Multi-Objective Optimization Methods for Optimal Funding Allocations to Mitigate Chemical and Biological Attacks

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Outline

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- ❑ **MIDST: Exploration Mode**
- ❑ **MIDST: Optimization mode**
- ❑ **Alterative Optimization Methods**
- ❑ **Case study**
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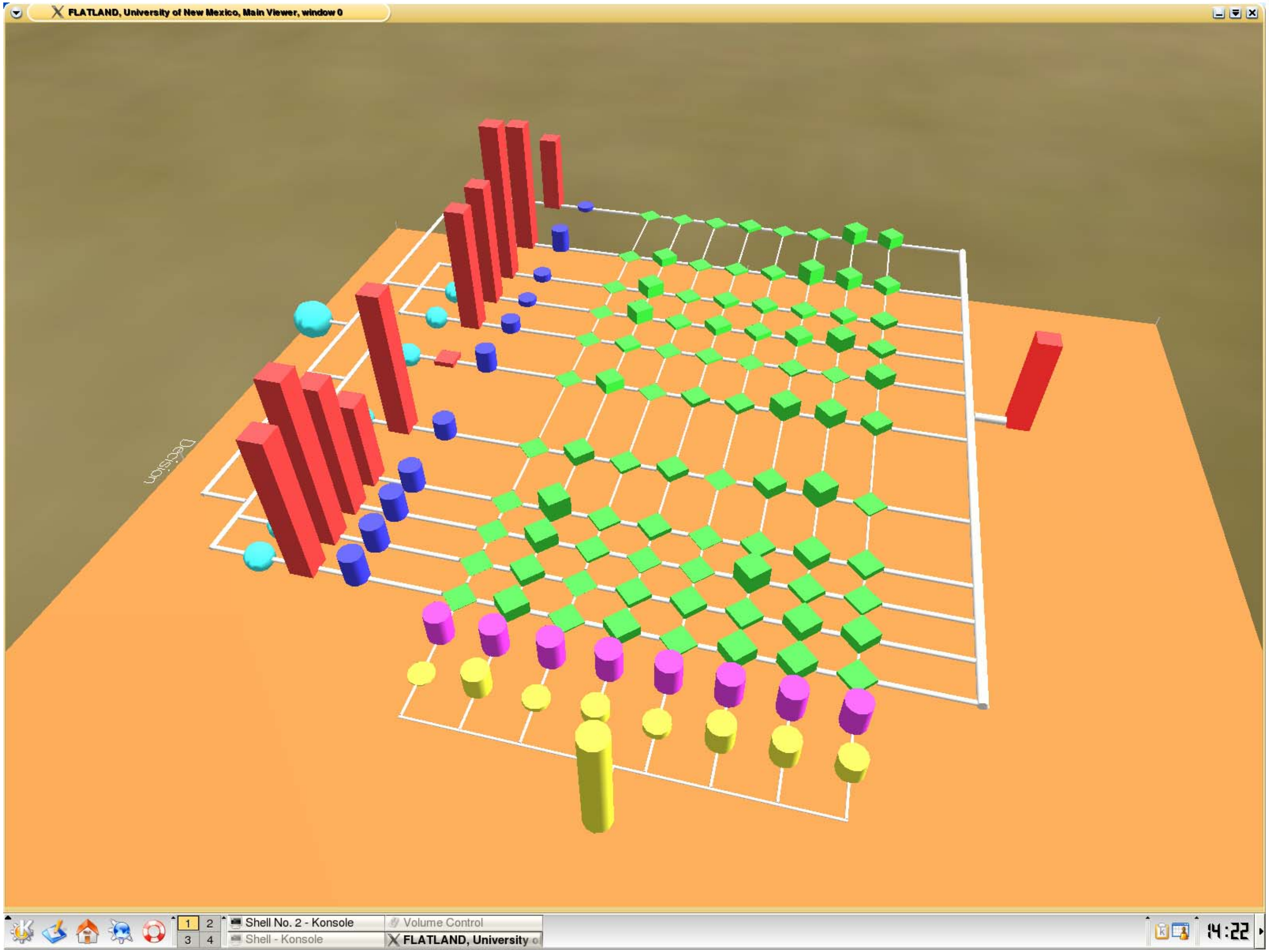
Introduction

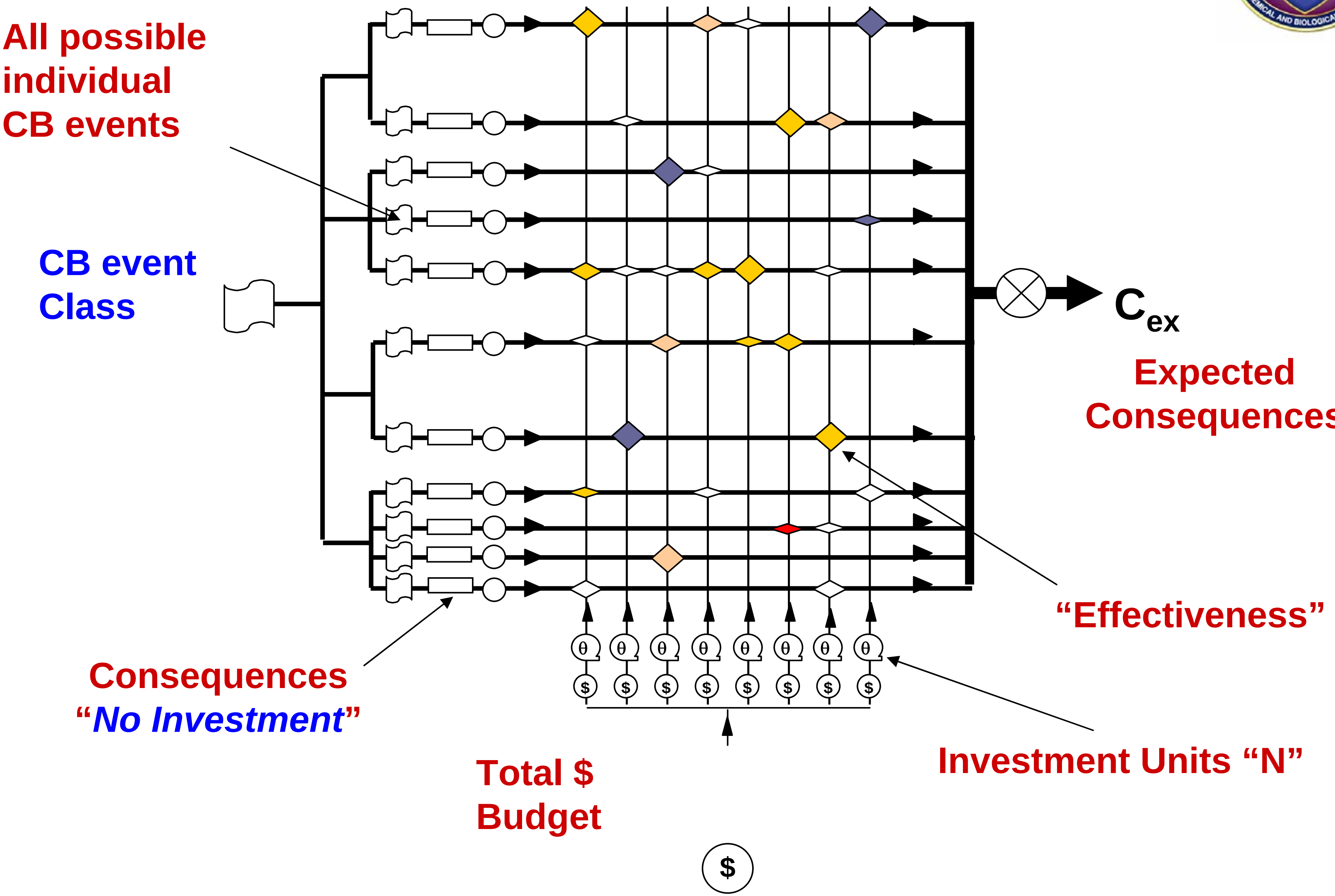
- **MIDST: Problem Statement**

What is the ***optimal*** budget ***$B*** and its distribution to ***N investment units*** in order to reduce the consequences of ***S number of CB events***?

