Combinatorial Testing:

Required Knowledge in a World Where Traversing Every Possible Path is Impossible

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Introductions

- Name
- Organization
- Job Title/Duties
- Experience in T&E, Combinatorial Testing, etc.

Agenda

- Some Basic Definitions
- Various Approaches to Testing Multiple Factors
- Design of Experiments (DOE): a Modern Approach to Combinatorial Testing
- Break
- Examples and Demonstration of a DOE
- Using DOE to Achieve Robust Designs
- DOE with Modeling and Simulation
- High Throughput Testing

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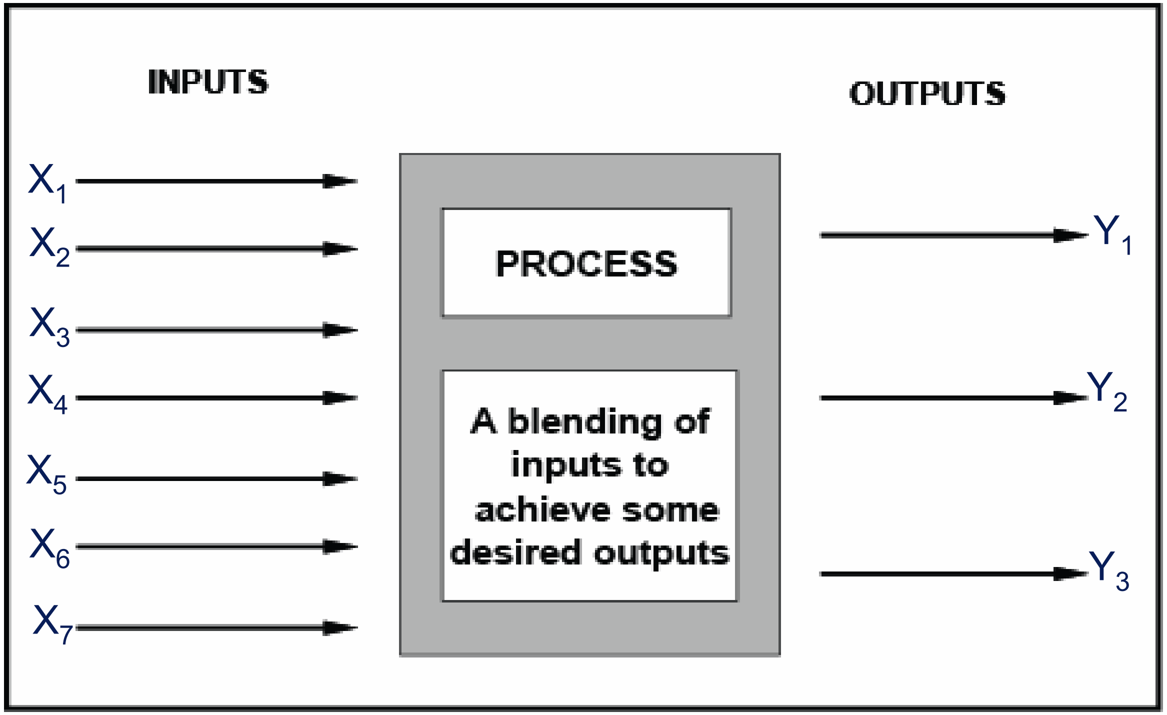
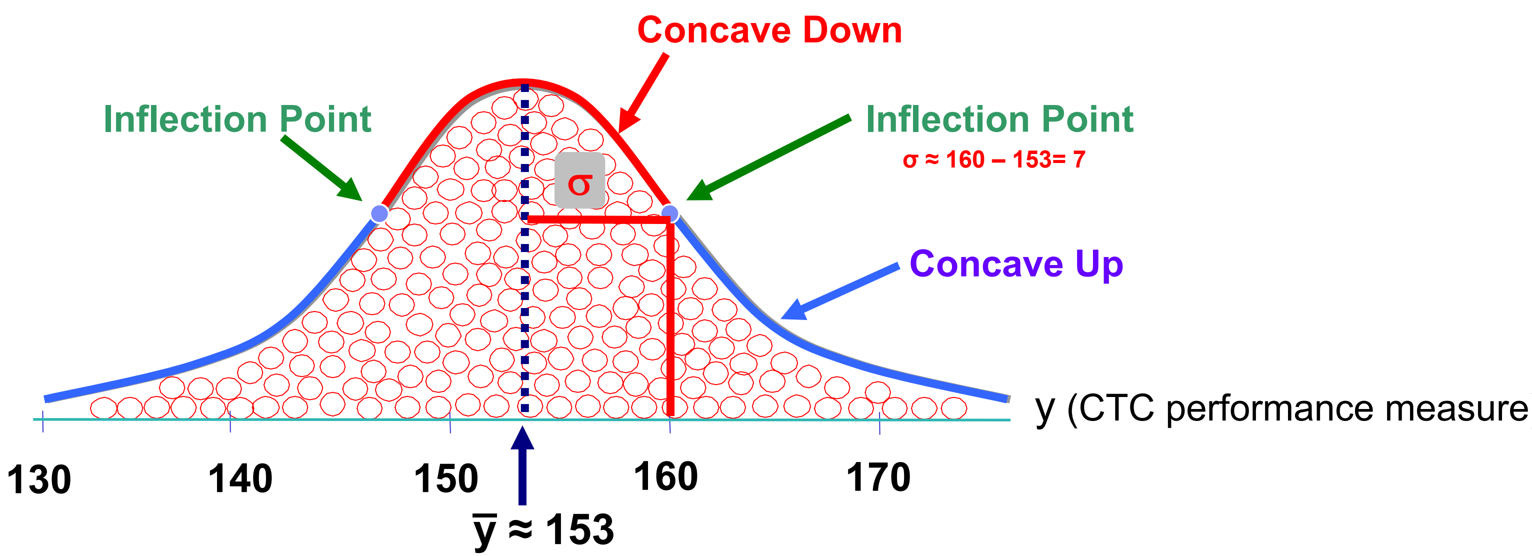
Definition of a Process



Graphical Meaning of $\bar{y}$ and σ

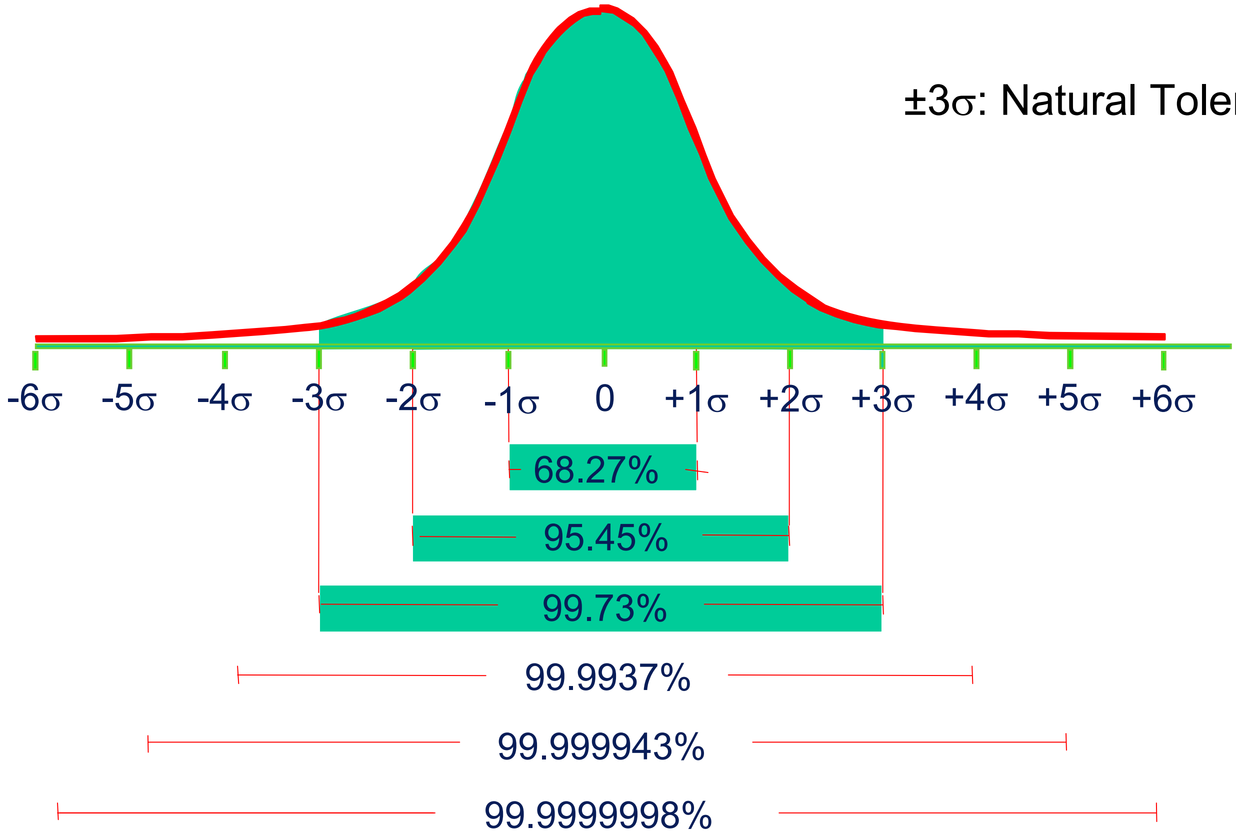
$\bar{y}$ = Average = Mean = Balance Point

σ = Standard Deviation



σ ≈ average distance of points from the centerline

Graphical View of Variation



Typical Areas under the Normal Curve

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Approaches to Testing Multiple Factors

- **Traditional Approaches**
  - One Factor at a Time (OFAT)
  - Oracle (Best Guess)
  - All possible combinations (full factorial)

- **Modern Approach**
  - Statistically designed experiments (DOE) … full factorial plus other selected DOE designs, depending on the situation

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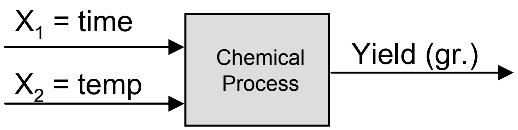
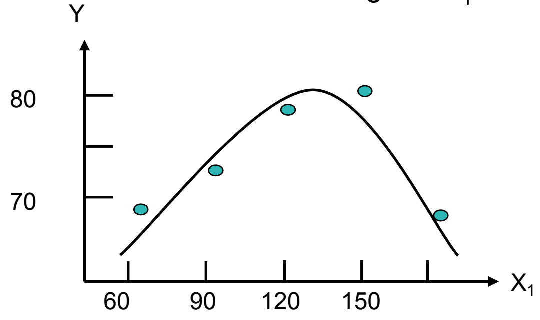
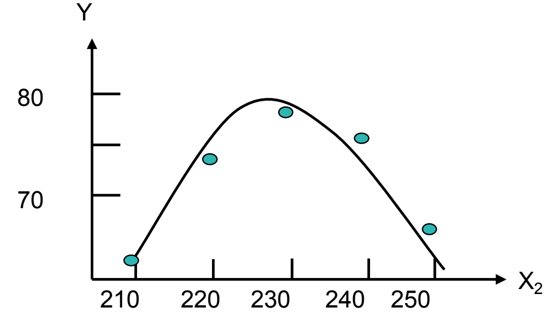
OFAT (One Factor at a Time)



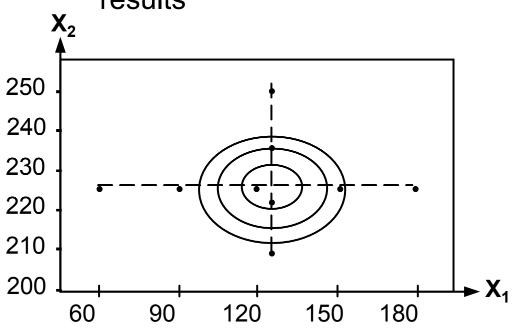
1. Hold X_2 constant and vary X_1
   Find the "best setting" for X_1



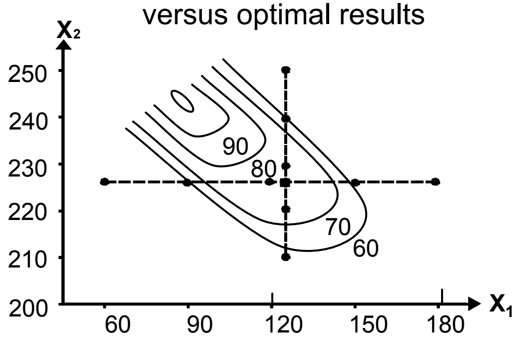
2. Hold X_1 constant at "best setting" and vary X_2.
   Find the "best setting" for X_2.



