



Multi-Objective Optimization Methods for Optimal Funding Allocations to Mitigate Chemical and Biological Attacks

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Outline



- Introduction**

- MIDST: Exploration Mode**

- MIDST: Optimization mode**

- Alterative Optimization Methods**

- Case study**

- Conclusions**



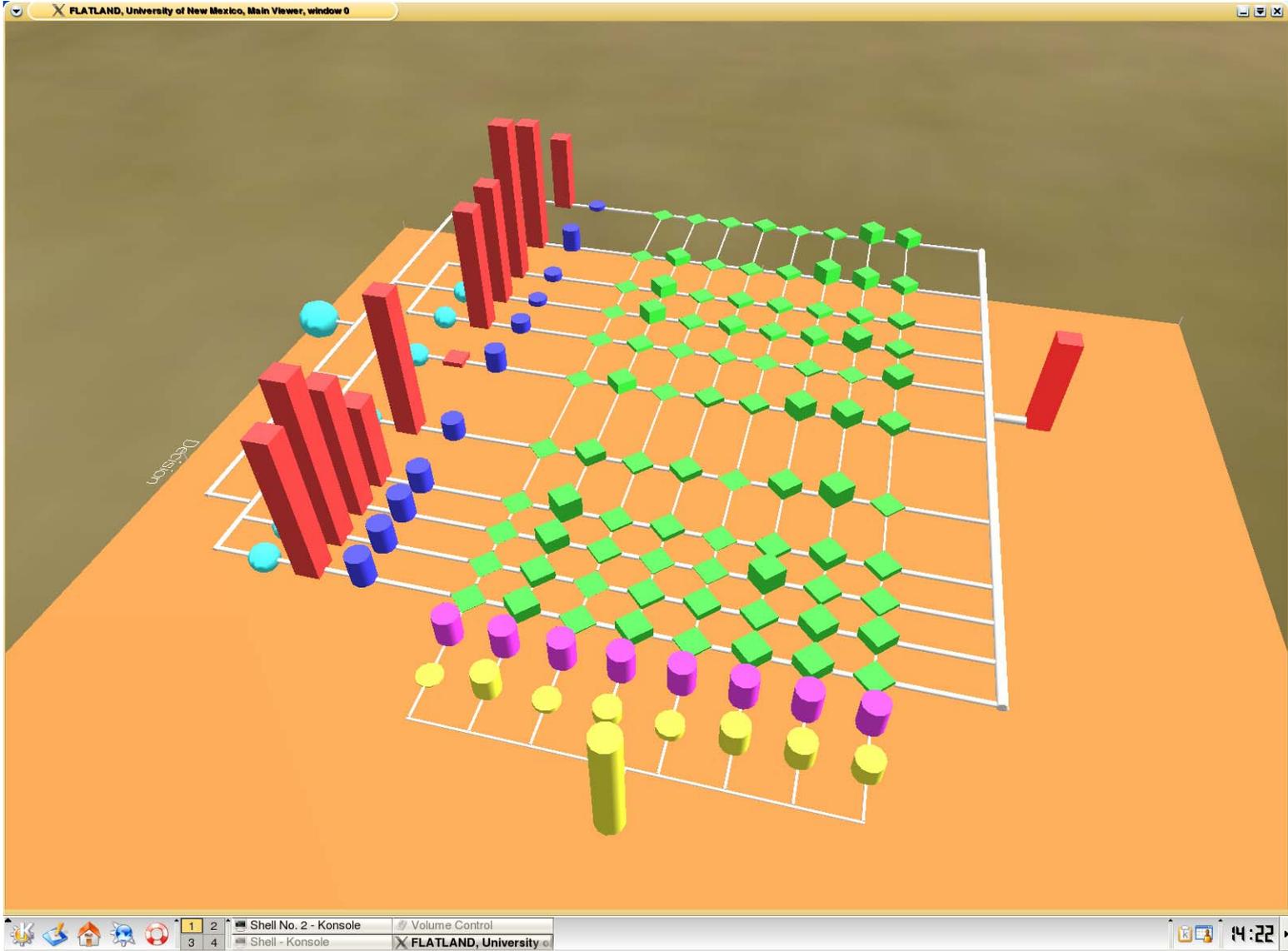
Introduction

□ MIDST: Problem Statement

What is the *optimal* budget $\$B$ and its distribution to N *investment units* in order to reduce the consequences of S *number of CB events*?



Introduction

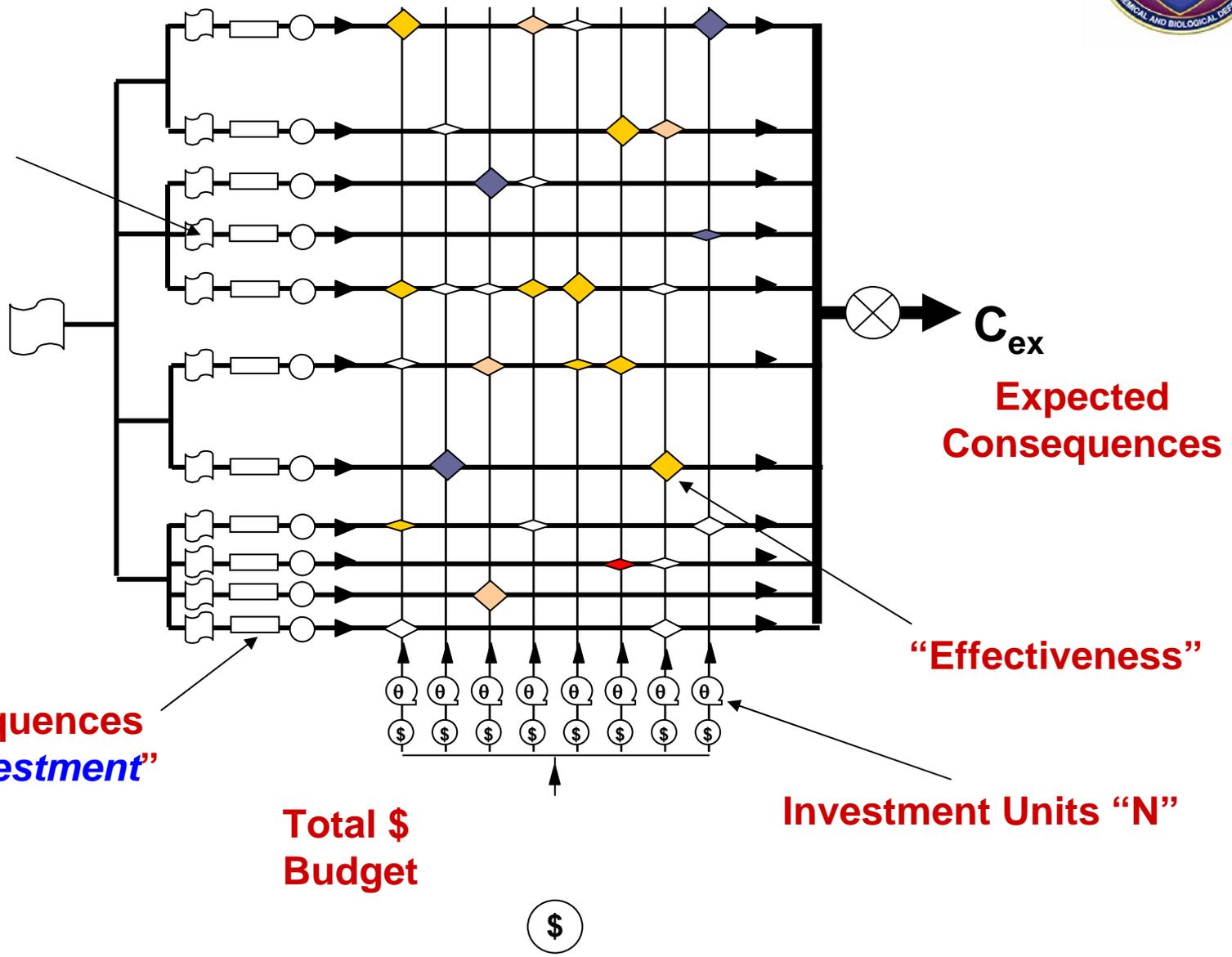




Introduction

All possible individual CB events

CB event Class



Consequences "No Investment"

Total \$ Budget

Investment Units "N"