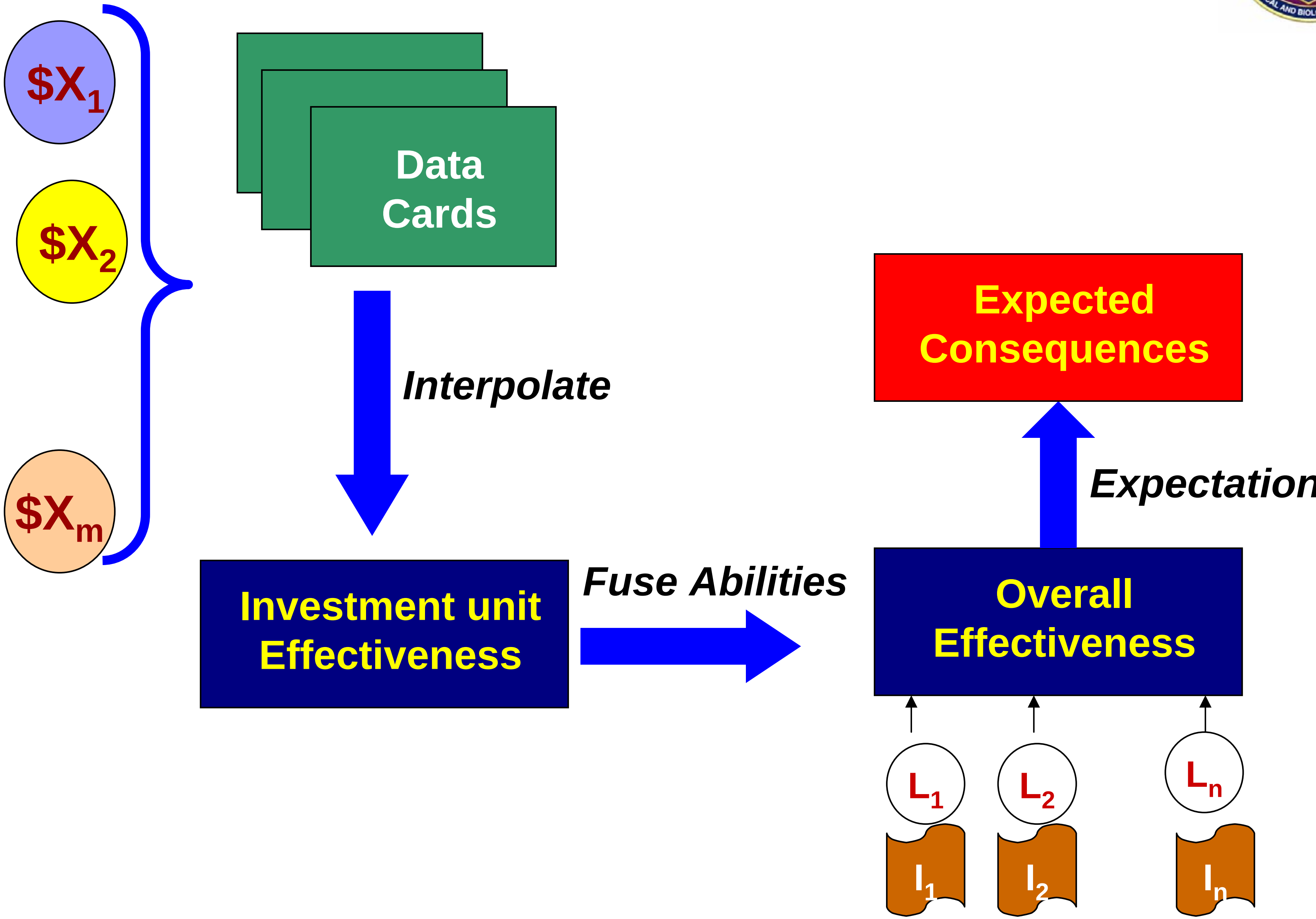
Introduction



Introduction



MIDST: Exploration Mode



Soliciting Information: Data Cards

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http://sandia.nmsu.edu/DTRA/remediation.php

DTRA DS-Control Page | DTRA DS- Remediation Effects

Predict outcome given Vaccines for bacterial agents (anthrax) remediation is in place:

	Effectiveness Prediction		
Funding Level	Casualties	Days to recover	Mission Disruption
Event Baseline	5%	30	50%
Threshold ($20M)	Highly effective	Highly effective	Highly effective
Productive ($37M)	Highly effective	Highly effective	Highly effective
Optimal ($50M)	Completly effective	Completly effective	Completly effective

Predict outcome given Vaccines for bacterial agents (plague) remediation is in place:

	Effectiveness Prediction		
Funding Level	Casualties	Days to recover	Mission Disruption
Event Baseline	5%	30	50%
Threshold ($30M)	Not effective	Not effective	Not effective
Productive ($50M)	Not effective	Not effective	Not effective
Optimal ($60M)	Not effective	Not effective	Not effective

Predict outcome given Nerve Agent bioscavengers remediation is in place:

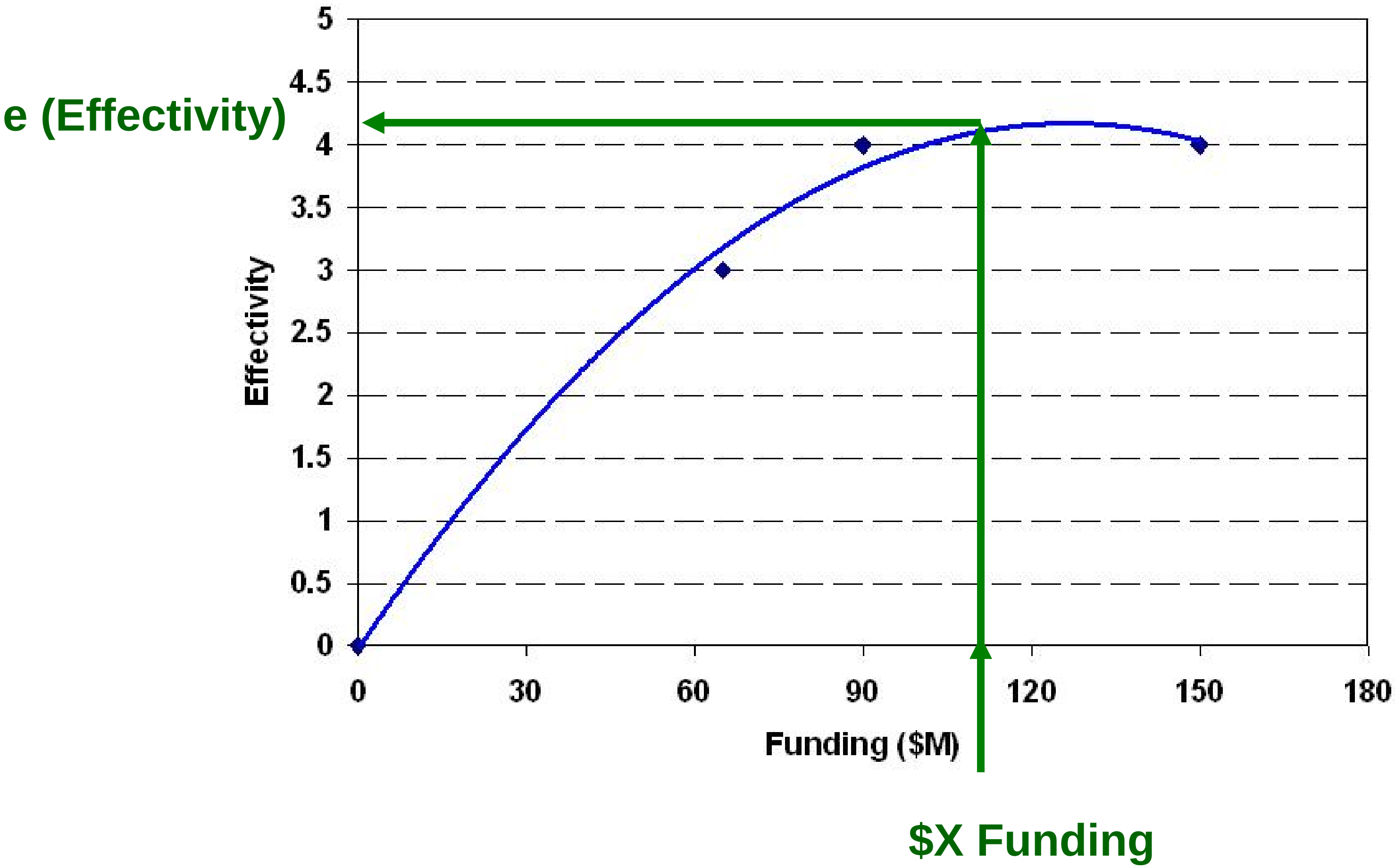
	Effectiveness Prediction		
Funding Level	Casualties	Days to recover	Mission Disruption
Event Baseline	5%	30	50%
Threshold ($166M)	Not effective	Not effective	Not effective
Productive ($166M)	Not effective	Not effective	Not effective
Optimal ($250M)	Not effective	Not effective	Not effective

Predict outcome given Multiagent Vaccines remediation is in place:

	Effectiveness Prediction		
Funding Level	Casualties	Days to recover	Mission Disruption
Event Baseline	5%	30	50%

Establishing effectivity function

- Polynomial or ***spline*** interpolation
- ***Multivariate*** interpolation (See ***Prasad et al.*** tommorow!)