3. One factor at a time results



4. One factor at a time results versus optimal results



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Oracle (Best Guess)

W = Wetting Agent (1=.07 ml; 2=none)

P = Plasticizer (1=1ml; 2=none)

E = Environment (1=Ambient Mixing; 2=Semi-Evacuated)

C = Cement (1=Portland Type III; 2=Calcium Aluminate)

A = Additive (1=No Reinforcement; 2=Steel)

Y = Strength of Lunar Concrete

Run	W	P	E	C	A	Y
1	1	2	1	1	1	5
2	1	1	1	1	1	6
3	2	2	1	1	1	5
4	2	1	1	1	2	6
5	1	2	2	2	2	7
6	1	1	2	2	2	8
7	2	2	2	2	2	10
8	2	1	2	2	1	11

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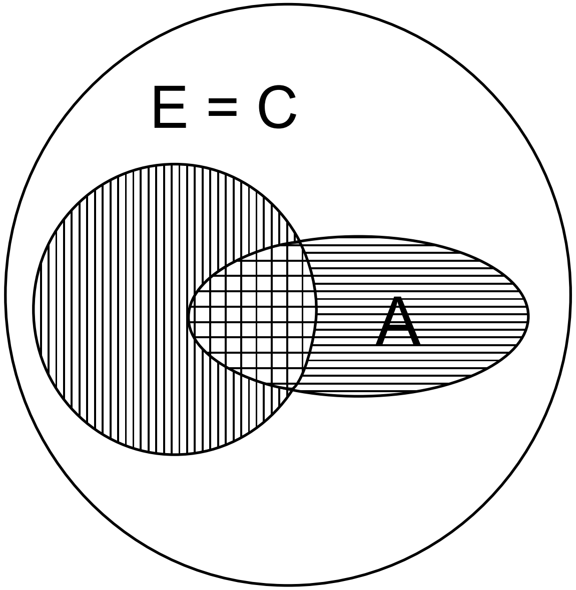
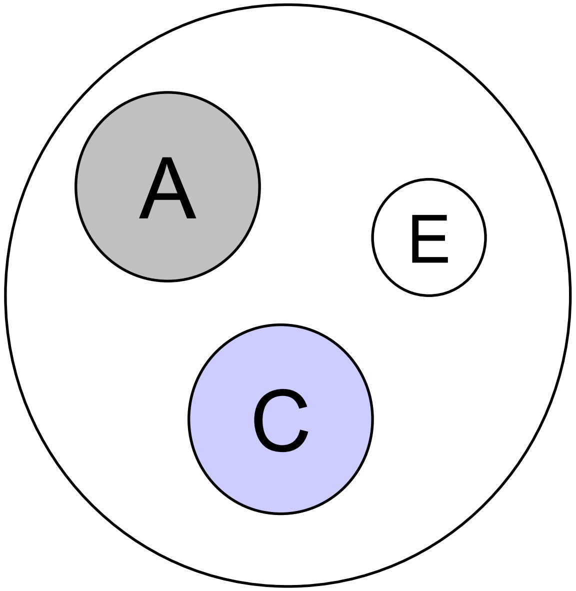
Evaluating the Effects of Variables on Y

What we have is:



What we need is a design to provide independent estimates of effects:



How do we obtain this independence of variables?

All Possible Combinations (Full Factorial)

MATRIX FORM

Example 1:

A (2 levels)
B (2 levels)

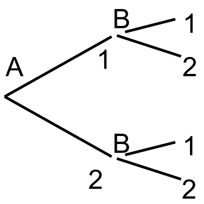
A	B
1	1
1	2
2	1
2	2

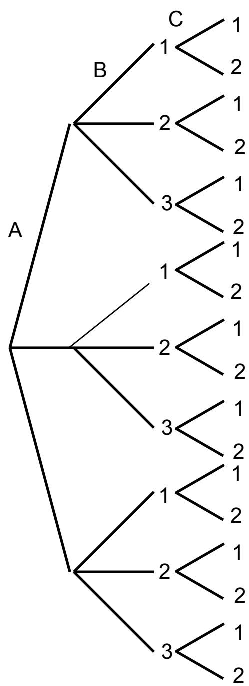
Example 2:

A (3 levels)
B (3 levels)
C (2 levels)

A	B	C
1	1	1
1	2	1
1	3	1
2	1	1
2	2	1
2	3	1
3	1	1
3	2	1
3	3	1
1	1	2
1	2	2
1	3	2
2	1	2
2	2	2
2	3	2
3	1	2
3	2	2
3	3	2

TREE DIAGRAM





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Design of Experiments (DOE)

- An optimal data collection methodology
- “Interrogates” the process
- Used to identify important relationships between input and output factors
- Identifies important interactions between process variables
- Can be used to optimize a process
- Changes “I think” to “I know”

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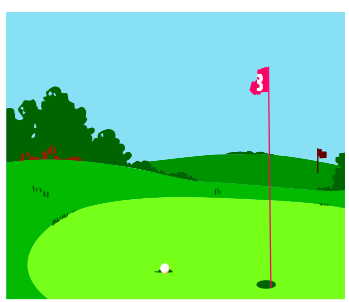
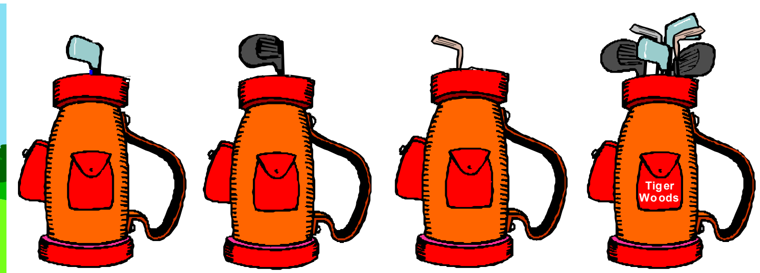
Important Contributions From:

	TAGUCHI	SHAININ	CLASSICAL	BLENDED APPROACH
Loss Function	*			*
Emphasis on Variance Reduction	*			*
Robust Designs	*			*
KISS	*	*		*
Simple Significance Tests		*		*
Component Swapping		*		
Multivariate Charts		*		*
Modeling			*	*
Sample Size			*	*
Efficient Designs			*	*
Optimization			*	*
Confirmation	*			*
Response Surface Methods			*	*





Which bag would a world class golfer prefer?

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Statistically Designed Experiments (DOE):
Orthogonal or Nearly Orthogonal Designs

- FULL FACTORIALS (for small numbers of factors)
- FRACTIONAL FACTORIALS
- PLACKETT - BURMAN
- LATIN SQUARES
- HADAMARD MATRICES
- BOX - BEHNKEN DESIGNS
- CENTRAL COMPOSITE DESIGNS

(PLACKETT - BURMAN, LATIN SQUARES, HADAMARD MATRICES: Taguchi Designs)

SIMPLE DEFINITION OF TWO-LEVEL ORTHOGONAL DESIGNS

Run	Actual Settings			Coded Matrix			Responses
	(5, 10) A: Time	(70, 90) B: Temp	(100, 200) C: Press	(A) Time	(B) Temp	(C) Press	
1	5	70	100	-1	-1	-1	
2	5	70	200	-1	-1	+1	
3	5	90	100	-1	+1	-1	
4	5	90	200	-1	+1	+1	
5	10	70	100	+1	-1	-1	
6	10	70	200	+1	-1	+1	
7	10	90	100	+1	+1	-1	
8	10	90	200	+1	+1	+1	

The Beauty of Orthogonality:
independent evaluation of effects

A Full Factorial Design for 3 Factors, Each at 2 Levels

Run	A	B	C	AB	AC	BC	ABC
1	-	-	-	+	+	+	-
2	-	-	+	+	-	-	+
3	-	+	-	-	+	-	+
4	-	+	+	-	-	+	-
5	+	-	-	-	-	+	+
6	+	-	+	-	+	-	-
7	+	+	-	+	-	-	-
8	+	+	+	+	+	+	+

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Building a Fraction of the Full Factorial

Given the number of factors (k) = 4

i) The full factorial has $n = 2^4 = 16$ runs

ii) A $^1/_2$ fraction will have $n = 2^4/2 = 2^{4-1} = 8$ runs

STEP 1: Build an 8 run design for 3 factors: A, B, and C

STEP 2: Alias (perfectly confound) the 4th factor, D, with the highest order interaction in Step 1

STEP 3: Determine Aliasing (confounding) Pattern

Run	A 1	B 2	AB 3	C 4	AC 5	BC 6	D = ABC 7
1	-	-	+	-	+	+	-
2	-	-	+	+	-	-	+
3	-	+	-	-	+	-	+
4	-	+	-	+	-	+	-
5	+	-	-	-	-	+	+
6	+	-	-	+	+	-	-
7	+	+	+	-	-	-	-
8	+	+	+	+	+	+	+
Avg (-)							
Avg (+)							
Δ							
$\Delta/2$							

$\hat{Y} =$

2^{5-1} DESIGN EXAMPLE

Run	Factors: A	B	C	D	AB	AC	BC	AD	BD	CD	DE ABC	CE ABD	BE ACD	AE BCD	E ABCD	Response: y_1	y_2	y_3
1																		
2																		
.																		
.																		
.																		
16																		

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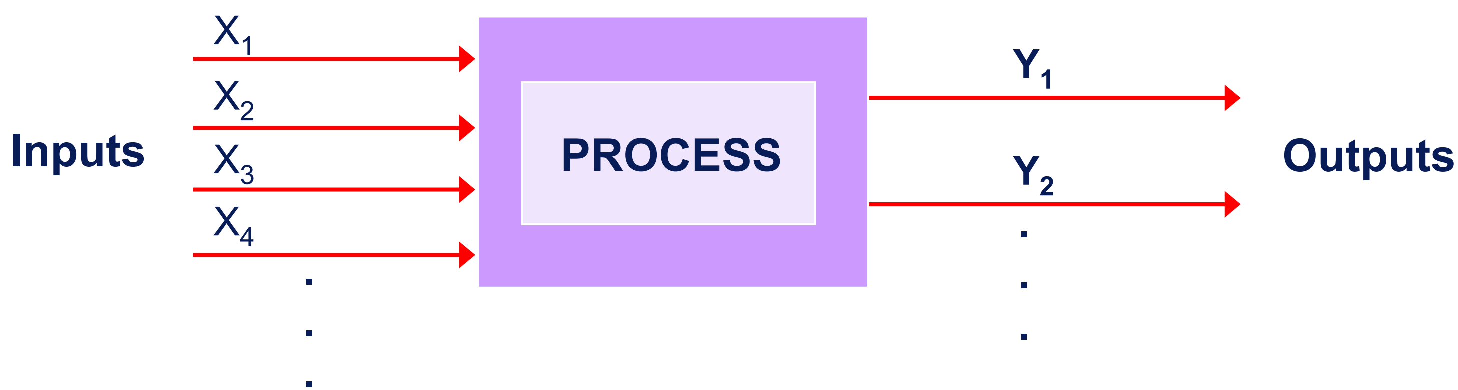
The Purpose of a Designed Experiment

Purposeful changes of the inputs (factors) in order to observe corresponding changes in the output (response).