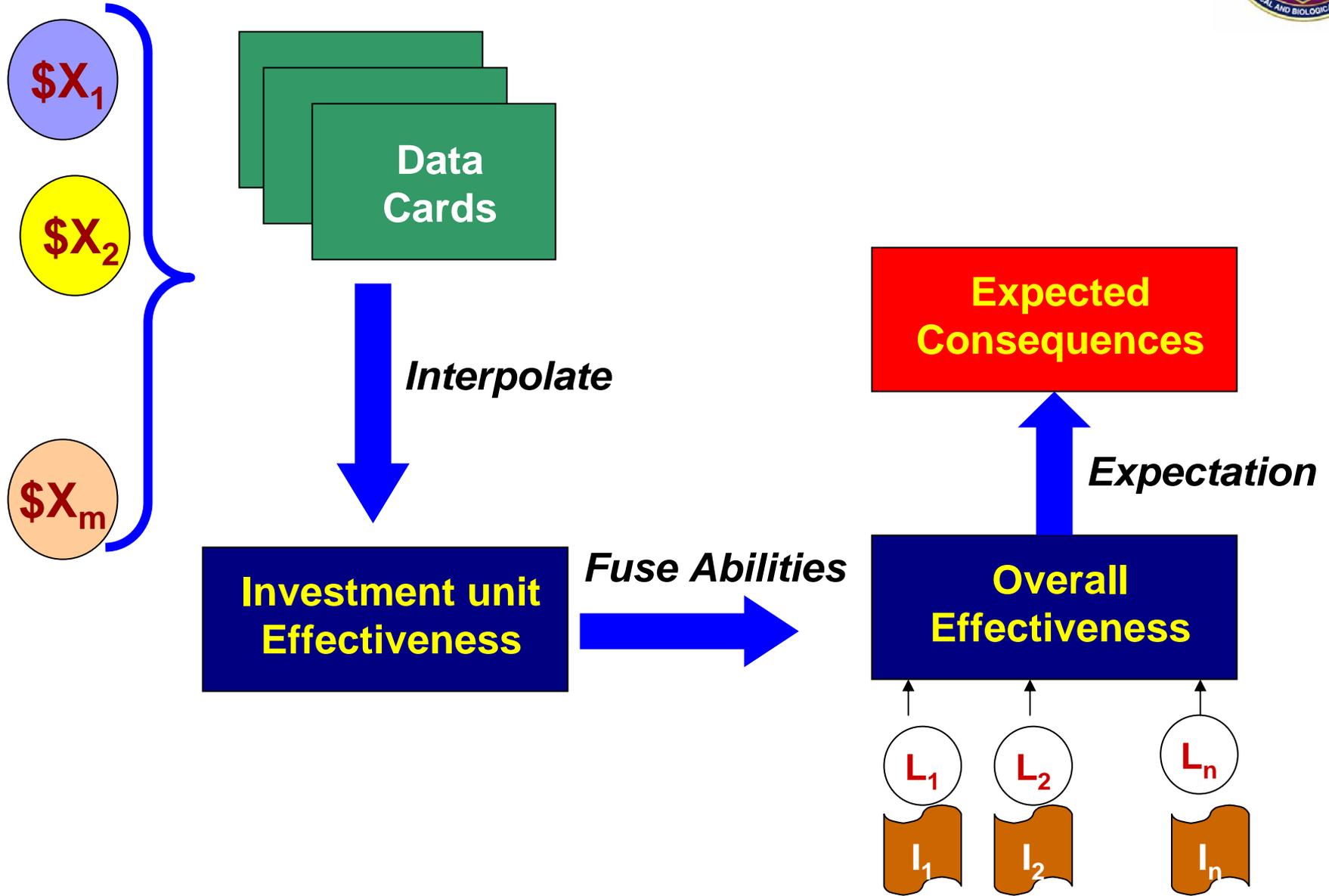
C_{ex}
Expected Consequences

"Effectiveness"





MIDST: Exploration Mode





Soliciting Information: Data Cards

DTRA DS- Home Page - Mozilla Firefox

File Edit View Go Bookmarks Tools Help

http://sandia.nmsu.edu/DTRA/

Getting Started Latest Headlines

DTRA DS-Control Page DTRA DS- Home Page

Decision Support Table-Top Exercise | CHEM-BIO DEFENSE

HOME EVENTS INVESTMENT AREAS REMEDIATION EFFECTS CONTROL OUTCOMES LOGIN



Is an ounce of prevention worth a pound of cure?

Login
Medical: Pretreatments

Password:

Done

Steps in the Exercise

Participants in the exercise will create online cards

Event selection – Ten Attack Event Cards are selected for the exercise. The exercise participant selecting these events can set the initial probability of each event. In the basic exercise, the ten event cards are fixed, but in more advanced versions of the exercise, exercise participants will be able to create custom event cards.

Capability selection – Capability cards for each mitigating technology are created. The exercise participant will create a capability card that describes a capability to be provided through the development of a new technology. Exercise participants will be able to specify three levels of funding: 1) a minimum funding level below which the technology could not be developed at all, 2) a maximum funding level over which additional funding would bring diminishing returns, 3) an optimal or planned funding level taking into account budget constraints.

Remediation assessment – Exercise participants predict the impact each capability will have on each event consequence. Exercise participants will estimate new consequence outcomes at each 10-year funding level, adjusting the funding levels if necessary and estimating the relationship between costs and consequences.

Outcome assessment – Exercise participants will run the exercise simulation which computes consequence outcomes for all events given the capability cards played. Exercise participants can look for capability programs that have the largest and/or least impact. At this point interactive adjustment to some of the



Soliciting Information: Data Cards

http://sandia.nmsu.edu/DTRA/remediation.php

DTRA DS-Control Page DTRA DS- Remediation Effects

Predict outcome given Vaccines for bacterial agents (anthrax) remediation is in place:

Funding Level	Casualties	Days to recover	Mission Disruption
Event Baseline	5%	30	50%
Threshold (\$20M)	Highly effective	Highly effective	Highly effective
Productive (\$37M)	Highly effective	Highly effective	Highly effective
Optimal (\$50M)	Completely effective	Completely effective	Completely effective

Predict outcome given Vaccines for bacterial agents (plague) remediation is in place:

Funding Level	Casualties	Days to recover	Mission Disruption
Event Baseline	5%	30	50%
Threshold (\$30M)	Not effective	Not effective	Not effective
Productive (\$50M)	Not effective	Not effective	Not effective
Optimal (\$60M)	Not effective	Not effective	Not effective

Predict outcome given Nerve Agent bioscavengers remediation is in place:

Funding Level	Casualties	Days to recover	Mission Disruption
Event Baseline	5%	30	50%
Threshold (\$166M)	Not effective	Not effective	Not effective
Productive (\$166M)	Not effective	Not effective	Not effective
Optimal (\$250M)	Not effective	Not effective	Not effective

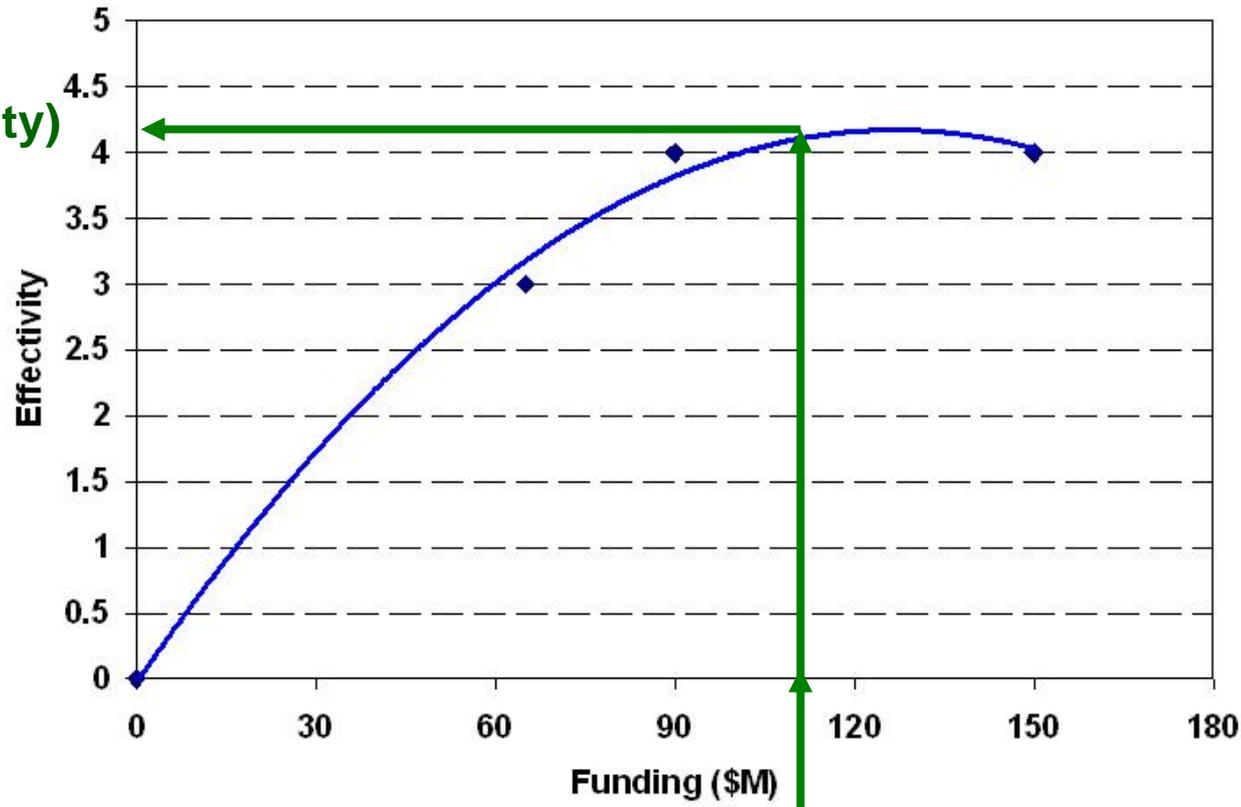
Predict outcome given Multiagent Vaccines remediation is in place:

Funding Level	Casualties	Days to recover	Mission Disruption
Event Baseline	5%	30	50%



Establishing effectivity function

- ❑ Polynomial or *spline* interpolation
- ❑ *Multivariate* interpolation (See *Prasad et al.* tomorrow!)



\$X Funding



Establishing effectivity function

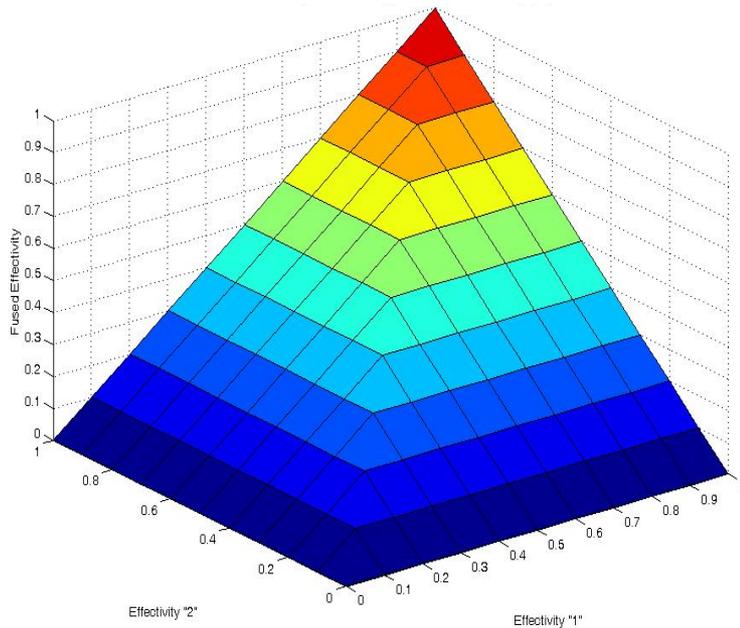
□ Using this method we establish the matrix of effectivity

$$\bar{e} = \left\{ \begin{array}{ccccc} e_{1,1} & e_{1,2} & \dots e_{1,i} & \dots & e_{1,N} \\ e_{2,1} & e_{2,2} & \dots e_{2,i} & \dots & e_{2,N} \\ e_{m,1} & e_{m,2} & \dots e_{m,i} & \dots & e_{m,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e_{S,1} & e_{S,2} & e_{S,i} & \dots & e_{S,N} \end{array} \right\}$$

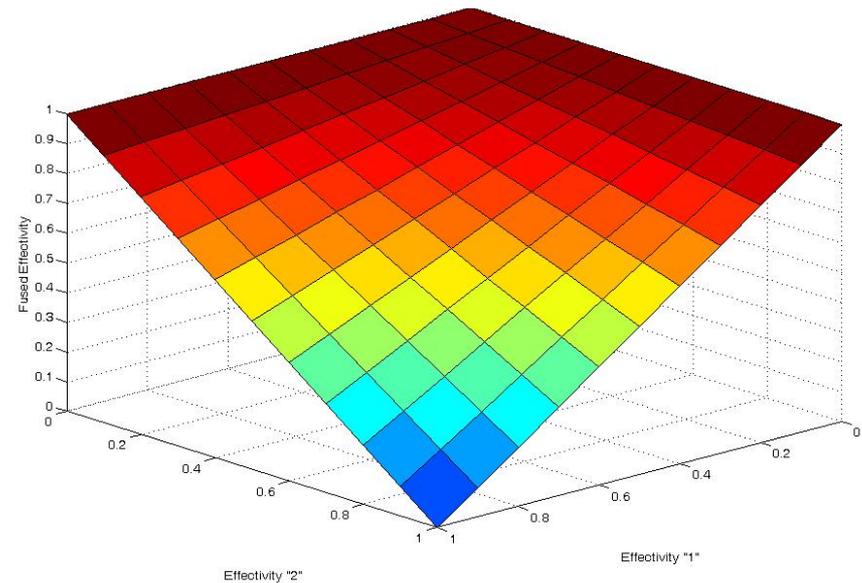
For N: investment units and S: CB events

Fusing Effectivities

- Considering the *interaction between IUs* on the final consequences we have to fuse these effectivities
- Many fusion operators exist. *Example 2D fusion:*



Very conservative



Very optimistic



Expected Consequences

□ The fusion operation results in

$$\hat{e}^{fN} = \left\{ e_1^{fN}, e_2^{fN}, e_3^{fN} \dots e_m^{fN} \dots e_S^{fN} \right\}$$

For S: CB events

□ The expected consequence for each CB event can be computed as

$$\overline{C}_m^k = \left(1 - \hat{e}_m^k \right) \overline{C}_m^0$$

For each CB event

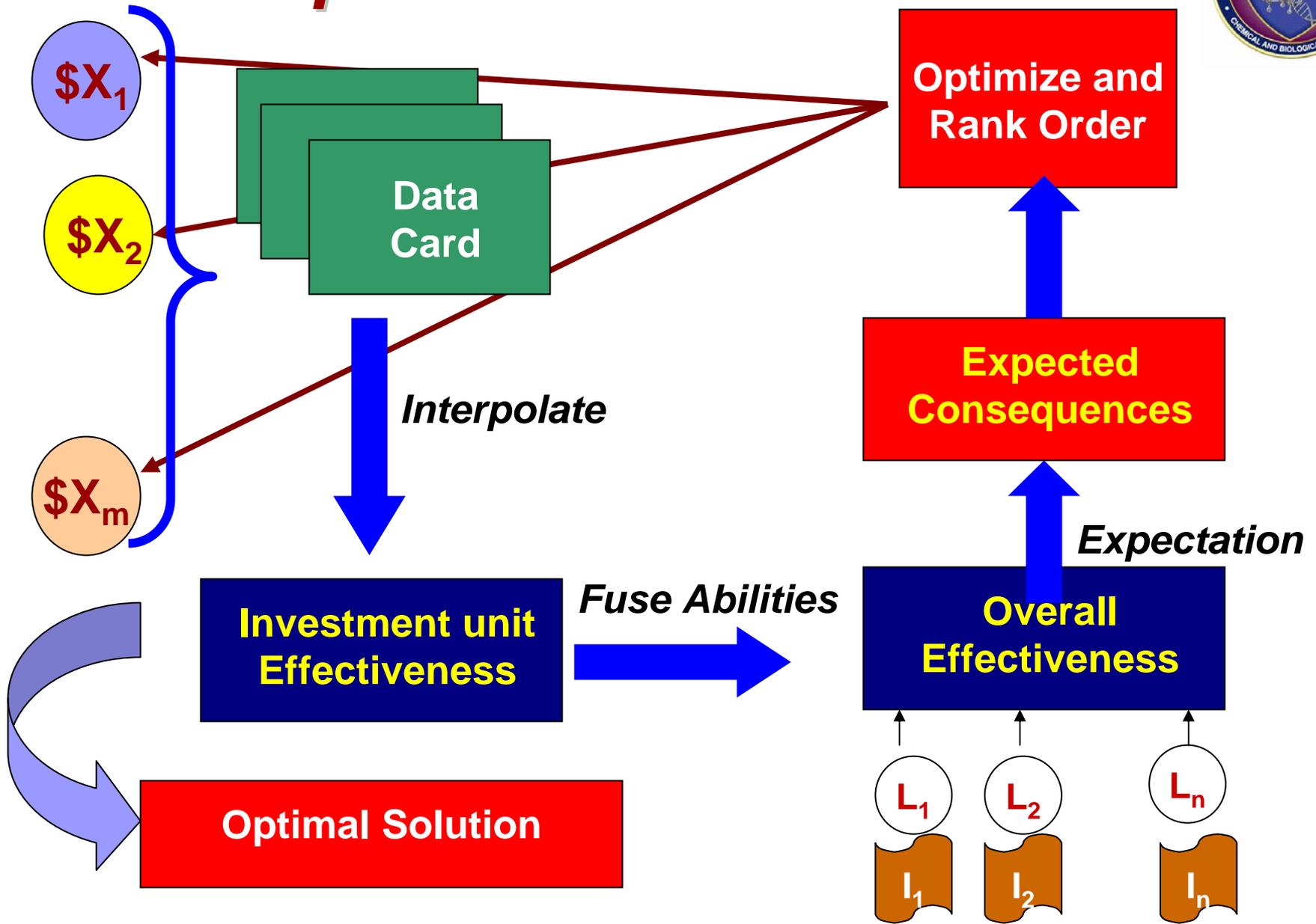
□ Considering the likelihoods of the CB events we can compute the overall expected consequences as