Establishing effectivity function

- Using this method we establish the matrix of effectivity

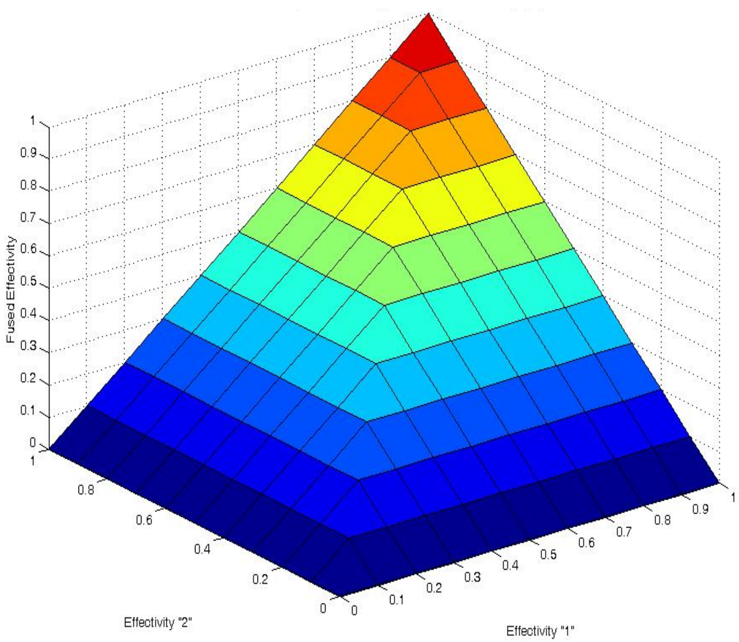
$$\bar{e} = \begin{Bmatrix} e_{1,1} & e_{1,2} & \ldots e_{1,i} & \ldots & e_{1,N} \\ e_{2,1} & e_{2,2} & \ldots e_{2,i} & \ldots & e_{2,N} \\ e_{m,1} & e_{m,2} & \ldots e_{m,i} & \ldots & e_{m,N} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ e_{S,1} & e_{S,2} & e_{S,i} & \ldots & e_{S,N} \end{Bmatrix}$$

For <u>N: investment units</u> and <u>S: CB events</u>

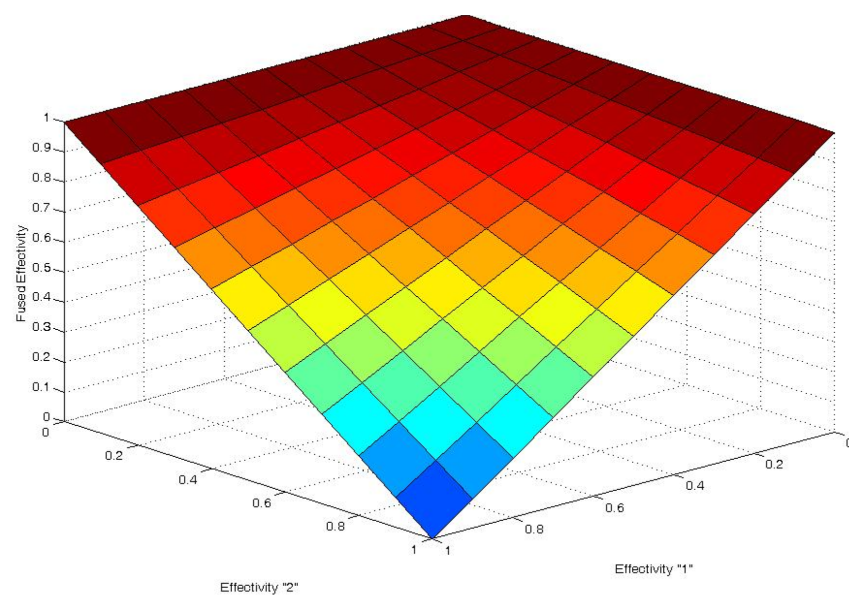
Fusing Effectivities

- Considering the ***interaction between IUs*** on the final consequences we have to fuse these effectivities

- Many fusion operators exist. ***Example 2D fusion:***



Very conservative



Very optimistic

Expected Consequences

- The fusion operation results in

$$\hat{e}^{fN} = \left\{ e_1^{fN}, e_2^{fN}, e_3^{fN} \ldots e_m^{fN} \ldots e_S^{fN} \right\}$$

For S: CB events

- The expected consequence for each CB event can be computed as

$$\overline{C_m^k} = \left(1 - \hat{e}_m^k\right) \overline{C_m^0}$$

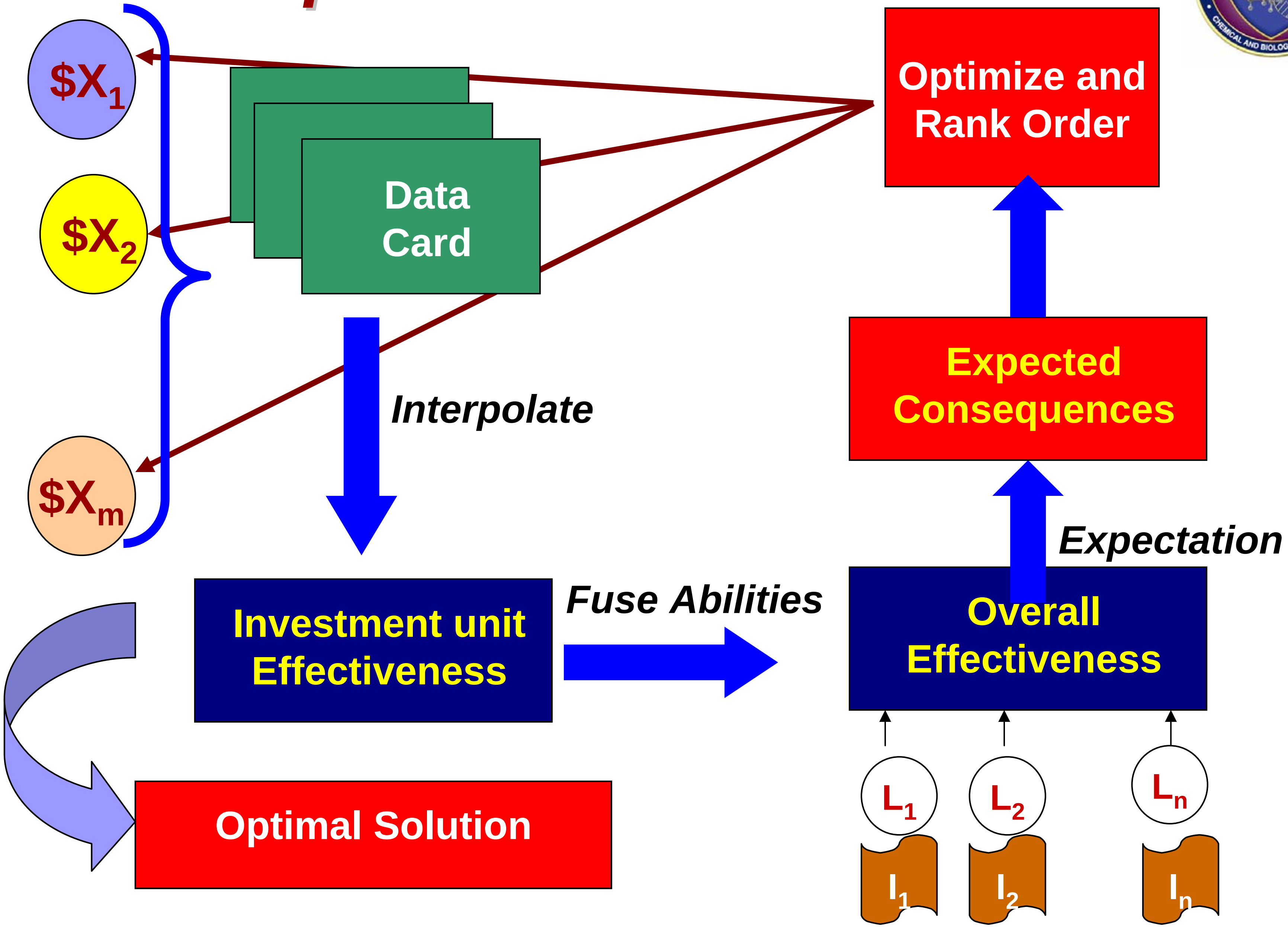
For each CB event

- Considering the likelihoods of the CB events we can compute the overall expected consequences as