Run	X_1 X_2 X_3 X_4	Y_1 Y_2	$\bar{Y}$	S_Y
1				
2				
3				
.				
.				

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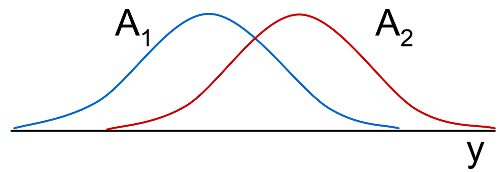
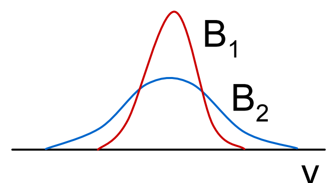
DOE Helps Determine How Inputs Affect Outputs

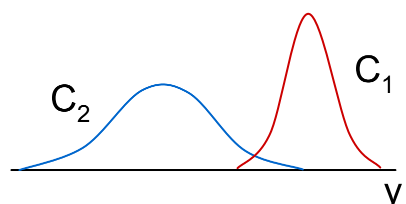
i) Factor A affects the average of y



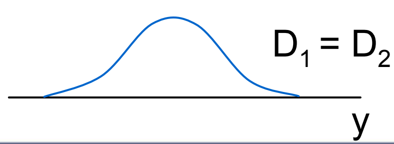
ii) Factor B affects the standard deviation of y



iii) Factor C affects the average and the standard deviation of y



iv) Factor D has no effect on y



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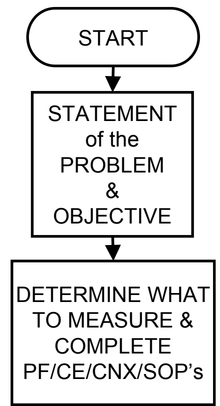
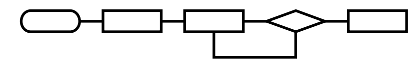
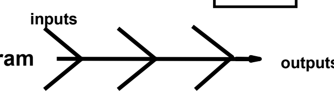
KISS Guidelines for Choosing an Experimental Design

KISS - Keep It Simple Statistically

Source: Steve Schmidt
Mark Kiemele
Cass Grandone



PF → **Process flow diagram** 

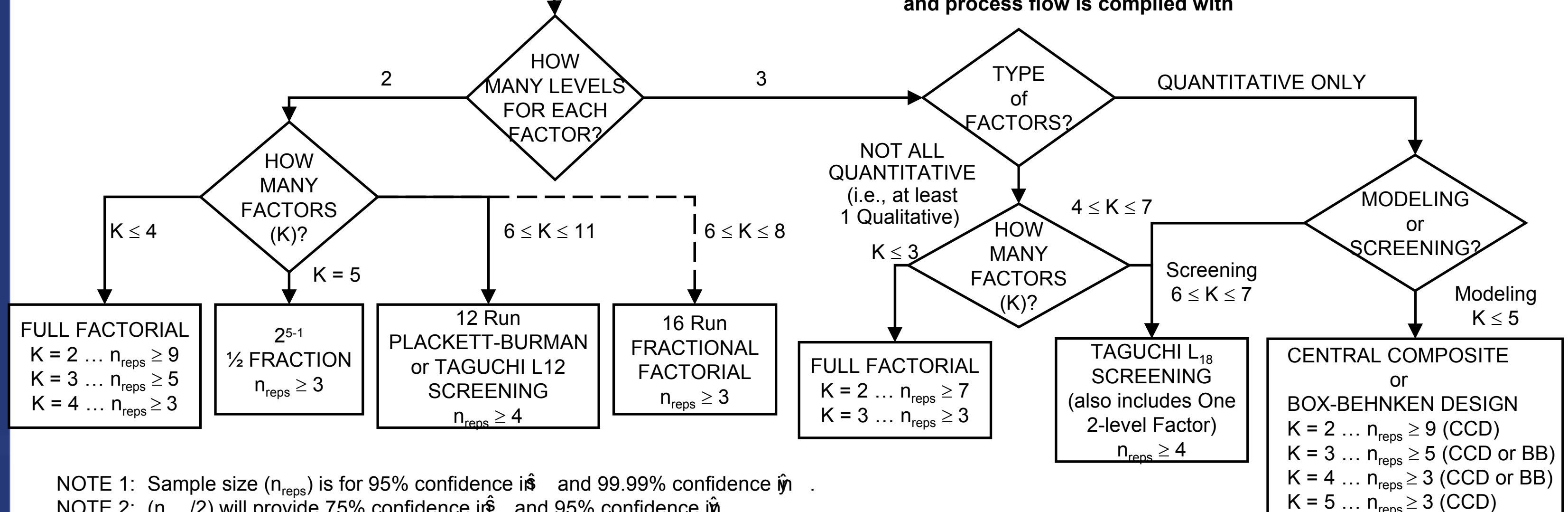
CE → **Cause-and-effect diagram** 

C → **Inputs on cause-and-effect to be held constant**

N → **Inputs on cause-and-effect that are noise or uncontrolled**

X → **Inputs (factors) on cause-and-effect identified for experimentation**

SOPs → **Standard operating procedures to insure all Cs are held constant and process flow is complied with**



NOTE 1: Sample size (n_{reps}) is for 95% confidence in $\hat{s}$ and 99.99% confidence in $\hat{y}$.

NOTE 2: (n_{reps}/2) will provide 75% confidence in $\hat{s}$ and 95% confidence in $\hat{y}$.

NOTE 3: The 12 Run Plackett-Burman or L12 is very sensitive to large numbers of interactions. If this is the case, you would be better off using the 16 Run Fractional Factorial or a smaller number of variables in 2 or more full factorial experiments.

NOTE 4: For more complete 2-level design options, see next page.

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Building a Screening Design

L_{12} Design											
Run	1	2	3	4	5	6	7	8	9	10	11
1	-	-	-	-	-	-	-	-	-	-	-
2	-	-	-	-	-	+	+	+	+	+	+
3	-	-	+	+	+	-	-	-	+	+	+
4	-	+	-	+	+	-	+	+	-	-	+
5	-	+	+	-	+	+	-	+	-	+	-
6	-	+	+	+	-	+	+	-	+	-	-
7	+	-	+	+	-	-	+	+	-	+	-
8	+	-	+	-	+	+	+	-	-	-	+
9	+	-	-	+	+	+	-	+	+	-	-
10	+	+	+	-	-	-	-	+	+	-	+
11	+	+	-	+	-	+	-	-	-	+	+
12	+	+	-	-	+	-	+	-	+	+	-

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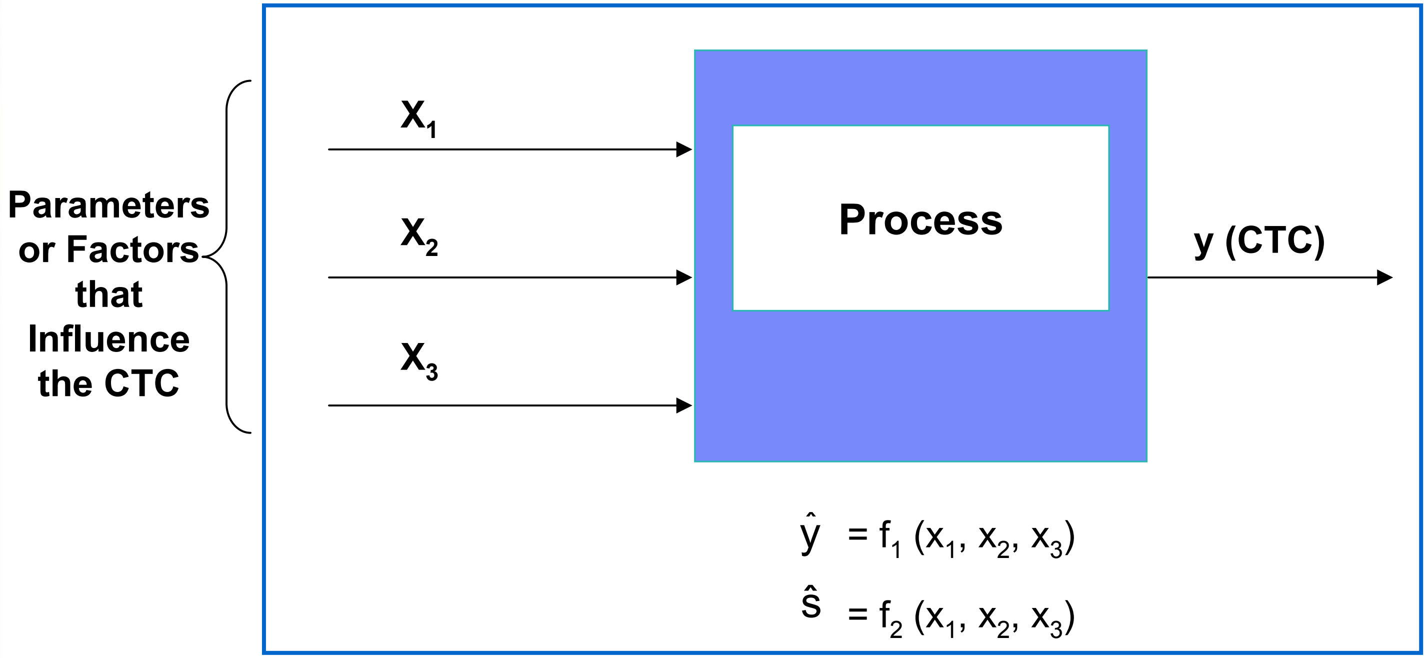
Transfer Functions

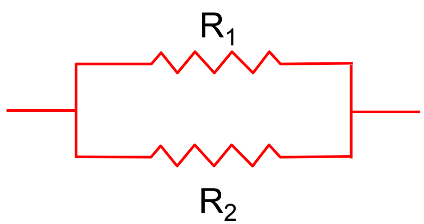


Where does the transfer function come from?