$$\overline{C}^k = \sum_{m=1}^S L_m \overline{C}_m^k$$

Vector of consequences at \$k investment



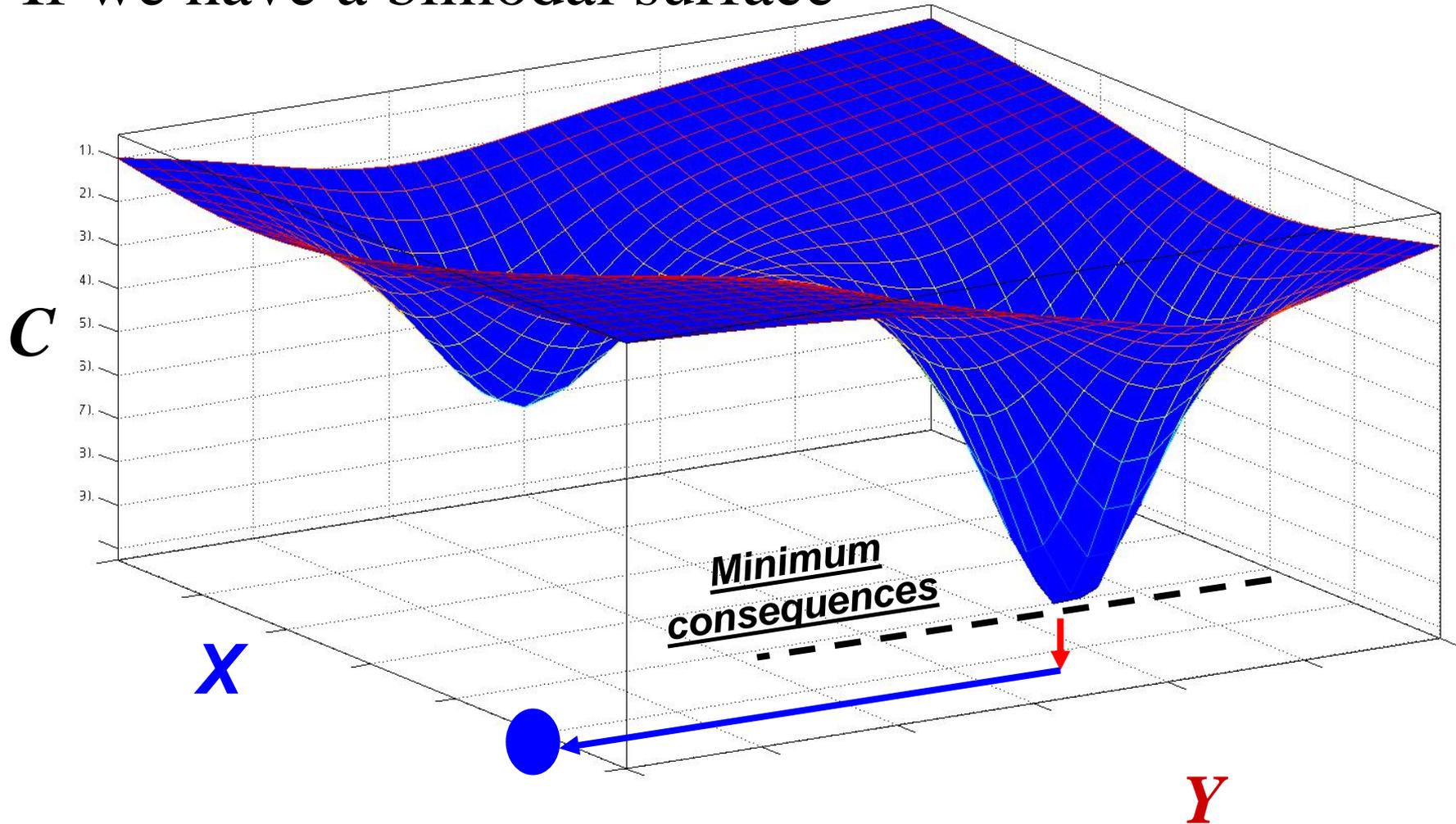
MIDST: Optimization Mode





What does optimization mean?

If we have a bimodal surface



We need to *identify* “ x ” that results in *minimum* “ C ”



Our optimization challenges are

- The surface of our *function is not bimodal*
- There *might be many local minima*
- There is *more than one objective* and *they are not necessary achievable all together*
- *Computing time, space* and accuracy resolution
- Practical interests



Methods

- To address the risk associated with the previously listed concerns/challenges, a group of optimization methods was examined

- ***Derivative based optimization***

- Gradient descent method
- Levenberg Marquardt
- Many other

- ***Non-derivative based optimization***

- Genetic algorithms
- Simulated annealing
- Many other



Derivative-free optimization

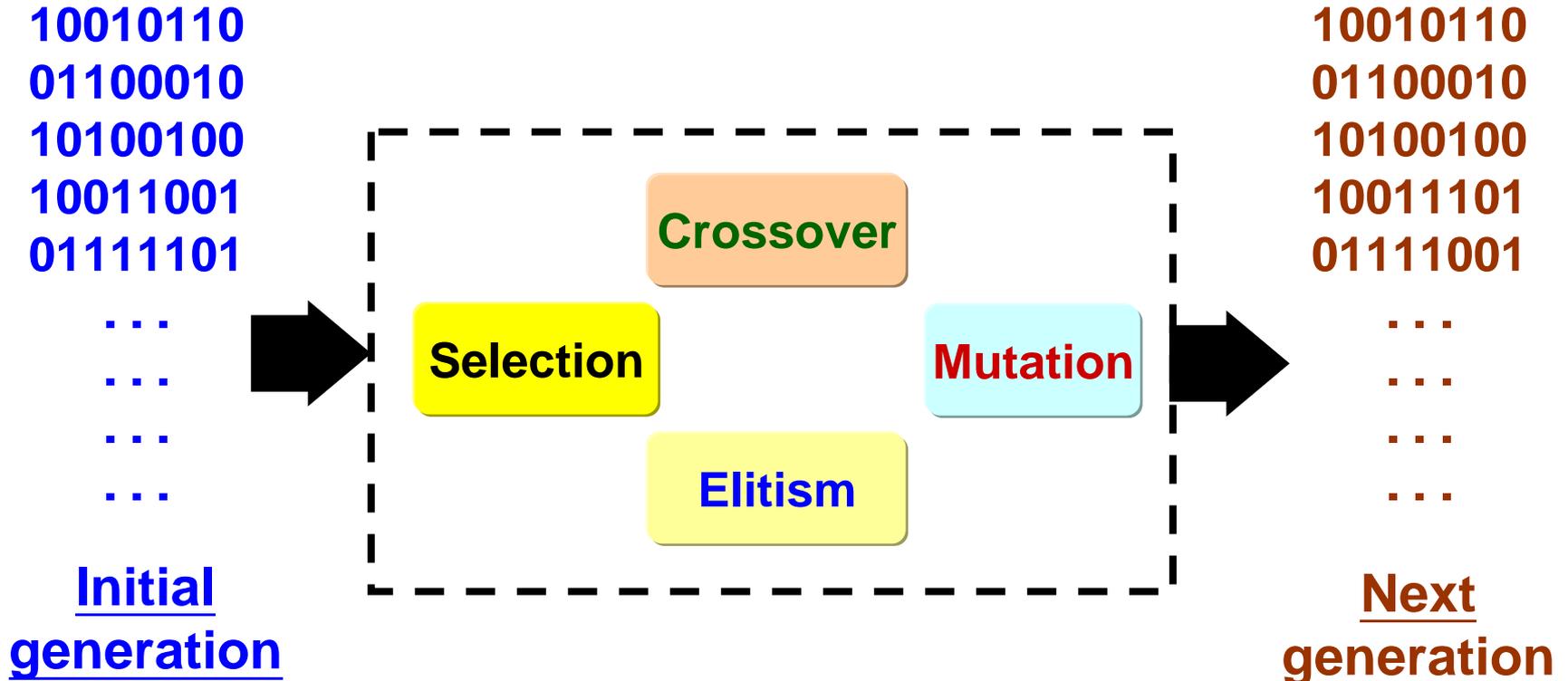
Genetic Algorithms (GA) mimics laws of ***Natural Evolution*** which emphasizes “***survival of the fittest***”.