$$\overline{C^k} = \sum_{m=1}^{S} L_m \overline{C_m^k}$$

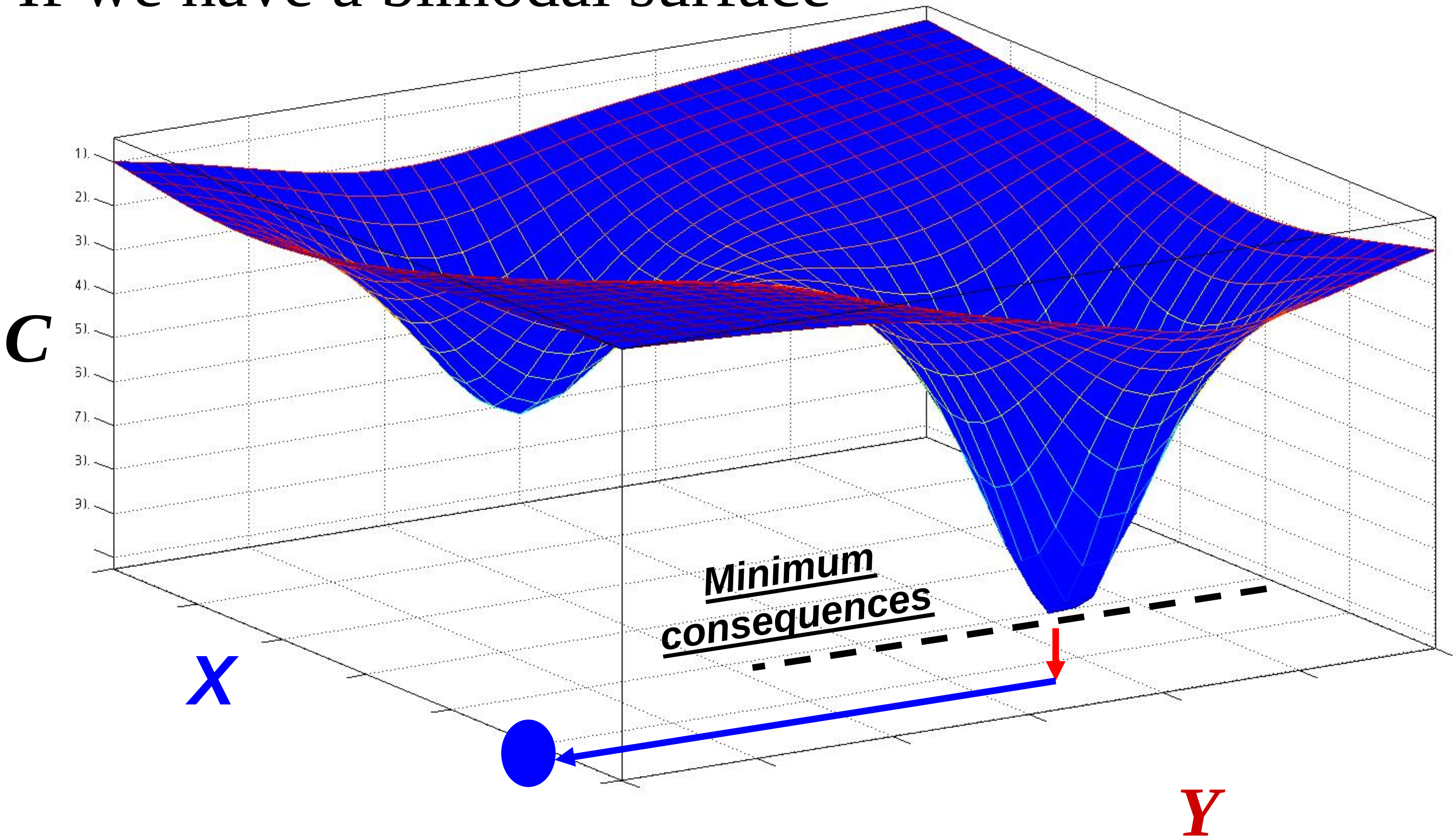
Vector of consequences at $k investment

MIDST: Optimization Mode



What does optimization mean?

If we have a bimodal surface



We need to ***identify* "x"** that results in ***minimum* "C"**

Our optimization challenges are

- The surface of our ***function is not bimodal***

-There ***might be many local minima***

-There is ***more than one objective*** and ***they are not necessary achievable all together***

- ***Computing time, space*** and accuracy resolution

- Practical interests

Methods

- To address the risk associated with the previously listed concerns/challenges, a group of optimization methods was examined

- ***Derivative based optimization***
  - Gradient descent method
  - Levenberg Marquadrt
  - Many other

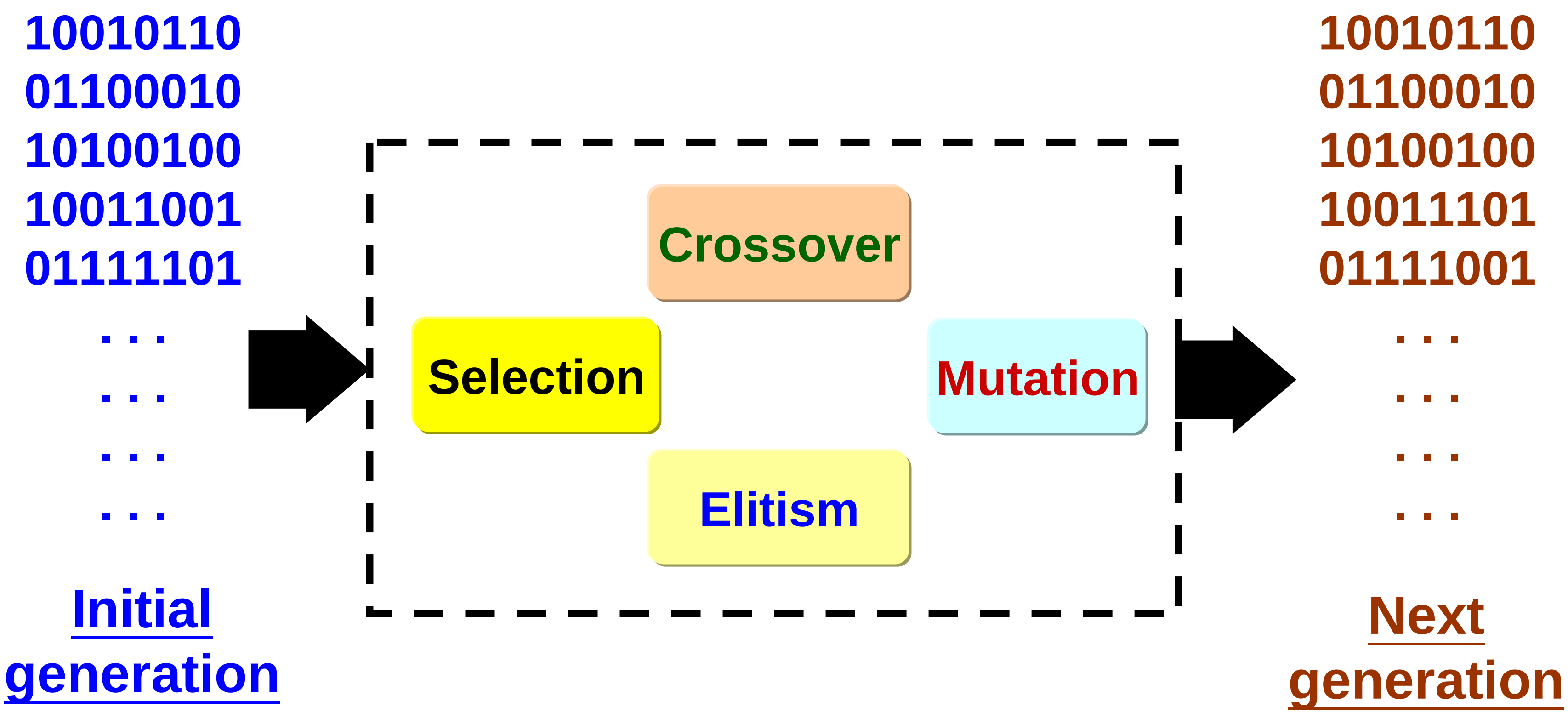
- ***Non-derivative based optimization***
  - Genetic algorithms
  - Simulated annealing
  - Many other

Derivative-free optimization

Genetic Algorithms (GA) mimics laws of ***Natural Evolution*** which emphasizes "***survival of the fittest***".