- **Exact transfer Function**
- **Approximations**
  - **DOE**
  - **Historical Data Analysis**
  - **Simulation**

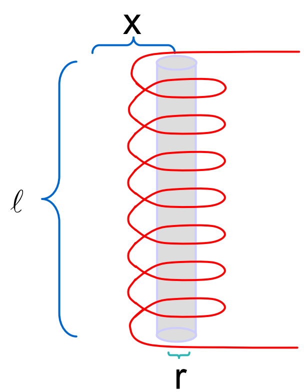
Exact Transfer Function

- Engineering Relationships
  - $V = IR$
  - $F = ma$



The equation for the impedance (Z) through this circuit is defined by:

$$Z = \frac{R_1 \cdot R_2}{R_1 + R_2}$$



The equation for magnetic force at a distance X from the center of a solenoid is:

$$H = \frac{NI}{2\ell}\left[\frac{.5\ell + x}{\sqrt{r^2 + (.5\ell + x)^2}} + \frac{.5\ell - x}{\sqrt{r^2 + (.5\ell - x)^2}}\right]$$

Where

- N : total number of turns of wire in the solenoid
- I : current in the wire, in amperes
- r : radius of helix (solenoid), in cm
- ℓ : length of the helix (solenoid), in cm
- x : distance from center of helix (solenoid), in cm
- H : magnetizing force, in amperes per centimeter

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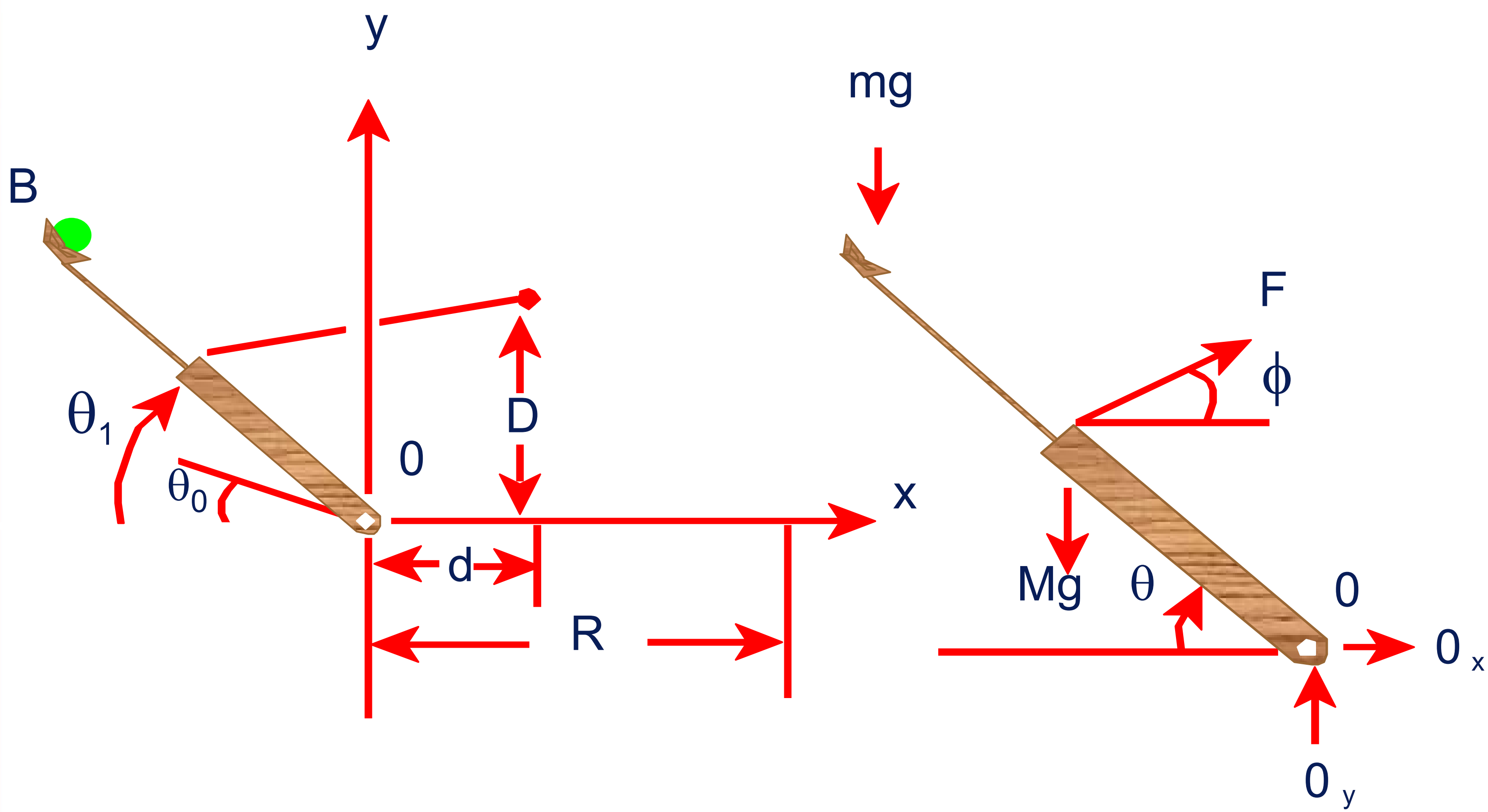
Catapulting Power into Test and Evaluation

Statapult ® Catapult

The Theoretical Approach



The Theoretical Approach (cont.)

$$I_0\ddot{\theta} = r_F F(\theta)\sin\theta\cos\varphi - (Mgr_G + mgr_B)\sin\theta \qquad \tan\phi = \frac{D - r_F\sin\theta}{d + r_F\cos\theta},$$

$$\frac{1}{2}I_0\dot{\theta}^2 = r_F\int_{\theta_0}^{\theta} F(\theta)\sin\theta\cos\varphi\, d\theta - (Mgr_G + mgr_B)(\sin\theta - \sin\theta_0)$$

$$\frac{1}{2}I_0\dot{\theta}_1^2 = r_F\int_{\theta_0}^{\theta_1} F(\theta)\sin\theta\cos\varphi\, d\theta - (Mgr_G + mgr_B)(\sin\theta_1 - \sin\theta_0).$$

$$x = v_B\cos\left(\frac{\pi}{2} - \theta_1\right)t - \frac{1}{2}r_B\cos\theta_1 \qquad y = r_B\sin\theta_1 + v_B\sin\left(\frac{\pi}{2} - \theta_1\right)t - \frac{1}{2}gt^2.$$

$$r_B\sin\theta_1 + (R + r_B\cos\theta_1)\tan\left(\frac{\pi}{2} - \theta_1\right) - \frac{g}{2V_B^2}\frac{(R + r_B\cos\theta_1)^2}{\cos^2\left(\frac{\pi}{2} - \theta_1\right)} = 0.$$

$$\frac{gI_0}{4r_B}\frac{(R + r_B\cos\theta_1)^2}{\cos^2\left(\frac{\pi}{2} - \theta_1\right)\left[r_B\sin\theta_1 + (R + r_B\cos\theta_1)\tan\left(\frac{\pi}{2} - \theta_1\right)\right]}$$

$$= r_F\int_{\theta_0}^{\theta_1} F(\theta)\sin\theta\cos\phi\, d\theta - (Mgr_G + mgr_B)(\sin\theta_1 - \sin\theta_0).$$

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Statapult® DOE Demo

(The Empirical Approach)

	Actual Factors		Coded Factors			Response Values			
Run	A	B	A	B	AB	Y_1	Y_2	$\bar{Y}$	S
1	144	2	-1	-1	+1				
2	144	3	-1	+1	-1				
3	160	2	+1	-1	-1				
4	160	3	+1	+1	+1				
		Avg –							
		Avg +							
		Δ							

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Minimizing the # of Factor Changes

(GRAY CODE SEQUENCE)

Problem: If changing factor settings is time consuming and/or expensive, using a Gray Code sequence to determine the sequence of runs may be useful. A Gray Code sequence orders the runs so that only 1 factor setting changes between runs and the most difficult to change factors are changed less frequently.

16-Run Design

Run	A	B	C	D
1	-	-	-	-
2	-	-	-	+
3	-	-	+	-
4	-	-	+	+
5	-	+	-	-
6	-	+	-	+
7	-	+	+	-
8	-	+	+	+
9	+	-	-	-
10	+	-	-	+
11	+	-	+	-
12	+	-	+	+
13	+	+	-	-
14	+	+	-	+
15	+	+	+	-
16	+	+	+	+

Gray Code by Run #
1
2
4
3
7
8
6
5
13
14
16
15
11
12
10
9

Cycling through the runs from top to bottom (or vice versa) will produce 15 changes:

- D will be changed 8 times.
- C will be changed 4 times.
- B will be changed 2 times.
- A will be changed 1 time.

Thus, the most difficult (or expensive) to change factors should be assigned to A, B, C, D, respectively.

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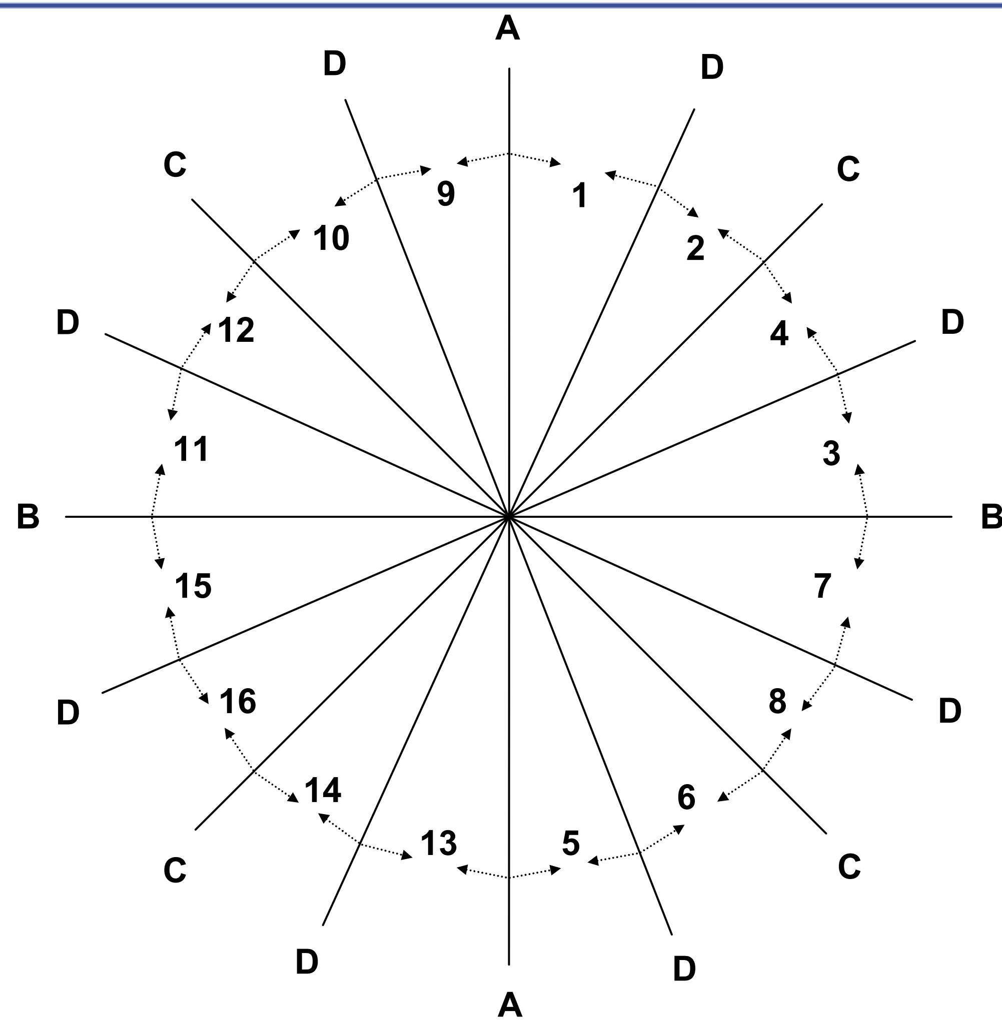
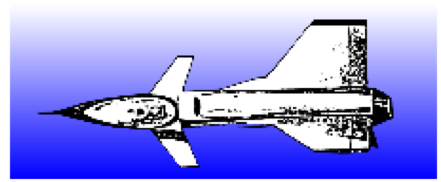
Test Sequence Generator