In GA a “***population***” that contains different possible solutions to the problem is created.



Genetic Algorithms



The process is repeated until *evolution happens*
“a solution is found!”



Multi-Objective Optimization

- It is practical to assume that the decision maker might have priorities on the different objectives ***casualties/mission disruption and time to recover.***
- In this case, usually ***there exist more than one optimal solution*** to the problem (Named ***Pareto solution***)
- Based on the preferences, these ***solutions can be rank ordered.***



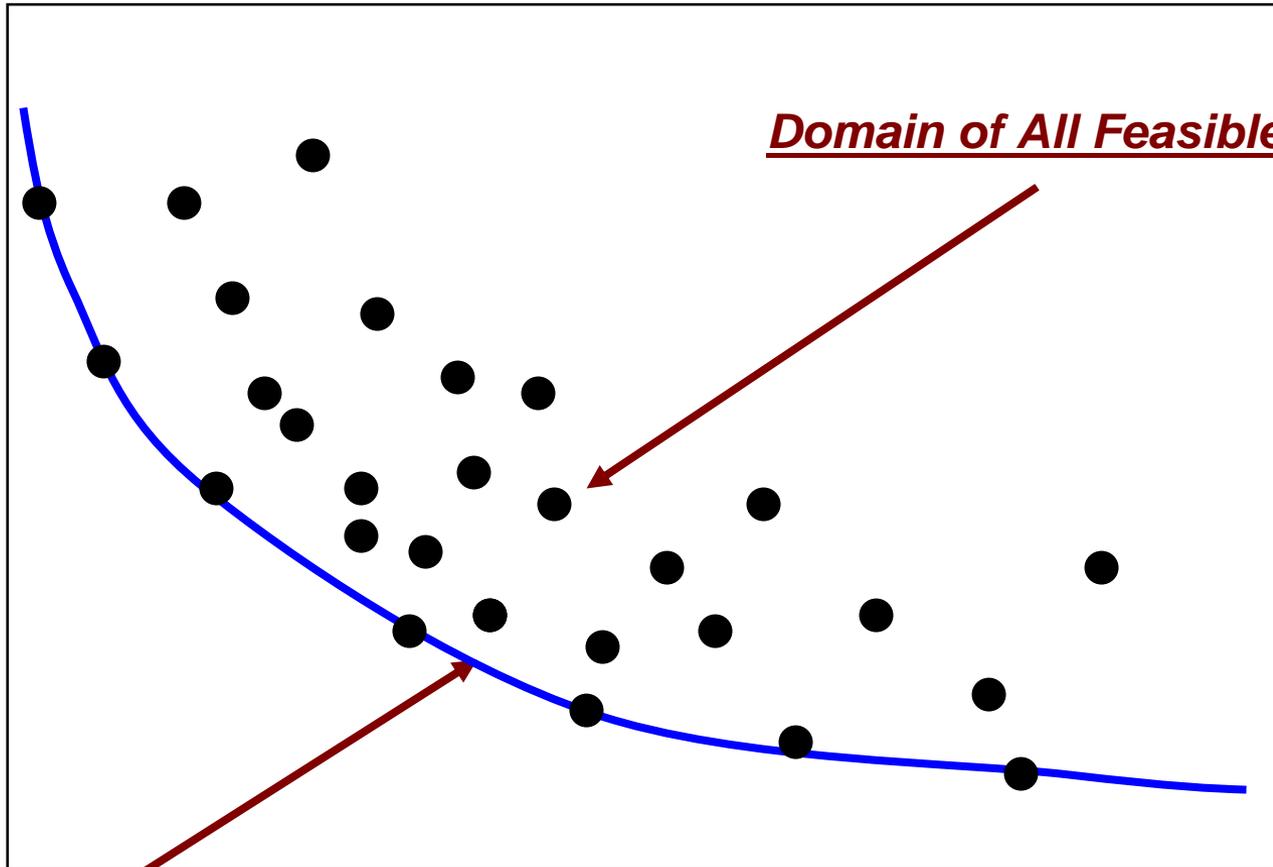
Multi-Objective Optimization

- Three major issues differentiate between single and multi-objective optimizations
 - *Multiple (three) goals instead of one*
 - *Dealing with multiple search spaces not one*
 - *Artificial fixes affect results*
- We are looking for a set of Pareto-optimal solutions



Multi-Objective Optimization

Mission Disruption (Objective 2)



Domain of All Feasible Solutions

Casualties (Objective 1)

Pareto Optimal Solutions



Multi-Objective Optimization Methods

- *Global criteria method*
 - Require *target values for the functions*
 - Can incorporate *weights for preferences*
- *Hierarchical optimization method*
 - Optimize the *top priority function*
 - Specify *constraints* to *prevent deteriorating* the *optimized function*
- *Multi-Objective Genetic Optimization (MOGA)*
 - Non-dominated Sorting Genetic Algorithm



Multi-Objective Optimization

Hierarchical Method

- Rank order the objective functions

$$f_{j-1}(\bar{x}) \leq \left(1 \pm \frac{\Sigma_j - 1}{100} \right) \cdot f_{j-1}(\bar{x}^{j-1})$$

- The $j-1$ function is used as *constraint in optimizing the j^{th}* function.

- Σ_j is a *lexicographic increment %*

- How much error is allowed in losing optimal solution for $(j-1)$ given more optimization in (j)



Multi-Objective Optimization

Global Criterion

- The *threshold vector* is defined by

$$f_i^0 = [f_1^0, f_2^0, f_3^0 \dots f_k^0]$$

$$f(\bar{x}) = \sum_{i=1}^k w_i \left(\frac{f_i^0 - f_i(\bar{x})}{f_i^0} \right)^P$$