In GA a "***population***" that contains different possible solutions to the problem is created.

Genetic Algorithms



The process is repeated until ***evolution happens***

"***a solution is found!***"

Multi-Objective Optimization

- It is practical to assume that the decision maker might have priorities on the different objectives ***casualties/mission disruption and time to recover***.

-In this case, usually ***there exist more than one optimal solution*** to the problem (Named ***Pareto solution***)

- Based on the preferences, these ***solutions can be rank ordered***.

Multi-Objective Optimization

- Three major issues differentiate between single and multi-objective optimizations

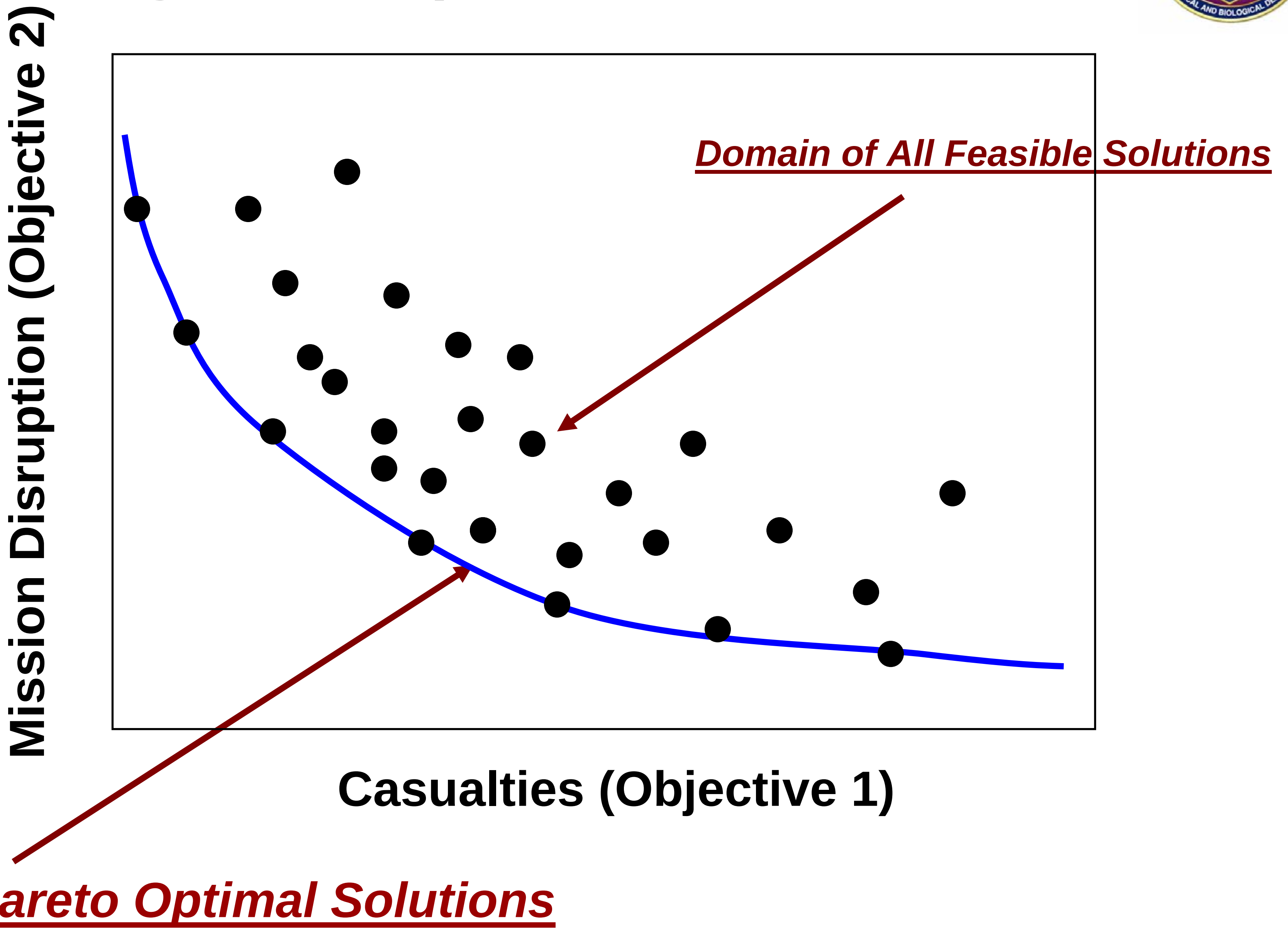
  - ***Multiple (three) goals instead of one***

  - ***Dealing with multiple search spaces not one***

  - ***Artificial fixes affect results***

- We are looking for a set of Pareto-optimal solutions

Multi-Objective Optimization



Multi-Objective Optimization Methods

- ***Global criteria method***
  - Require ***target values for the functions***
  - Can incorporate ***weights for preferences***

- ***Hierarchical optimization method***
  - Optimize the ***top priority function***
  - Specify ***constraints*** to ***prevent deteriorating*** the ***optimized function***

- ***Multi-Objective Genetic Optimization (MOGA)***
  - Non-dominated Sorting Genetic Algorithm

Multi-Objective Optimization

Hierarchical Method

- Rank order the objective functions

$$f_{j-1}(\bar{x}) \le \left(1 \pm \frac{\Sigma_j - 1}{100} \right) \cdot f_{j-1}(\bar{x}^{j-1})$$

-The j-1 function is used as ***constraint in optimizing the jth*** function.

- Σ_j is a ***lexicographic increment %***

– How much error is allowed in losing optimal solution for (j-1) given more optimization in (j)

Multi-Objective Optimization

Global Criterion

- The ***threshold vector*** is defined by

$$f_i^0 = \left[f_1^0, f_2^0, f_3^0 \ldots f_k^0 \right]$$

$$\boxed{f(\bar{x}) = \sum_{i=1}^{k} w_i \left(\frac{f_i^0 - f_i(\bar{x})}{f_i^0} \right)^P}$$

P is ***integer 1 or 2***

w can also be implemented to represent preferences as ***weights***

Multi-Objective Optimization

Non-dominated Sorting Genetic Algorithm (NSGA)

- While similar to GA, NSGA ***sorts the population*** according to ***non-domination principles***.