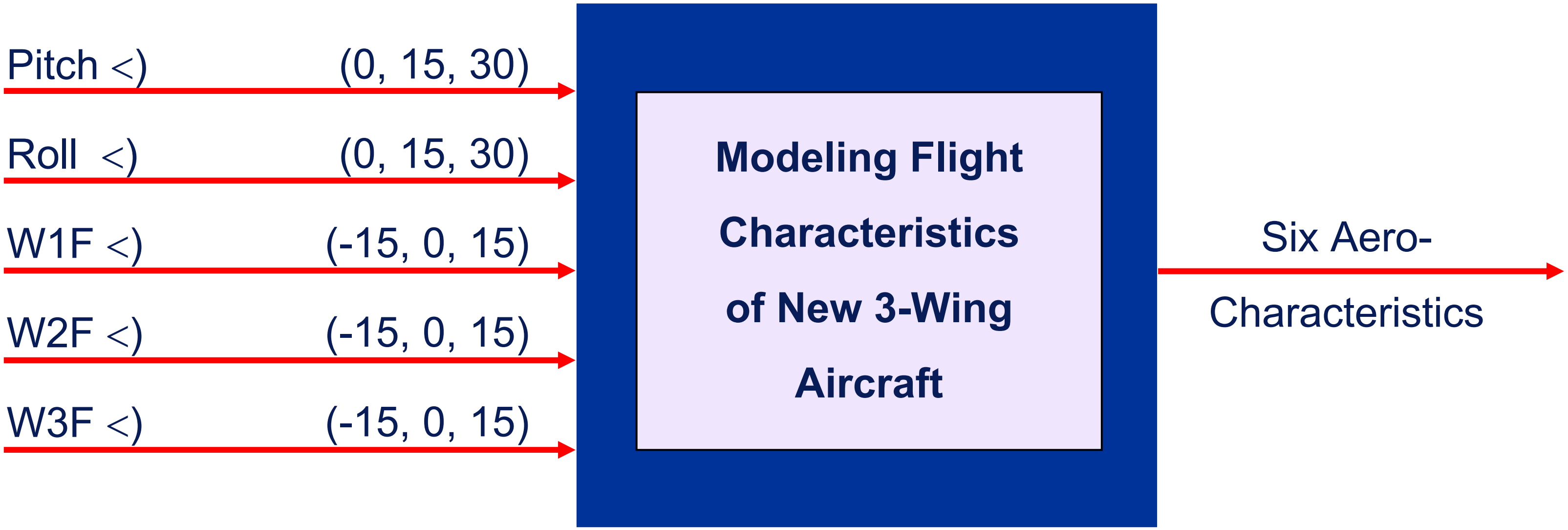


Gray Code Sequence Generator (Wheel)
by Run Number for 16 Runs and 4 Factors

Value Delivery: Reducing Time to Market for New Technologies



INPUT **OUTPUT**



- **Total # of Combinations = $3^5 = 243$**
- **Central Composite Design: n = 30**

Patent Holder: Dr. Bert Silich

Central Composite Designs
(Box-Wilson Designs)

CCD for K=3 Factors

	Run	A	B	C
		FACTOR		
	1	-	-	-
	2	-	-	+
	3	-	+	-
F	4	-	+	+
	5	+	-	-
	6	+	-	+
	7	+	+	-
	8	+	+	+
	9	0	0	0
C	10	0	0	0
	11	0	0	0
	12	$-\alpha$	0	0
	13	$+\alpha$	0	0
A	14	0	$-\alpha$	0
	15	0	$+\alpha$	0
	16	0	0	$-\alpha$
	17	0	0	$+\alpha$

F = factorial portion

C = centerpoint portion

A = axial portion

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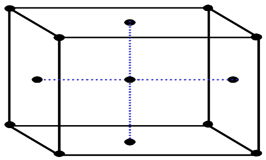
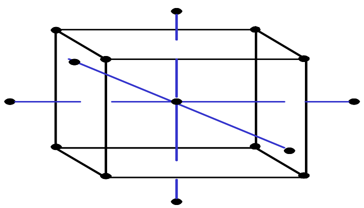
Suggested Values for α and Center Points
(Central Composite Designs)

- **Face-centered Design (α = 1)**
  - Hard limits (restrictions) on factor settings
  - Cannot take factor settings beyond ±1 (coded values)
  - Predictions made within the "cube"
  - Recommended number of center points = 2



- **Spherical Design ($\alpha = \sqrt{k}$)**

  Rotatable Design ($\alpha = (n_F)^{1/4}$)
  - No hard limits (constraints) on factor settings
  - Able to go beyond ±1 coded settings
  - Predictions slightly beyond the "cube" (in case the optimum lies just outside)
  - Orthogonality can be an issue, so a larger number of center points is recommended (between 3 and 6)
    - n_F is the number of runs in the factorial part of the design
    - k is the number of factors



Aircraft Equations

C_L = .233 + .008(P)^2 + .255(P) + .012(R) - .043(WD1) - .117(WD2) + .185(WD3) + .010(P)(WD3) - .042(R)(WD1) + .035(R)(WD2) + .016(R)(WD3) + .010(P)(R) - .003(WD1)(WD2) - .006(WD1)(WD3)

C_D = .058 + .016(P)^2 + .028(P) - .004(WD1) - .013(WD2) + .013(WD3) + .002(P)(R) - .004(P)(WD1) - .009(P)(WD2) + .016(P)(WD3) - .004(R)(WD1) + .003(R)(WD2) + .020(WD1)^2 + .017(WD2)^2 + .021(WD3)^2

C_Y = -.006(P) - .006(R) + .169(WD1) - .121(WD2) - .063(WD3) - .004(P)(R) + .008(P)(WD1) - .006(P)(WD2) - .008(P)(WD3) - .012(R)(WD1) - .029(R)(WD2) + .048(R)(WD3) - .008(WD1)^2

C_M = .023 - .008(P)^2 + .004(P) - .007(R) + .024(WD1) + .066(WD2) - .099(WD3) - .006(P)(R) + .002(P)(WD2) - .005(P)(WD3) + .023(R)(WD1) - .019(R)(WD2) - .007(R)(WD3) + .007(WD1)^2 - .008(WD2)^2 + .002(WD1)(WD2) + .002(WD1)(WD3)

C_{YM}= .001(P) + .001(R) - .050(WD1) + .029(WD2) + .012(WD3) + .001(P)(R) - .005(P)(WD1) - .004(P)(WD2) - .004(P)(WD3) + .003(R)(WD1) + .008(R)(WD2) - .013(R)(WD3) + .004(WD1)^2 + .003(WD2)^2 - .005(WD3)^2

C_e = .003(P) + .035(WD1) + .048(WD2) + .051(WD3) - .003(R)(WD3) + .003(P)(R) - .005(P)(WD1) + .005(P)(WD2) + .006(P)(WD3) + .002(R)(WD1)

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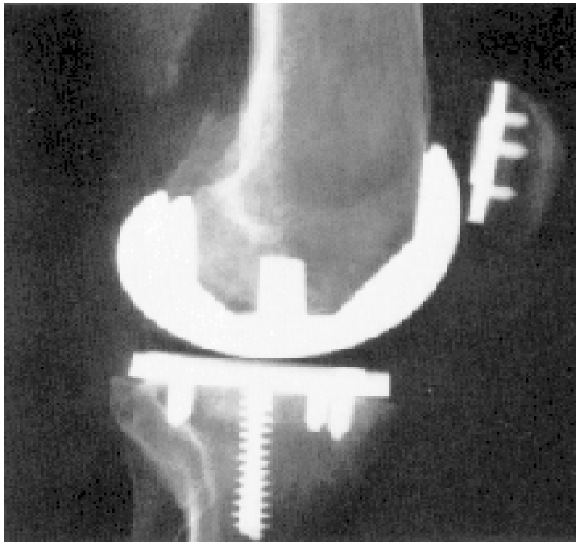
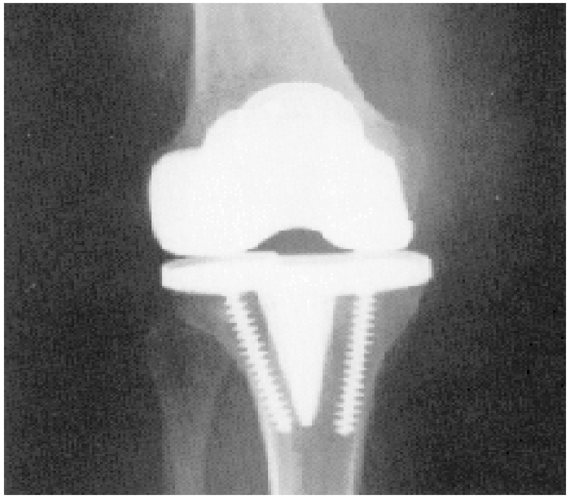
Fusing Titanium and Cobalt-Chrome





Courtesy Rai Chowdhary

Hierarchical Transfer Functions

$$Y = \text{Gross Margin} = \frac{\text{Gross Profit}}{\text{Gross Revenue}}$$

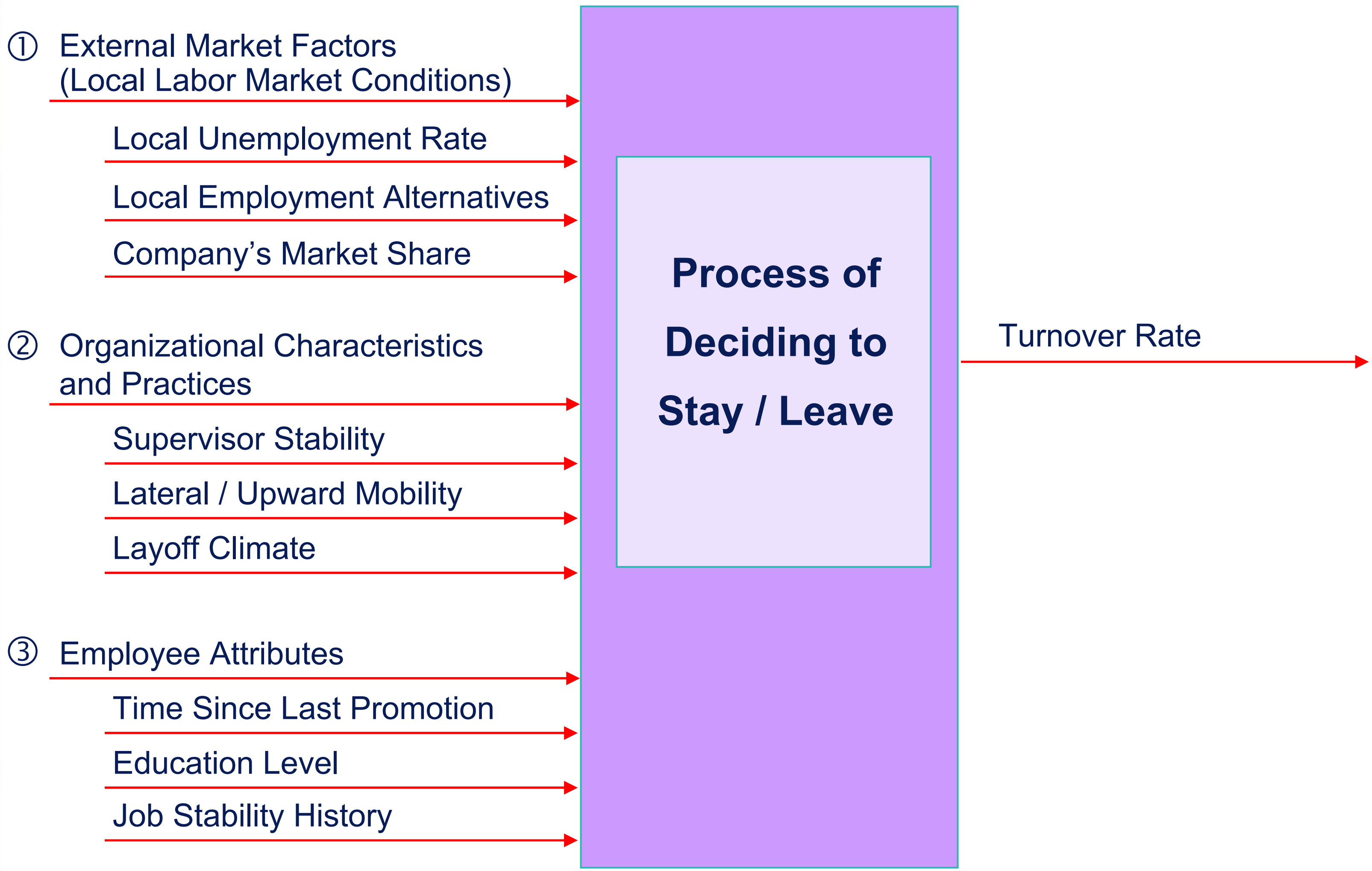
$$Y = f(y_1, y_2, y_3, y_4, y_5, y_6)$$

$$= \frac{\underbrace{(Rev_{equip}}_{y_1} - \underbrace{COG)}_{y_2} + \underbrace{(Rev_{post\ sales}}_{y_3} \underbrace{- Cost_{post\ sales})}_{y_4} + \underbrace{(Rev_{fin}}_{y_5} - \underbrace{Cost_{fin})}_{y_6}}{y_1 + y_3 + y_5}$$

$$\underbrace{Cost_{post\ sales}}_{y_4} = f(\underbrace{\text{field cost}}_{x_1}, \underbrace{\text{remote services}}_{x_2}, \underbrace{\text{suppliers}}_{x_3})$$

$$x_1 = f(\text{direct labor, freight, parts, depreciation})$$

Modeling The Drivers of Turnover



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DOE "Market Research" Example