P is *integer 1 or 2*

w can also be implemented to represent preferences as *weights*



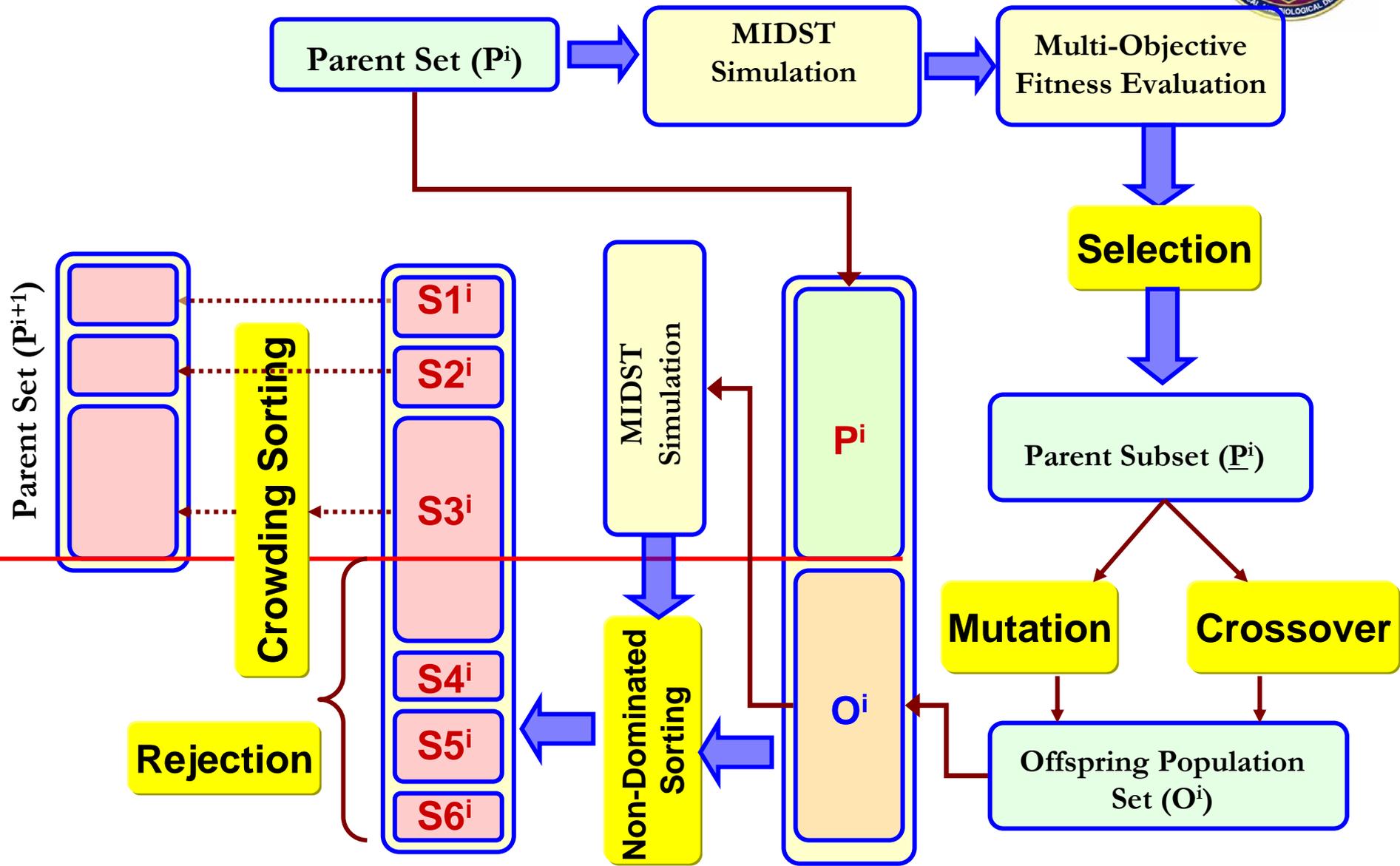
Multi-Objective Optimization

Non-dominated Sorting Genetic Algorithm (NSGA)

- While similar to GA, NSGA *sorts the population* according to *non-domination principles*.
- Population is classified into *a number of mutually exclusive classes*
- Highest fitness is assigned to *class* that are *closest to the Pareto-optimal front*
- The use of non-dominated sorting *allows diversity to solutions* and thus *guarantees reaching the Pareto-front*.
- ***NSGA*** also includes *elitism principles* which allows it to find higher number of Pareto-solutions.



NSGA-II





Merits and shortcomings

- Derivative based

- If the *space is continuum*, it converges very fast and an optimal solution is guaranteed
- If too many *local minima exist*, the algorithm might be *trapped* and *cannot find global minima*

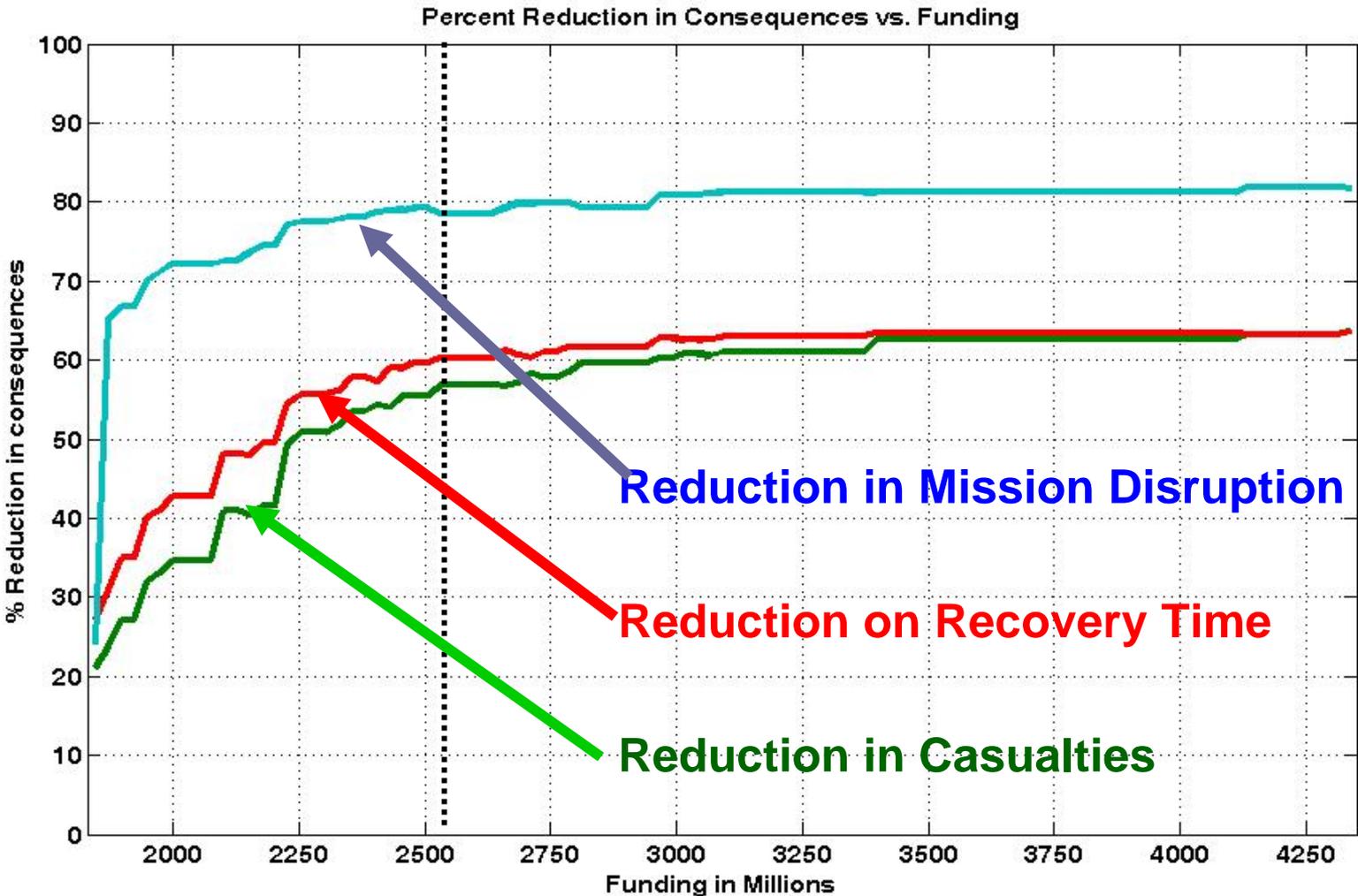
- Non-derivative based

- If the *space is non-continuum*, GA will be able to find the solution
- Whether *local minima exist or not*, it will converge.
- GA is *better equipped with some aiding optimization* technique to narrow search domain