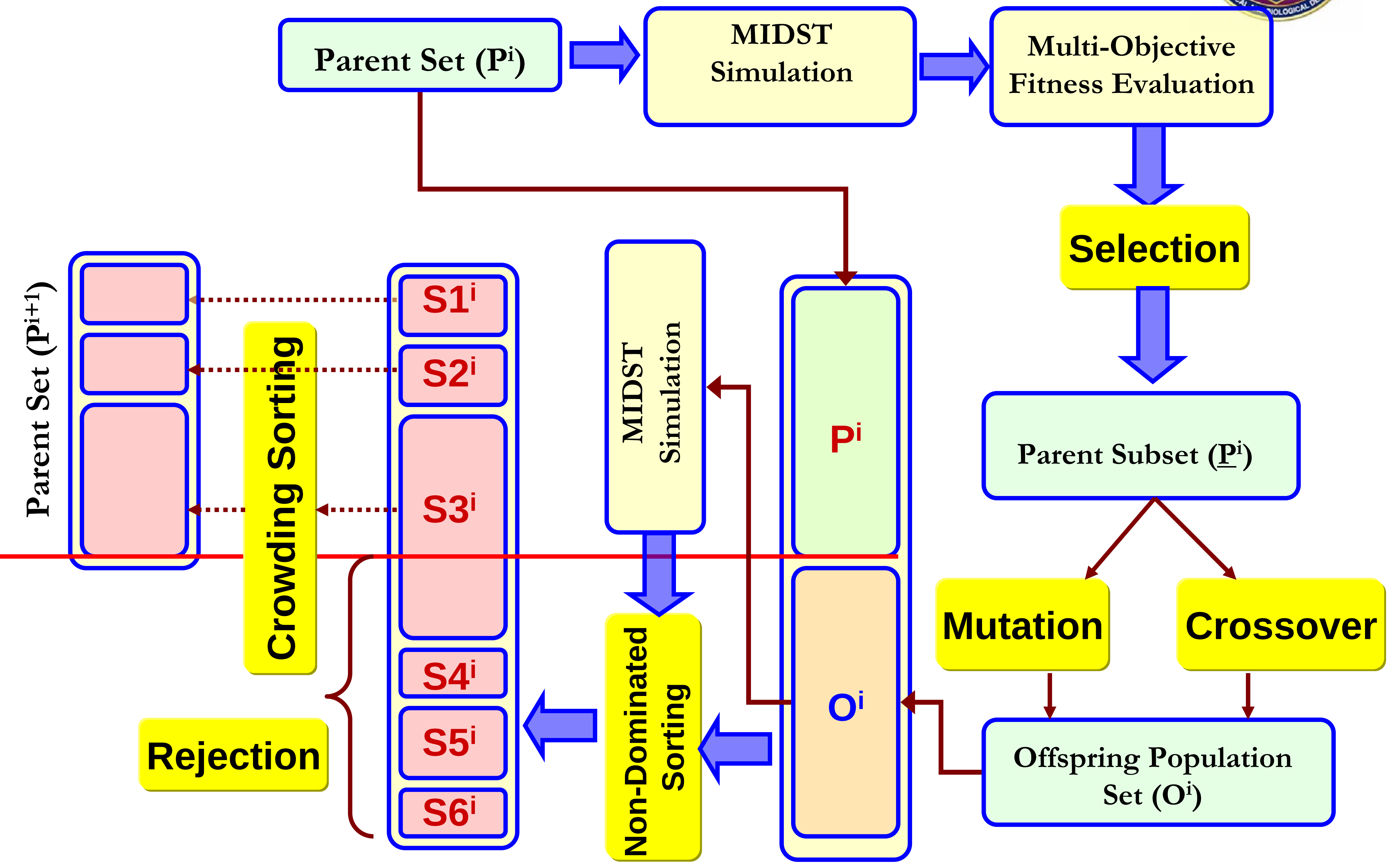
- Population is classified into ***a number of mutually exclusive classes***

- Highest fitness is assigned to ***class*** that are ***closest to the Pareto-optimal front***

- The use of non-dominated sorting ***allows diversity to solutions*** and thus ***guarantees reaching the Pareto-front***.

-***NSGA*** also includes ***elitism principles*** which allows it to find higher number of Pareto-solutions.

NSGA-II



Parent Set (Pi)
MIDST Simulation
Multi-Objective Fitness Evaluation
Selection
Parent Subset (Pi)
Mutation
Crossover
Offspring Population Set (Oi)
Pi
Oi
MIDST Simulation
Non-Dominated Sorting
S1i
S2i
S3i
S4i
S5i
S6i
Crowding Sorting
Rejection
Parent Set (Pi+1)

Merits and shortcomings

- ***Derivative based***

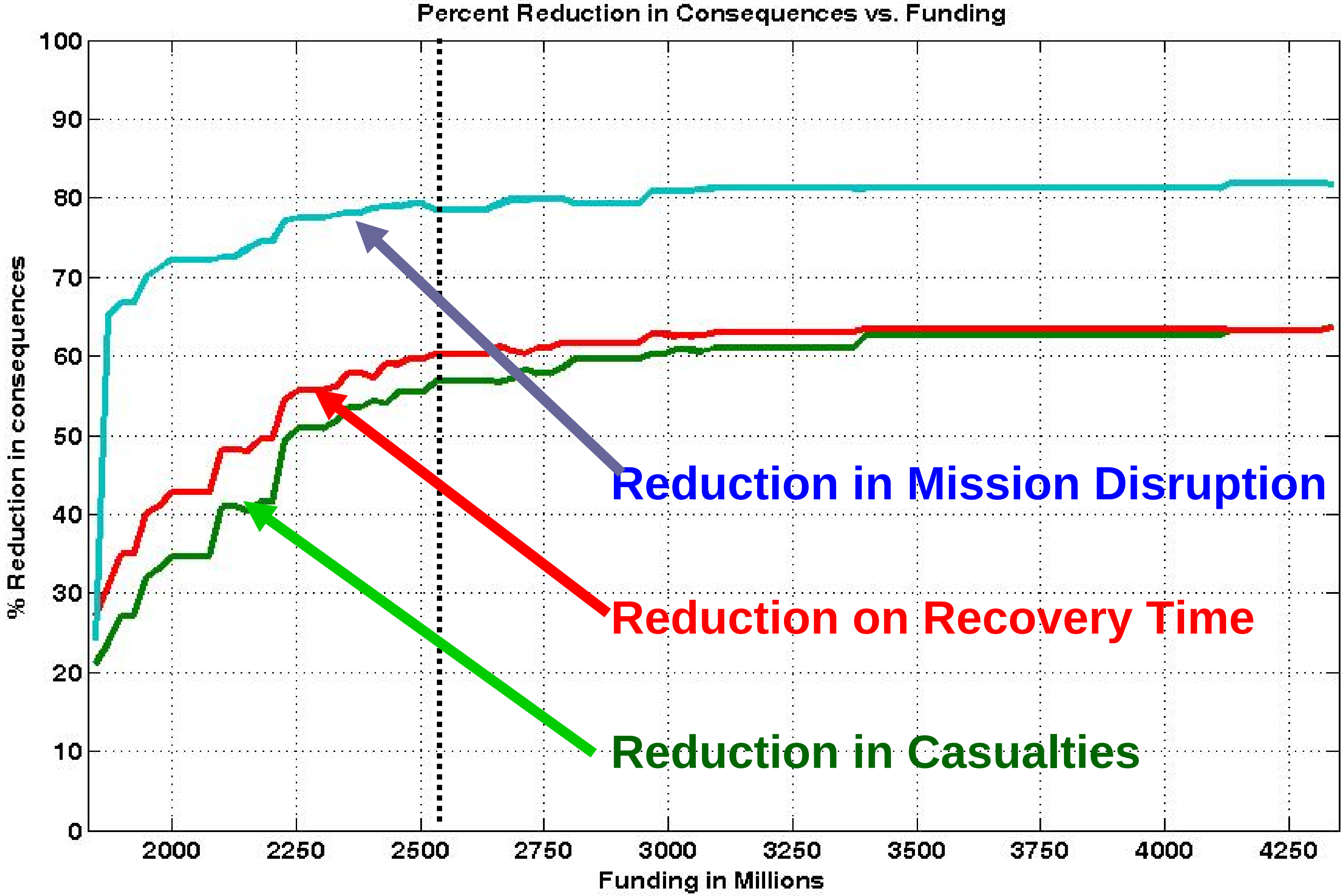
  - If the ***space is continuum***, it converges very fast and an optimal solution is guaranteed
  - If too many ***local minima exist***, the algorithm might be ***trapped*** and ***cannot find global minima***

- ***Non-derivative based***

  - If the ***space is non-continuum***, GA will be able to find the solution
  - Whether ***local minima exist or not***, it will converge.
  - GA is ***better equipped with some aiding optimization*** technique to narrow search domain