Suppose that, in the auto industry, we would like to investigate the following automobile attributes (i.e., factors), along with accompanying levels of those attributes:

A: Brand of Auto:	-1 = foreign		+1 = domestic
B: Auto Color:	-1 = light	0 = bright	+1 = dark
C: Body Style:	-1 = 2-door	0 = 4-door	+1 = sliding door/hatchback
D: Drive Mechanism:	-1 = rear wheel	0 = front wheel	+1 = 4-wheel
E: Engine Size:	-1 = 4-cylinder	0 = 6-cylinder	+1 = 8-cylinder
F: Interior Size:	-1 ≤ 2 people	0 = 3-5 people	+1 ≥ 6 people
G: Gas Mileage:	-1 ≤ 20 mpg	0 = 20-30 mpg	+1 ≥ 30 mpg
H: Price:	-1 ≤ $20K	0 = $20-$40K	+1 ≥ $40K

In addition, suppose the respondents chosen to provide their preferences to product profiles are taken based on the following demographic:

J: Age:	-1 ≤ 25 years old	+1 ≥ 35 years old
K: Income:	-1 ≤ $30K	+1 ≥ $40K
L: Education:	-1 < BS	+1 ≥ BS

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DOE "Market Research" Example (cont.)

Question: Choose the best design for evaluating this scenario

Answer: L_{18} design with attributes A - H in the inner array and factors J, K, and L in the outer array, resembling an L_{18} robust design, as shown below:

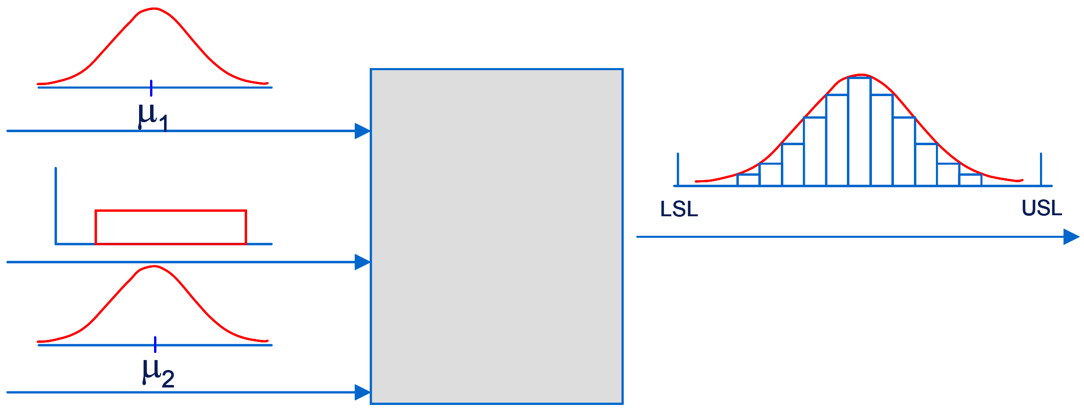
									L	-	+	-	+	-	+	-	+		
									K	-	-	+	+	-	-	+	+		
									J	-	-	-	-	+	+	+	+		
Run*	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**		y_1	y_2	y_3	y_4	y_5	y_6	y_7	y_8	$\bar{y}$	s
1	-	-	-	-	-	-	-	-											
2	-	-	0	0	0	0	0	0											
3	-	-	+	+	+	+	+	+											
4	-	0	-	-	0	0	+	+											
5	-	0	0	0	+	+	-	-											
6	-	0	+	+	-	-	0	0											
7	-	+	-	0	-	+	0	+											
8	-	+	0	+	0	-	+	-											
9	-	+	+	-	+	0	-	0											
10	+	-	-	+	+	0	0	-											
11	+	-	0	-	-	+	+	0											
12	+	-	+	0	0	-	-	+											
13	+	0	-	0	+	-	+	0											
14	+	0	0	+	-	0	-	+											
15	+	0	+	-	0	+	0	-											
16	+	+	-	+	0	+	-	0											
17	+	+	0	-	+	-	0	+											
18	+	+	+	0	-	0	+	-											

Segmentation of the population or

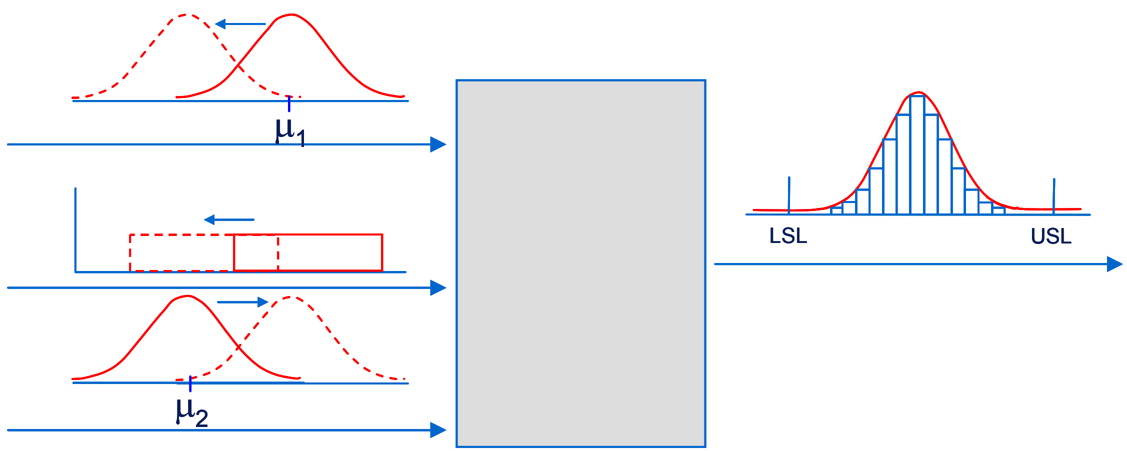
Respondent Profiles

* 18 different product profiles

Robust Design



Process of finding the optimal location parameters (i.e., means) of the input variables to minimize dpm.



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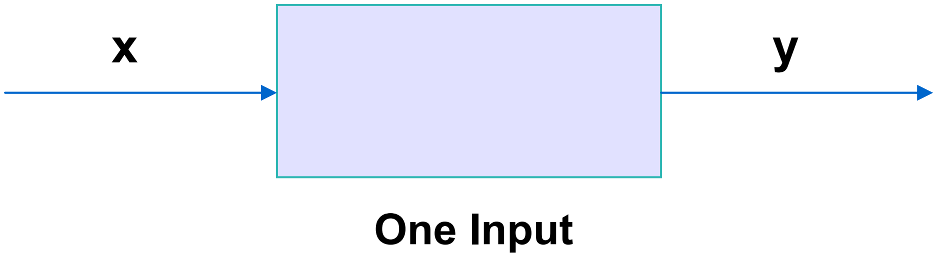
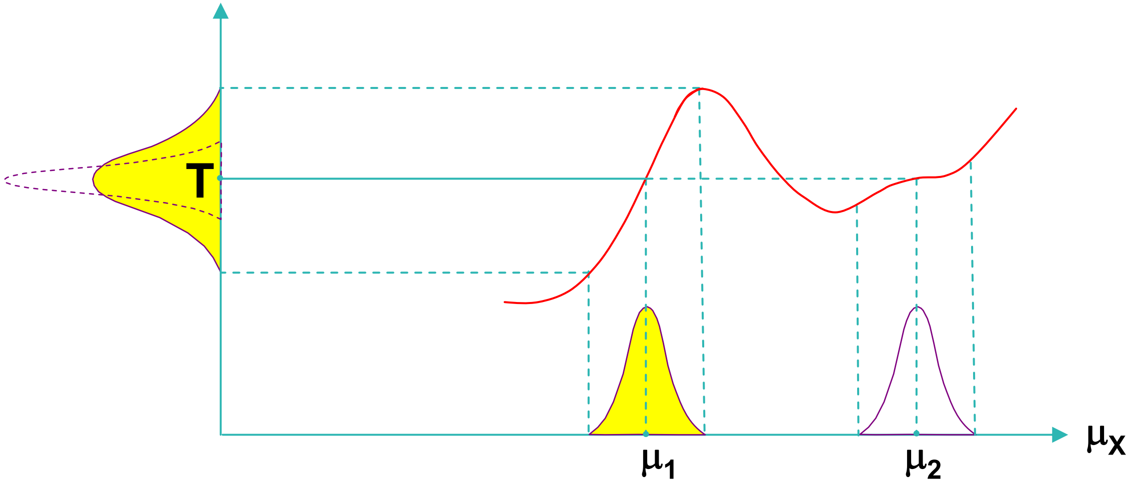
Why Robust Design?





If μ_X varies, should we select μ_1 or μ_2 to hit y = T?