Case study

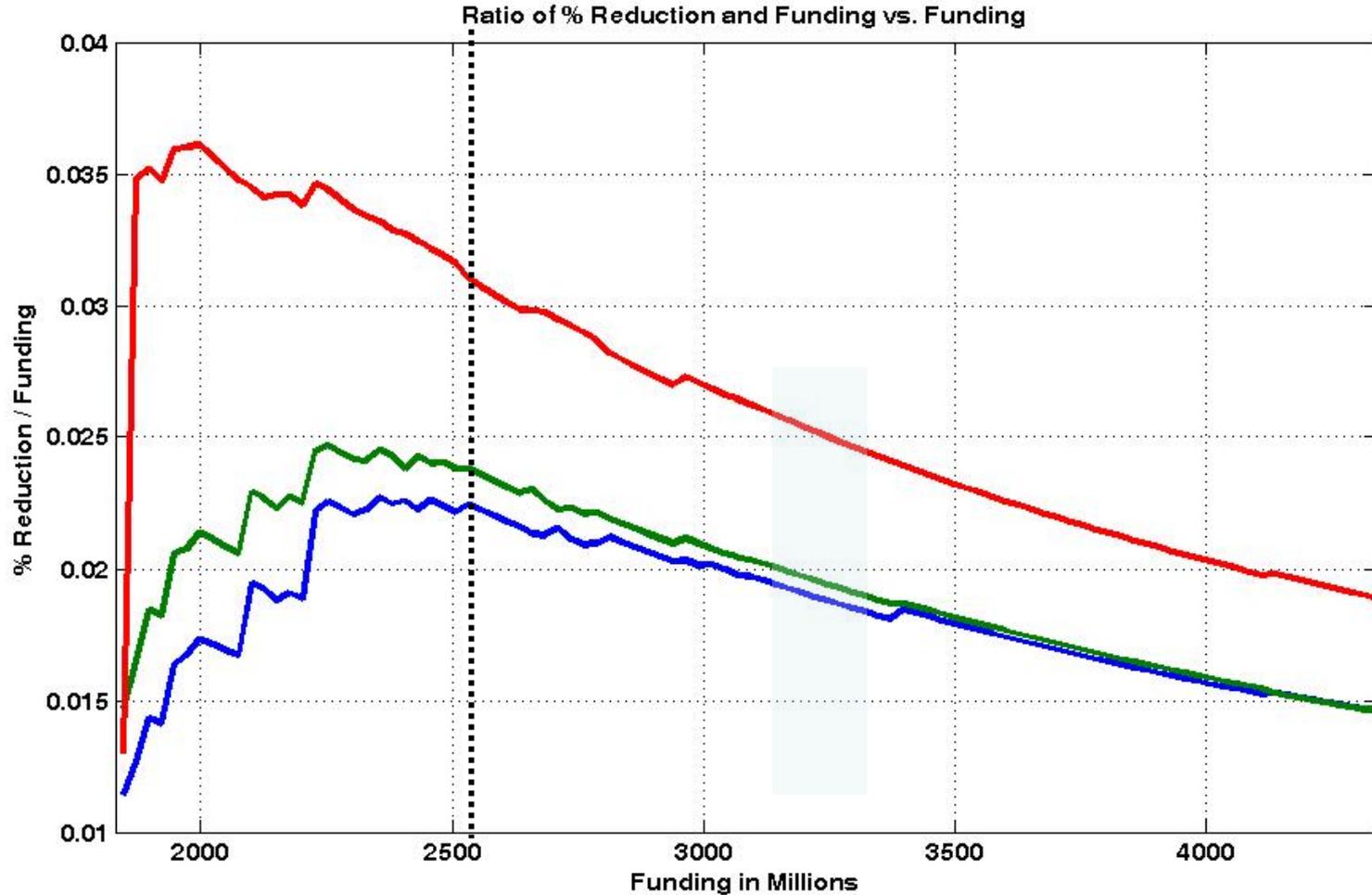
- For a given group of data cards and inputs we identified





Case study

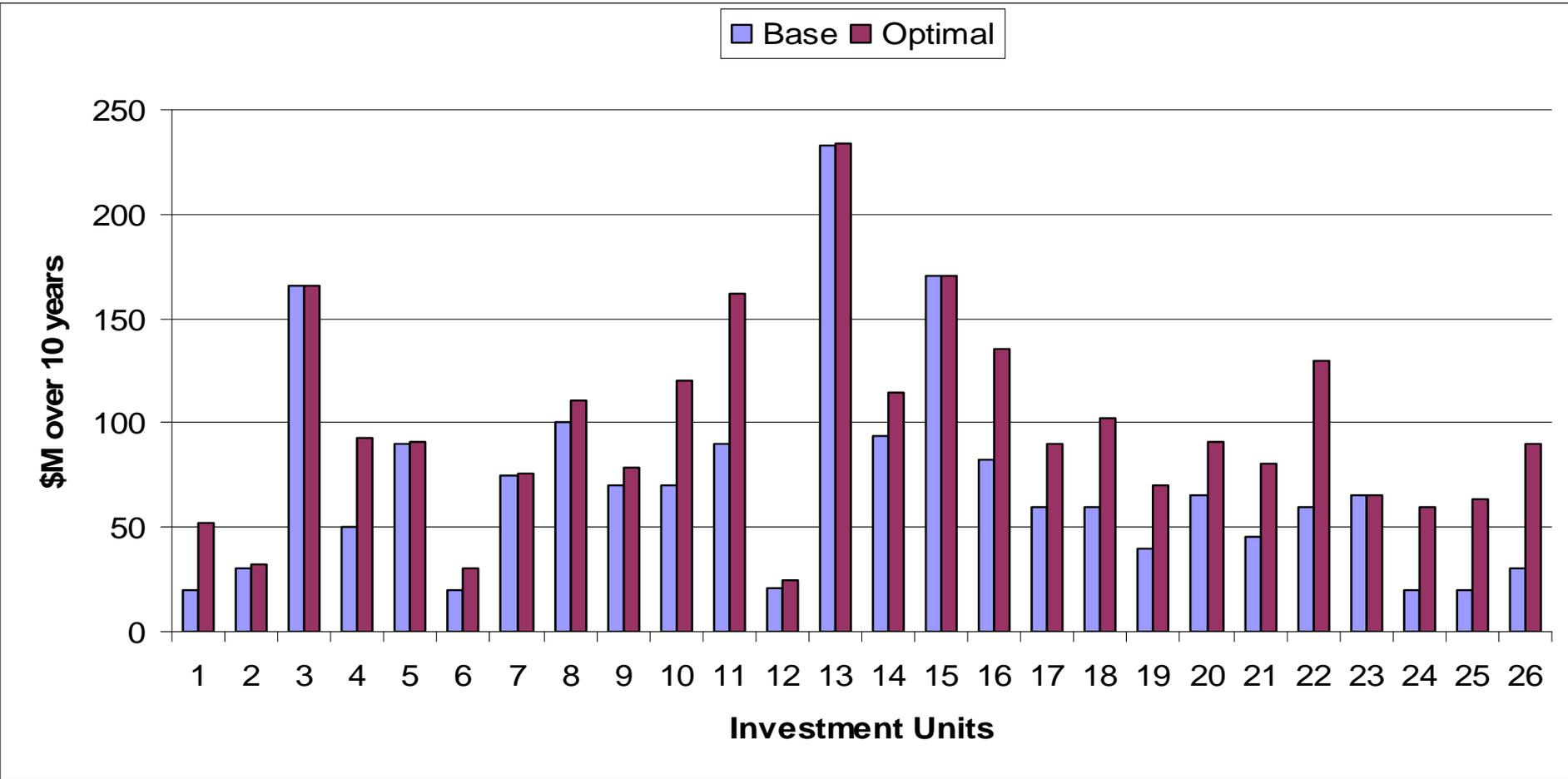
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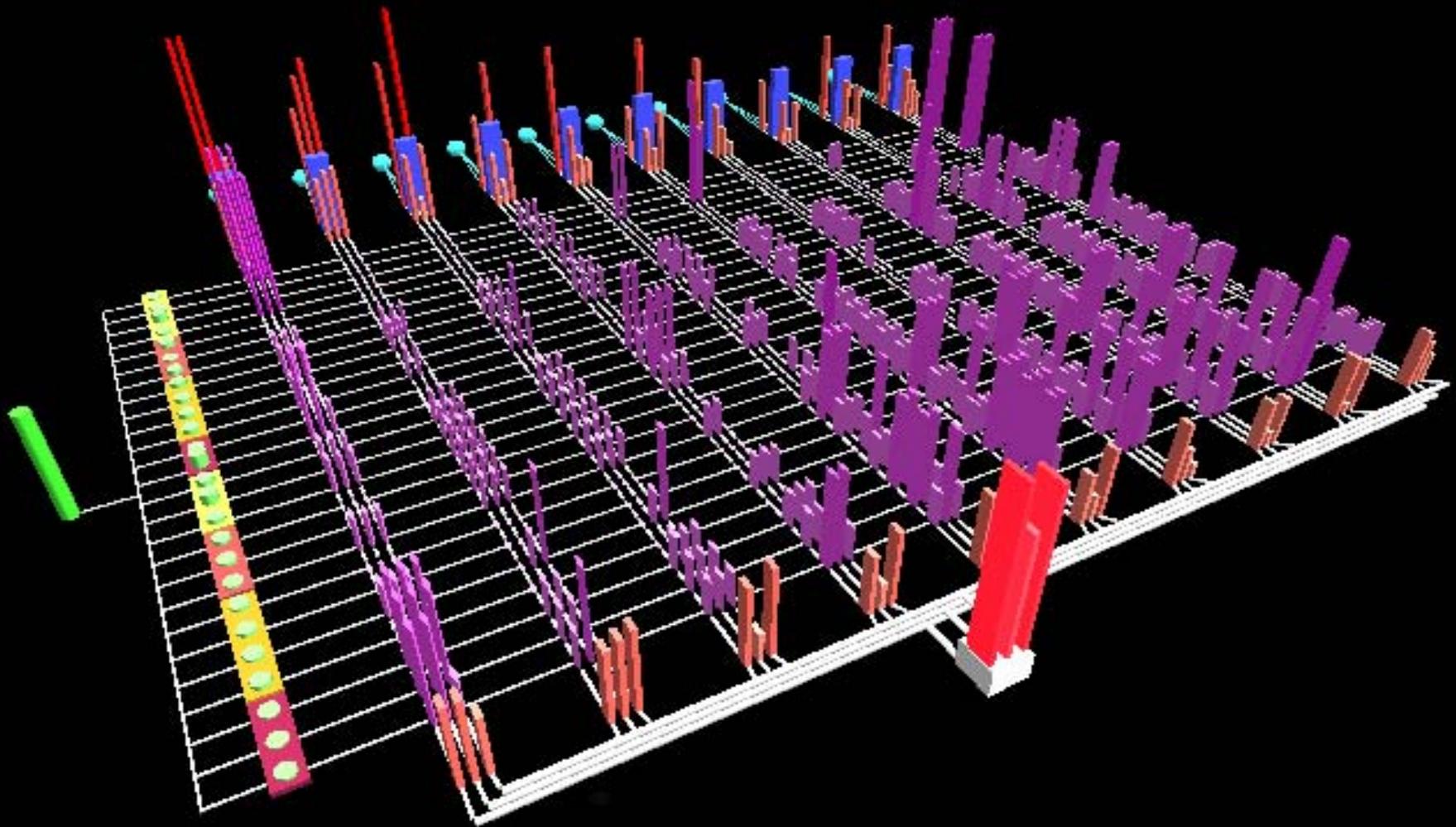