Case study

- For a given group of data cards and inputs we identified



Case study

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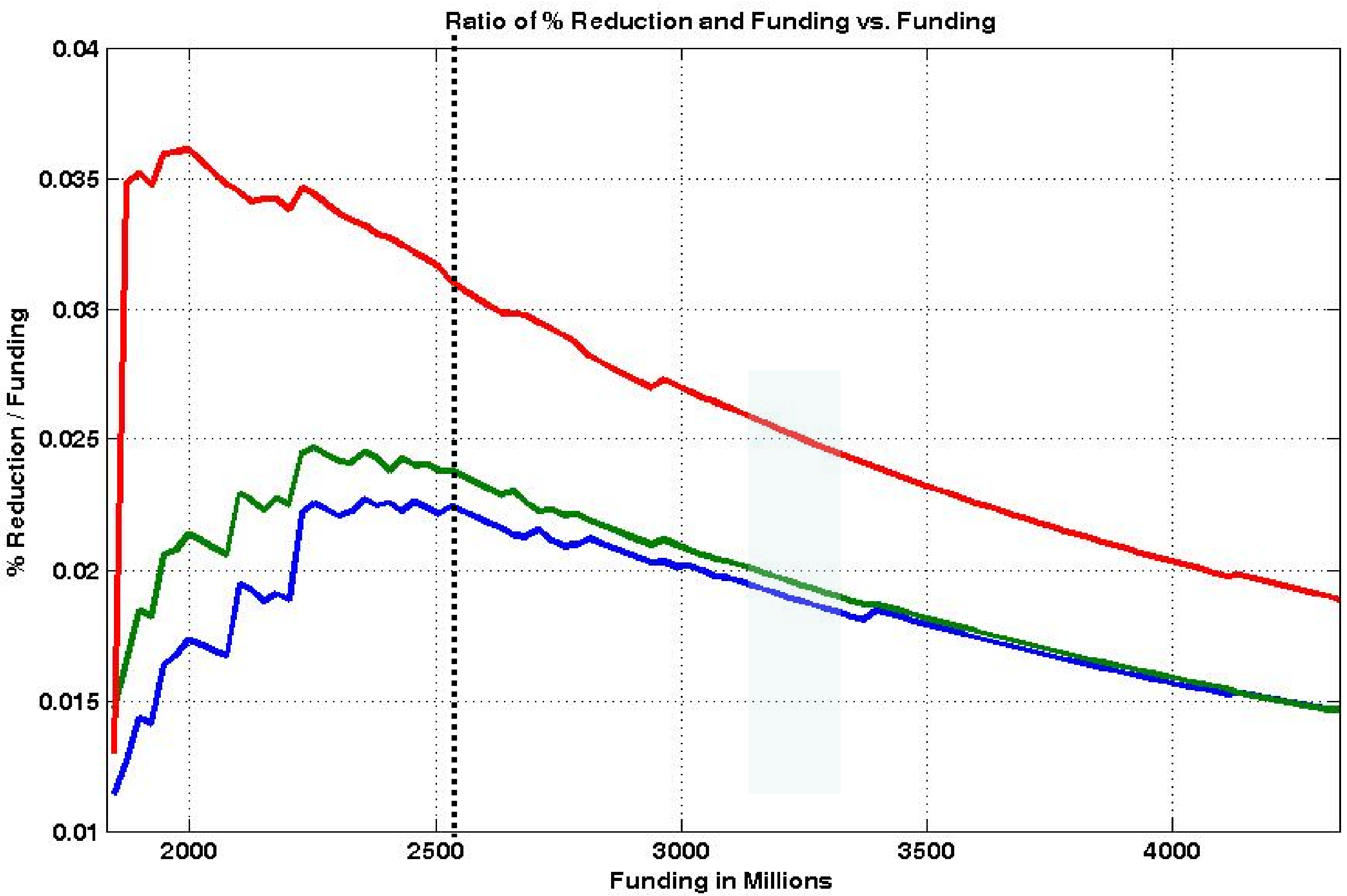
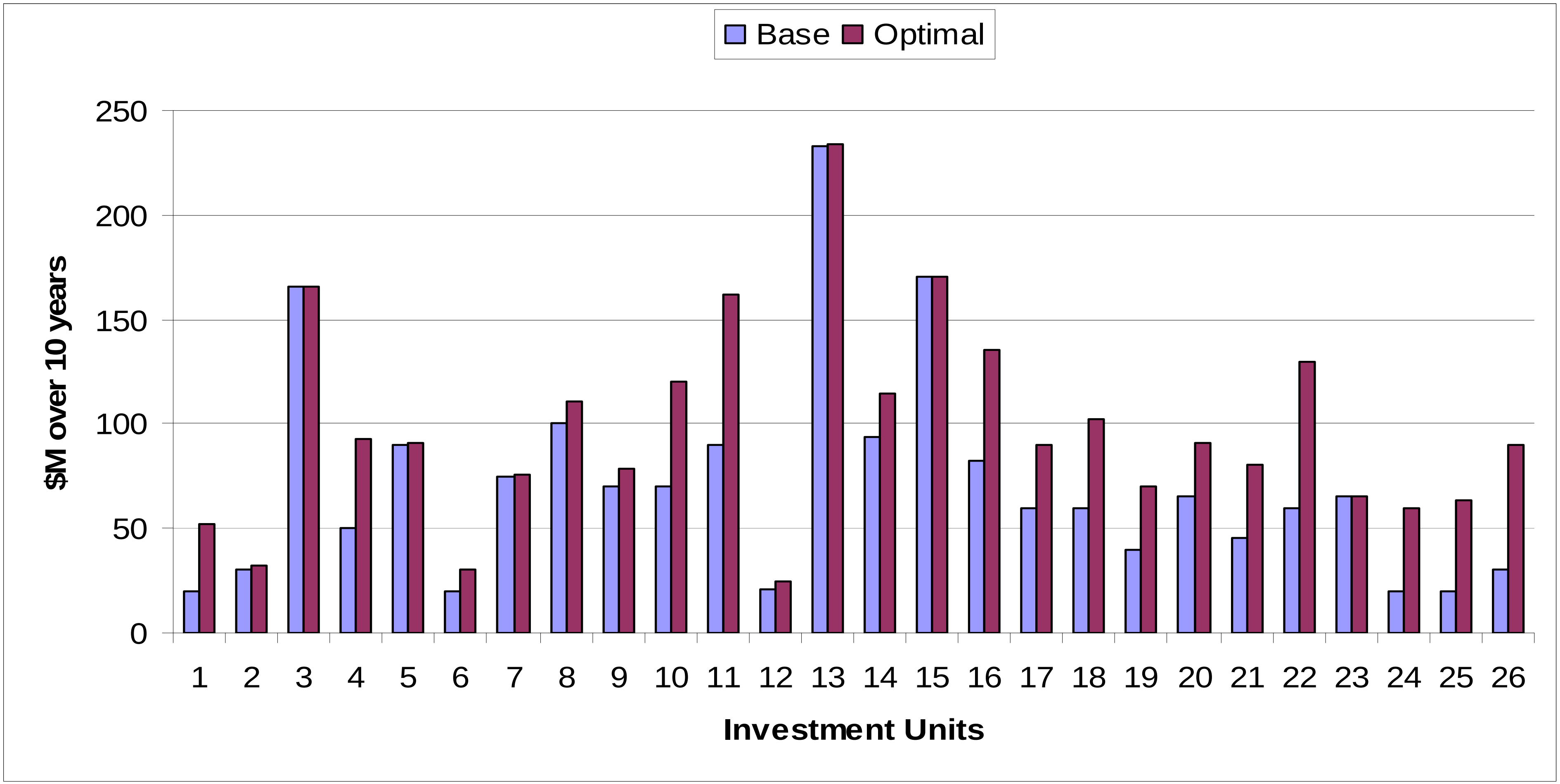
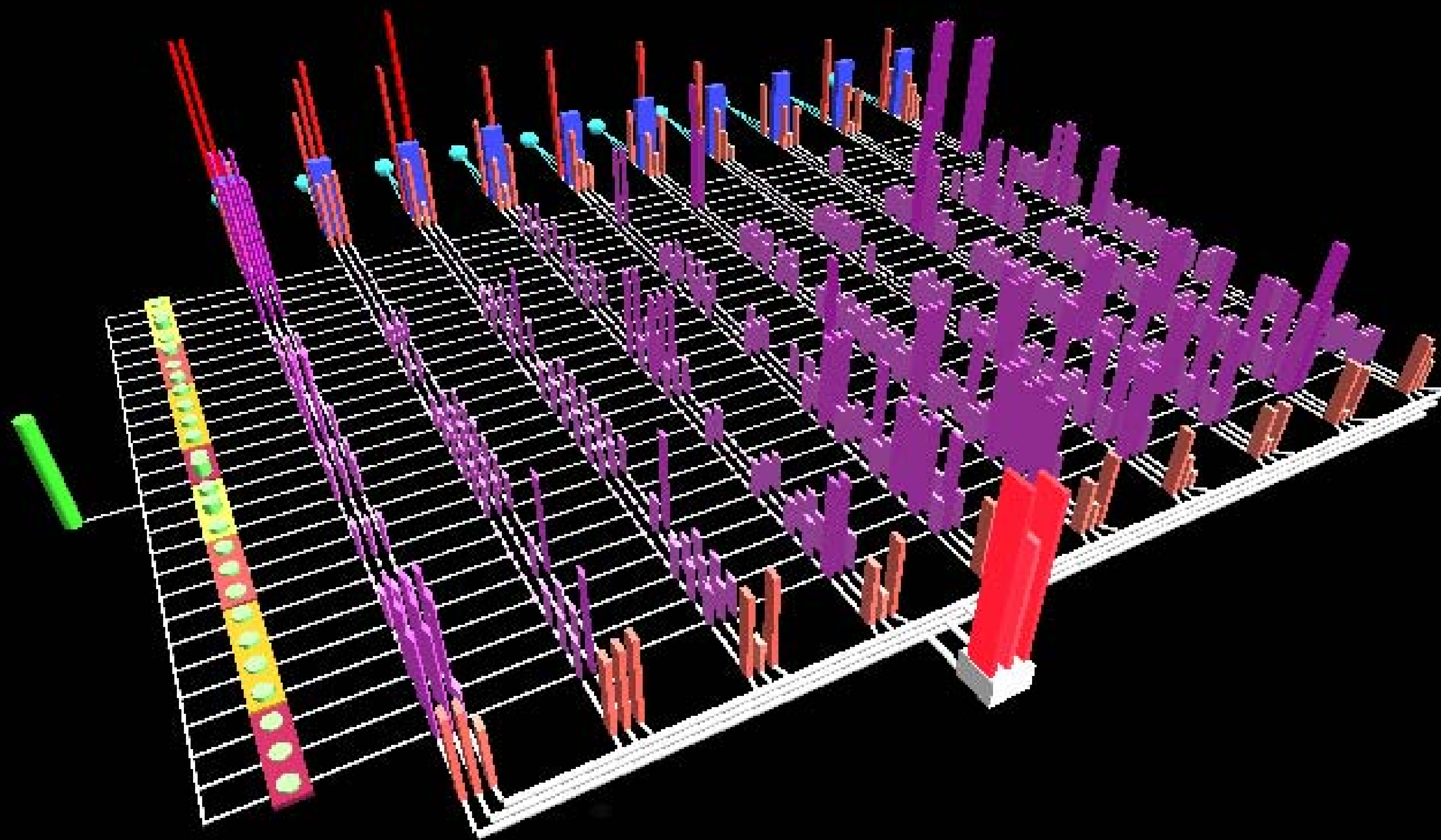