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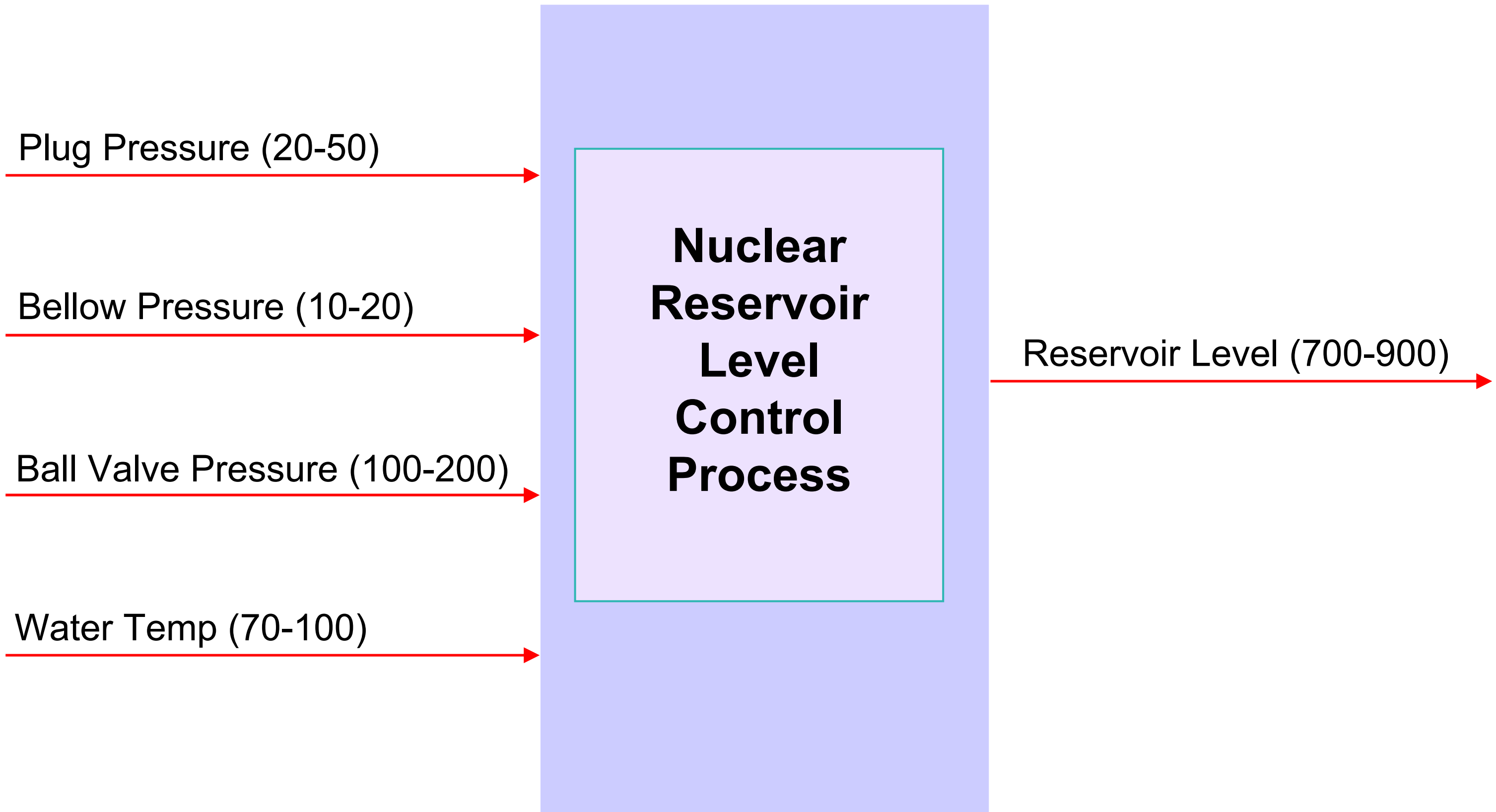
Robust (Parameter) Design Simulation Example



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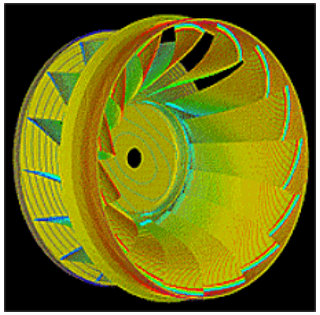
Examples of Simulation and High Performance Computing (HPC)

Power

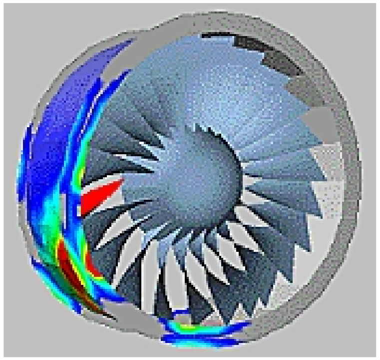


Simulation of stress and vibrations of turbine assembly for use in nuclear power generation

Automotive

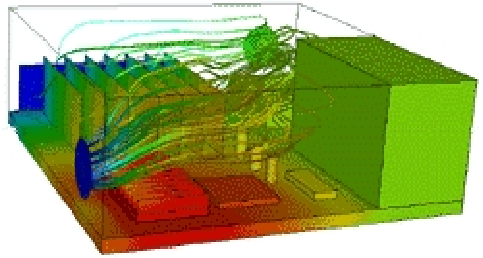
Simulation of underhood thermal cooling for decrease in engine space and increase in cabin space and comfort

Aerospace



Evaluation of dual bird-strike on aircraft engine nacelle for turbine blade containment studies

Electronics



Evaluation of cooling air flow behavior inside a computer system chassis

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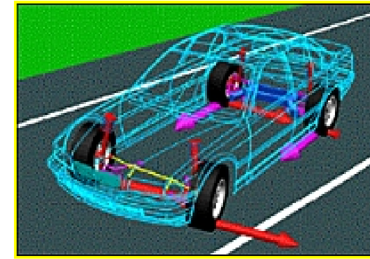
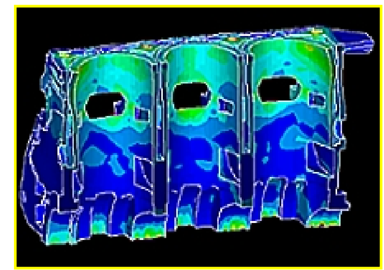
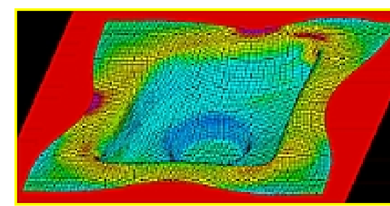
Examples of Computer Aided Engineering (CAE) and Simulation Software



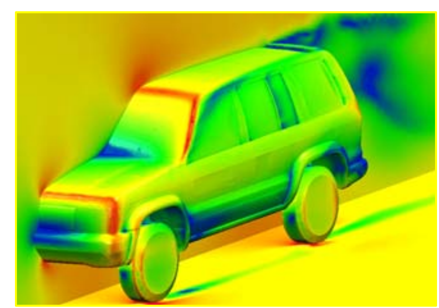
Mechanical motion: Multibody kinetics and dynamics
- ADAMS®
- DADS



Implicit Finite Element Analysis: Linear and nonlinear statics, dynamic response
- MSC.Nastran™, MSC.Marc™
- ANSYS®
- Pro MECHANICA
- ABAQUS® Standard and Explicit
- ADINA



Explicit Finite Element Analysis : Impact simulation, metal forming
- LS-DYNA
- RADIOSS
- PAM-CRASH®, PAM-STAMP



General Computational Fluid Dynamics: Internal and external flow simulation
- STAR-CD
- CFX-4, CFX-5
- FLUENT®, FIDAP™
- PowerFLOW®

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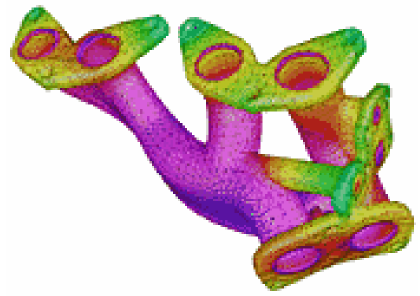
Examples of Computer Aided Engineering (CAE) and Simulation Software (cont.)

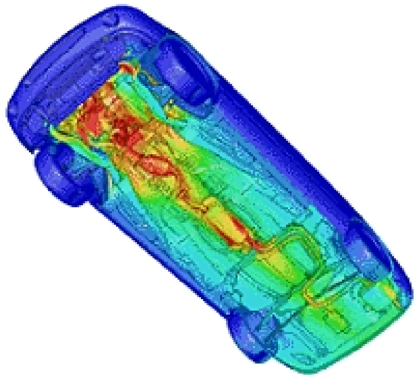
Preprocessing: Finite Element Analysis and Computational Fluid Dynamics mesh generation

- ICEM-CFD
- Gridgen
- Altair® HyperMesh®
- I-deas®
- MSC.Patran
- TrueGrid®
- GridPro
- FEMB
- ANSA



Postprocessing: Finite Element Analysis and Computational Fluid Dynamics results visualization

- Altair® HyperMesh®
- I-deas
- MSC.Patran
- FEMB
- EnSight
- FIELDVIEW
- ICEM CFD Visual3 2.0 (PVS)
- COVISE



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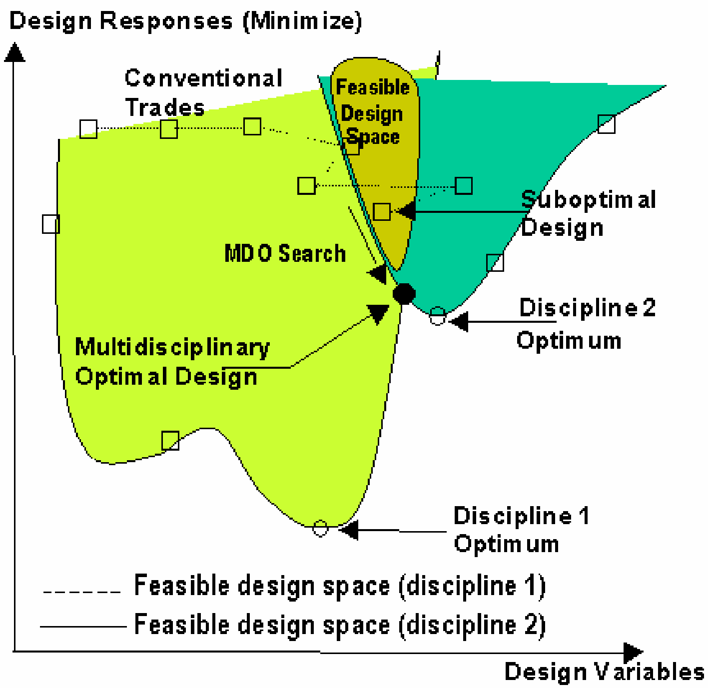
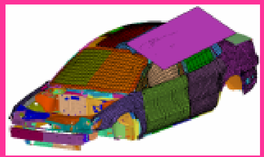
Multidisciplinary Design Optimization (MDO): A Design Process Application





- CFD
- Structures
- Performance
- Controls
- Cost

- NVH
- Safety
- Durability
- Controls Stability
- Cost

Key Elements of MDO

- Massive Computational Problem;
- Solution by decomposition effective for complex systems;
- Multiprocessor computing simplifies MDO solutions conceptually & enables solutions previously intractable;
- Aids in the management of the design process.

Mastery of interactions between the disciplines (or, subsystems) is as important as the methods & tools used within a single discipline

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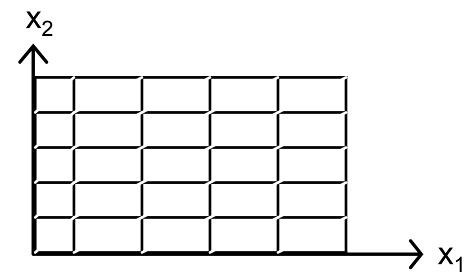
Latin Hypercube Sampling

- Method to populate the design space when using deterministic simulation models or when many variables are involved.

- Design space has k variables (or dimensions).

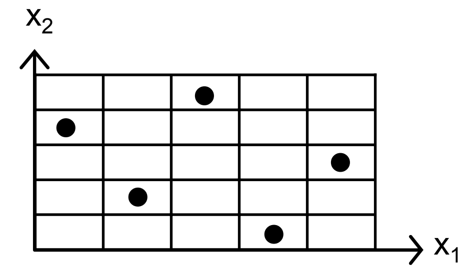
  Ex: Assume k = 2



- Suppose a sample of size n is to be taken; Stratify the design space into n^k cells.

  Ex: Assume n = 5; $n^k = 5^2 = 25$

  Note: there are n strata for each of the k dimensions.



- Each of the n points is sampled such that each marginal strata is represented only once in the sample.

  Note: each sample point has its own unique row and column.