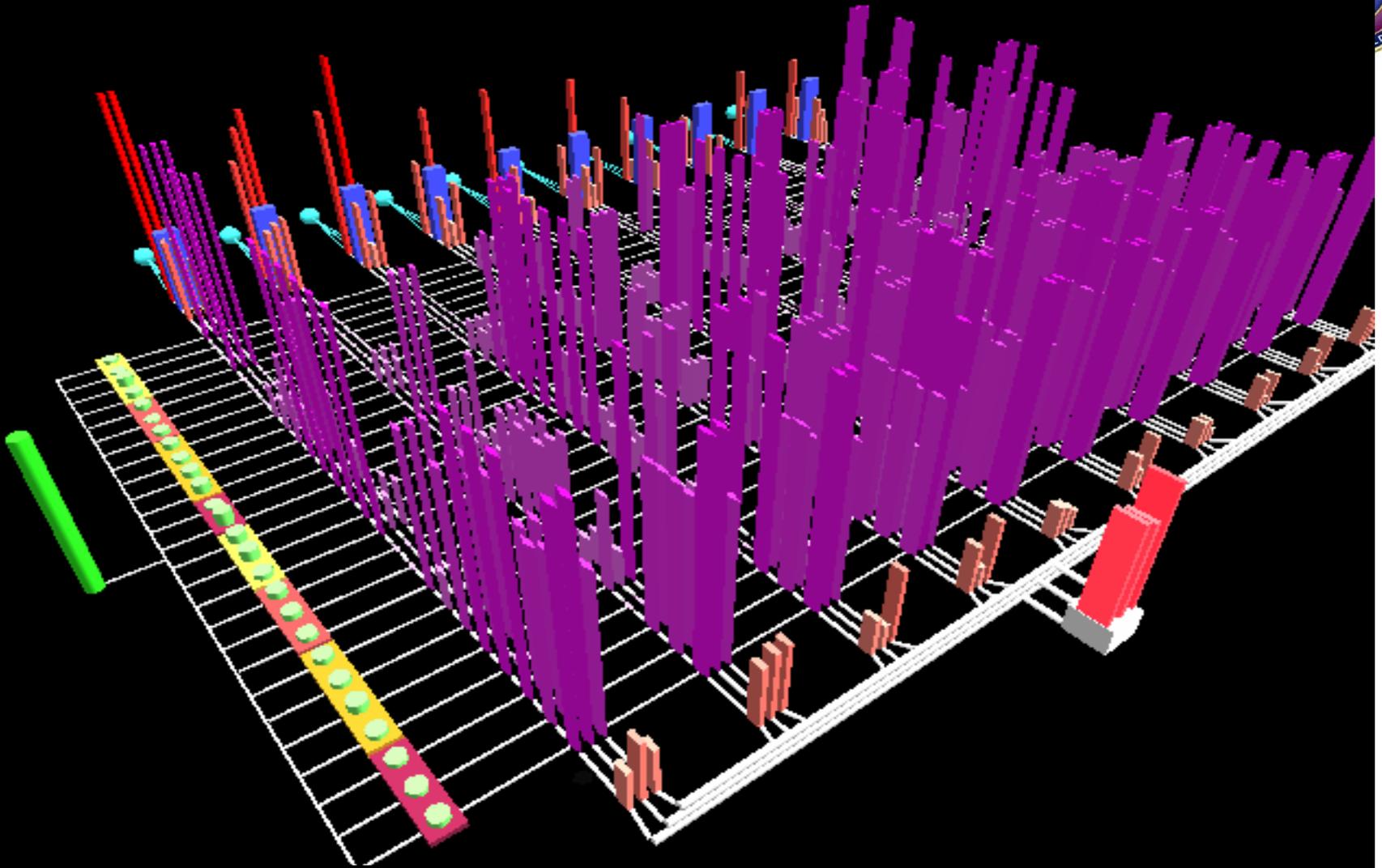
Case study

- At the optimal level, we can identify the funding portfolio





Portfolio for Base Funding $C^1 = 21$, $C^2 = 21$, $C^3 = 42$



Portfolio for Optimal Funding $C^1 = 11$, $C^2 = 12$, $C^3 = 12$



Conclusions

- We demonstrated the possible use of multi-objective genetic optimization for allocation of funding for investment units to reduce consequences of CB events
 - Classical gradient based versus gradient free optimization techniques have been examined in search for Pareto solutions
 - The presented work is part of MIDST: A robust mathematical framework that can be used to help decision makers for funding allocations considering multiple objectives and priorities
- Research is currently on-going to integrate fuzzy rank ordering module as part of the optimization process.



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Questions



Derivative-based optimization

Gradient descent method

- Assumes ***continuous*** and ***differentiable function***

$$\theta_{new} = \theta_{old} + \eta G g$$

- g is the derivative of the objective function

$$g(\theta) = \nabla E(\theta) = \left[\frac{\partial E(\theta)}{\partial \theta_1} \quad \frac{\partial E(\theta)}{\partial \theta_2} \quad \dots \quad \frac{\partial E(\theta)}{\partial \theta_n} \right]^T$$

- G is a positive definite matrix
- η is the step size



Derivative-based optimization

Levenberg-Marquardt (LM) method

- A ***modified version of classical Newton's method***. It also assumes ***continuous*** and ***differentiable function***

$$\theta_{new} = \theta_{old} - \eta (H + \lambda I)^{-1} g$$

- g is the gradient, I is the identity matrix, ***λ is some nonnegative value and H is the Hessian matrix***

$$H(\theta) = \nabla^2 E(\theta) = \begin{bmatrix} \frac{\partial^2 E(\theta)}{\partial \theta_1^2} & \frac{\partial^2 E(\theta)}{\partial \theta_2^2} & \dots & \frac{\partial^2 E(\theta)}{\partial \theta_n^2} \end{bmatrix}^T$$

- η is the step size as defined before



Derivative-based optimization

