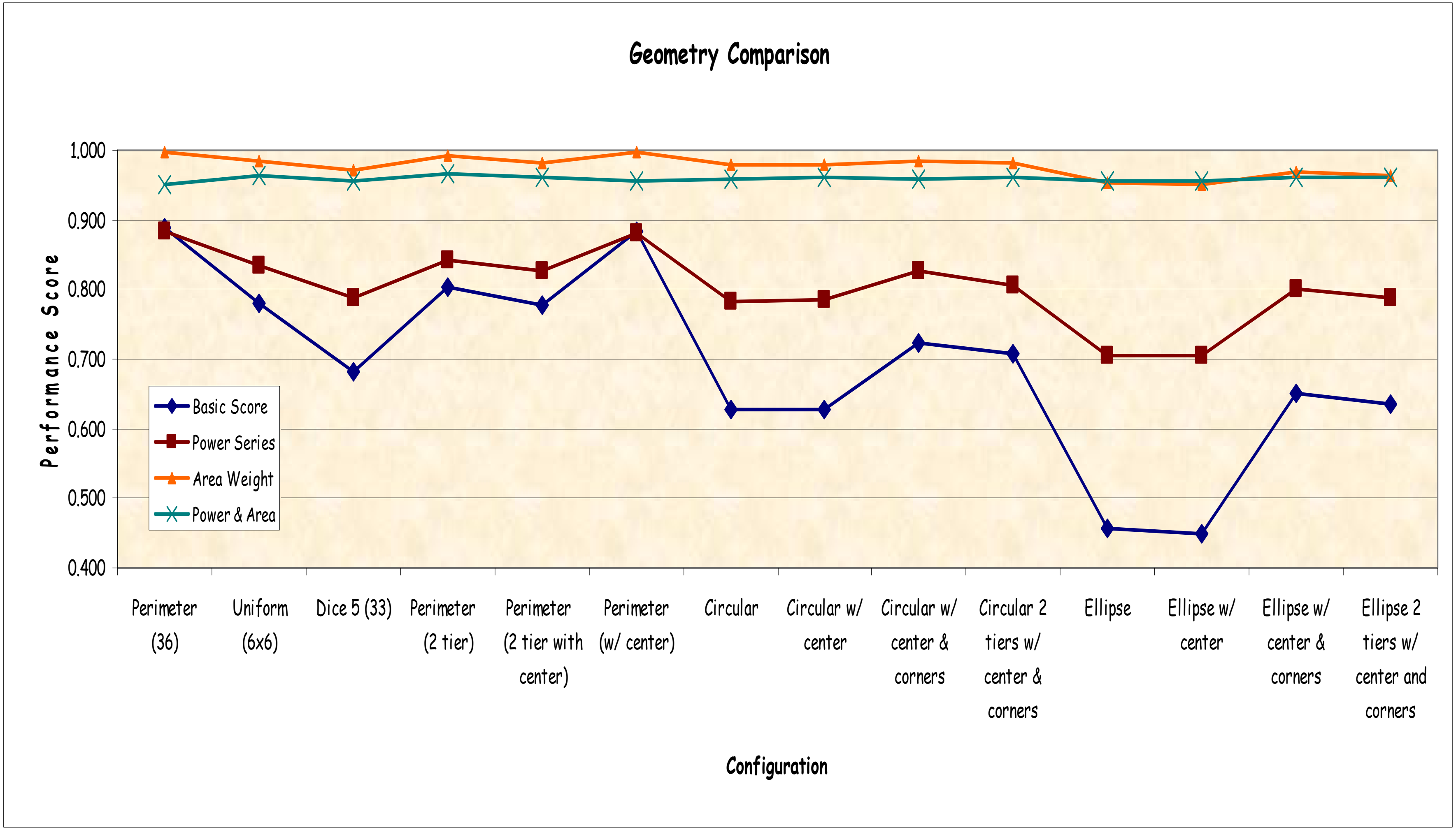

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Observations

- Dice 5 configurations offer no advantage over uniform arrays
- Configurations that conform to defended region “work better” than configurations that don’t conform
- Perimeter geometries and uniform arrays have a crossing point as number of sensors is increased
- Scoring system must take into account tactical motivations, false alarms, forensics, etc.
- Optimization using Tabu search should be able to optimize margin, spacing and number of sensors for a given area, especially with warm start provided by this tool

Future Areas for Study

- Optimization of sensor placement
  - Spacing (wind), geometry (spiral), margin, number, cost, performance
  - More realistic sensor performance/ Mixed sensitivity
- Chemical versus Bio plume size consideration
  - Topology, terrain, day/night, etc.
- Quantitative specification of perimeter/uniform cross-over point
- Non-rectangular defended regions