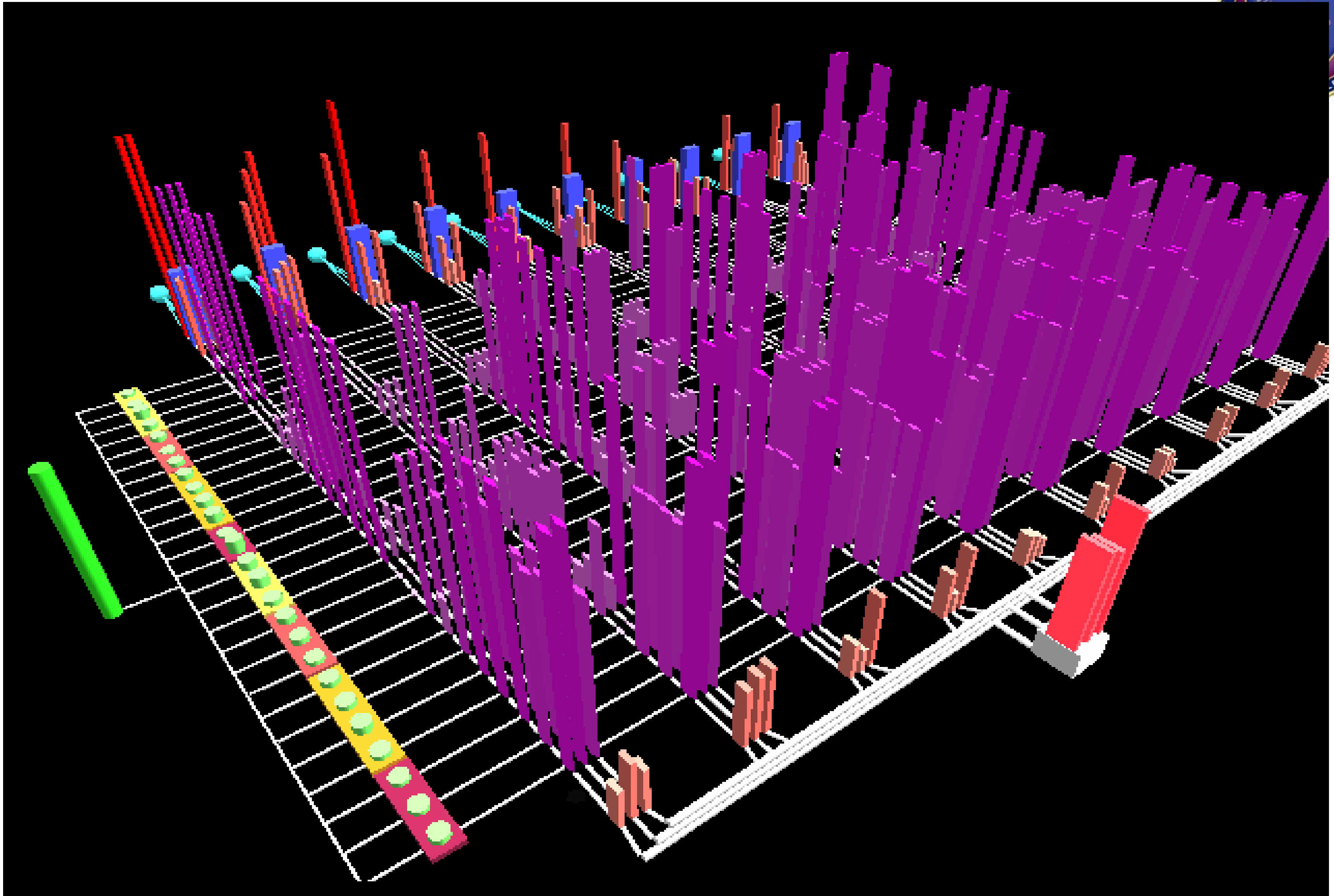
Case study

- At the optimal level, we can identify the funding portfolio





Portfolio for Base Funding $C^1 = 21$, $C^2 = 21$. $C^3 = 42$



Portfolio for Optimal Funding $C^1 = 11$, $C^2 = 12$. $C^3 = 12$

Conclusions

-We demonstrated the possible use of multi-objective genetic optimization for allocation of funding for investment units to reduce consequences of CB events

- Classical gradient based versus gradient free optimization techniques have been examined in search for Pareto solutions

- The presented work is part of MIDST: A robust mathematical framework that can be used to help decision makers for funding allocations considering multiple objectives and priorities

Research is currently on-going to integrate fuzzy rank ordering module as part of the optimization process.

Acknowledgment

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Questions

Derivative-based optimization

Gradient descent method

- Assumes ***continuous*** and ***differentiable function***

$$\theta_{new} = \theta_{old} + \eta\, G\, g$$

-g is the derivative of the objective function

$$g(\theta) = \nabla E(\theta) = \left[\frac{\partial E(\theta)}{\partial \theta_1} \quad \frac{\partial E(\theta)}{\partial \theta_2} \quad \cdots\cdots \quad \frac{\partial E(\theta)}{\partial \theta_n} \right]^T$$

- G is a positive definite matrix

- η is the step size

Derivative-based optimization

Levenberg-Marquardt (LM) method

- A ***modified version of classical Newton's method***. It also assumes ***continuous*** and ***differentiable function***

$$\theta_{new} = \theta_{old} - \eta \left(H + \lambda I \right)^{-1} g$$

- g is the gradient, I is the identity matrix, ***λ is some nonnegative value and H is the Hessian matrix***

$$H(\theta) = \nabla^2 E(\theta) = \left[\frac{\partial^2 E(\theta)}{\partial \theta_1^{\ 2}} \quad \frac{\partial^2 E(\theta)}{\partial \theta_2^{\ 2}} \quad .... \quad \frac{\partial^2 E(\theta)}{\partial \theta_n^{\ 2}} \right]^T$$

– η is the step size as defined before

Derivative-based optimization