Applying Modeling and Simulation to Automotive Vehicle Design

IDENTIFY CTCs, CDPs → Many, Many x's → **SCREENING DESIGN (DOE PRO)** → The critical few CDP's

Examples of CTCs:

NASTRAN RADIOSS MADYMO

Integrated processes with high fidelity CAE analyses on HPC servers

Examples of Critical Design Parameters (CDPs or Xs):

Safety CTCs with constraints specified by FMVSS (Federal Motor Vehicle Safety Standards)

- y_1 = weight of vehicle
- y_2 = cost of vehicle
- y_3 = frontal head impact
- y_4 = frontal chest impact
- y_5 = toe board intrusion
- y_6 = hip deflection
- y_7 = rollover impact
- y_8 = side impact

(y_3 – y_8): RADIOSS, DYNA, MADYMO

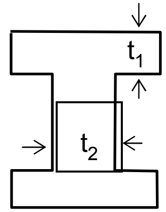
no federal requirements on these CTCs

- y_9 = internal aerodynamics (airflow)
- y_{10} = external aerodynamics (airflow)
- y_{11} = noise
- y_{12} = vibration (e.g., steering wheel)
- y_{13} = harshness (e.g., over bumps, shocks)
- y_{14} = durability (at 100K miles)

(y_9 – y_{10}): CFD

(y_{11} – y_{14}): NASTRAN

- x_1 = roof panel material
- x_2 = roof panel thickness
- x_3 = door pillar dimensions ⇒ i beam
- x_4 = shape/geometry
- x_5 = windshield glass
- x_6 = hood material, sizing and thickness
- x_7 = under hood panel material, sizing and thickness



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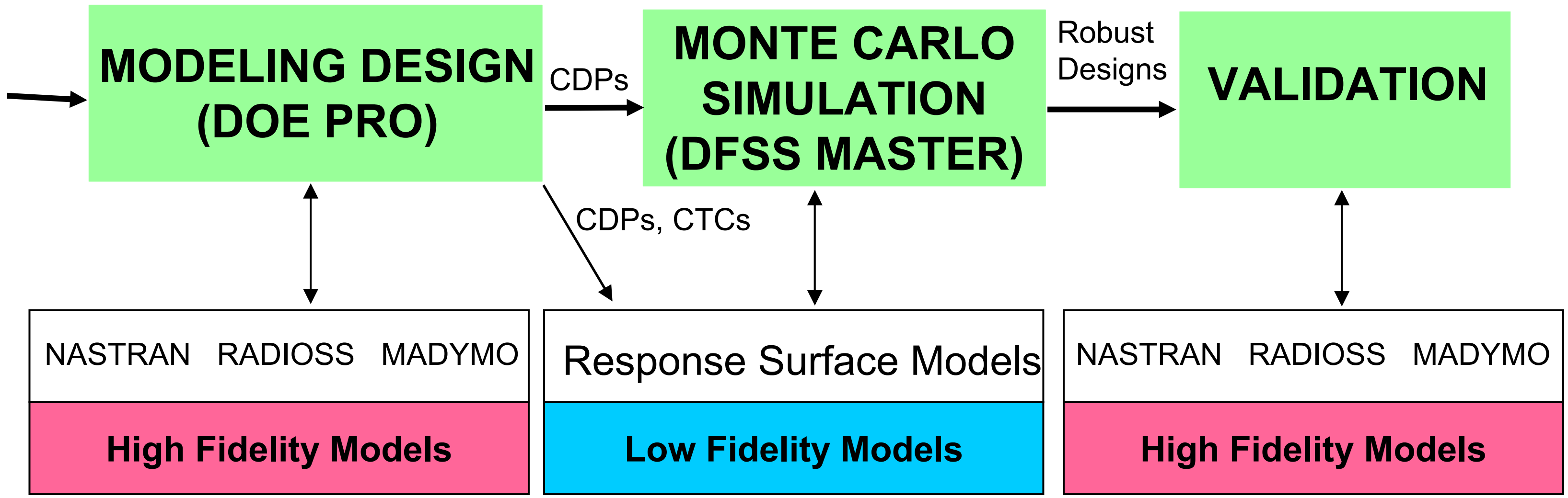
Applying Modeling and Simulation to Automotive Vehicle Design (cont.)



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Introduction to High Throughput Testing (HTT)

- A recently developed technique based on combinatorics
- Used to test myriad combinations of many factors (typically qualitative) where the factors could have many levels
- Uses a minimum number of runs or combinations to do this
- Software (e.g., ProTest) is needed to select the minimal subset of all possible combinations to be tested so that all n-way combinations are tested.
- HTT is not a DOE technique, although the terminology is similar
- A run or row in an HTT matrix is, like DOE, a combination of different factor levels which, after being tested, will result in a successful or failed run
- HTT has its origins in the pharmaceutical business where in drug discovery many chemical compounds are combined together (combinatorial chemistry) at many different strengths to try to produce a reaction.
- Other industries are now using HTT, e.g., software testing, materials discovery, interoperability testing, IT (see IT example on next page)

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HTT Example

- An IT function in a company wanted to test all 2-way combinations of a variety of computer configuration-related options or levels to see if they would function properly together.
- Here are the factors with each of their options:

Motherboards (5)	:	Gateway, ASUS, Micronics, Dell, Compaq
RAM (3)	:	128 MB, 256 MB, 512 MB
BIOS (3)	:	Dell, Award, Generic
CD (3)	:	Generic, Teac, Sony
Monitor (5)	:	Viewsonic, Sony, KDS, NEC, Generic
Printer (3)	:	HP, Lexmark, Cannon
Voltage (2)	:	220, 110
Resolution (2)	:	800x600, 1024x768

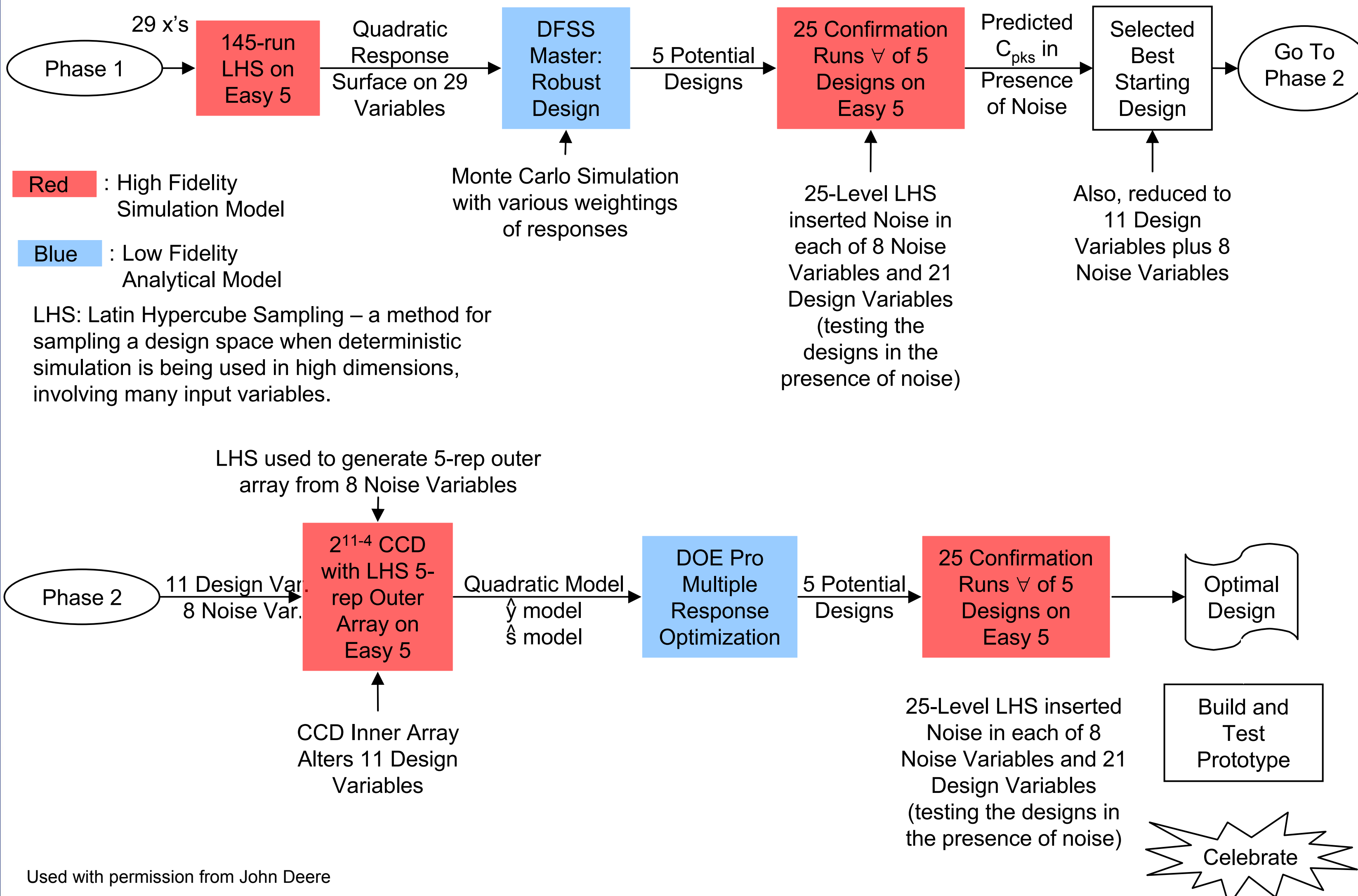
- How many total combinations are there?
- What is the minimum number of these combinations we will have to test (and which ones are they) in order to determine if every 2-way combination (e.g., Dell Bios with Teac CD) will indeed work properly together?
- To answer this question, we used Pro-Test software. The answer is 25 runs and those 25 combinations are shown on the next page.

High Throughput Testing (HTT)

(for all two-way combinations)

	5 Levels	3 Levels	3 Levels	3 Levels	5 Levels	3 Levels	2 Levels	2 Levels
	Motherboard	**RAM**	**BIOS**	**CD**	**Monitor**	**Printer**	**Voltage**	**Resolution**
Case 1	ASUS	256 MB	Dell	Generic	Viewsonic	Lexmark	110 V	800 x 600
Case 2	Compaq	512 MB	Dell	Teac	Sony	HP	220 V	1024 x 768
Case 3	Gateway	128 MB	Generic	Sony	KDS	Cannon	220 V	800 x 600
Case 4	Dell	128 MB	Award	Teac	NEC	Cannon	110 V	1024 x 768
Case 5	Micronics	256 MB	Generic	Teac	Generic	Lexmark	220 V	1024 x 768
Case 6	Gateway	256 MB	Award	Sony	Sony	HP	110 V	1024 x 768
Case 7	Micronics	512 MB	Award	Generic	Viewsonic	Cannon	220 V	1024 x 768
Case 8	ASUS	512 MB	Generic	Teac	KDS	HP	220 V	1024 x 768
Case 9	Compaq	128 MB	Award	Generic	Generic	HP	110 V	800 x 600
Case 10	Micronics	512 MB	Generic	Teac	Sony	Lexmark	110 V	800 x 600
Case 11	Dell	256 MB	Award	Generic	KDS	Lexmark	110 V	1024 x 768
Case 12	Gateway	512 MB	Dell	Sony	Generic	Lexmark	110 V	1024 x 768
Case 13	Compaq	256 MB	Generic	Sony	Viewsonic	Cannon	220 V	1024 x 768
Case 14	ASUS	128 MB	Dell	Sony	NEC	Cannon	220 V	800 x 600
Case 15	Micronics	128 MB	Dell	Sony	KDS	Lexmark	220 V	800 x 600
Case 16	Gateway	128 MB	Generic	Teac	Viewsonic	HP	110 V	800 x 600
Case 17	Dell	128 MB	Dell	Sony	Sony	Cannon	110 V	1024 x 768
Case 18	ASUS	256 MB	Award	Sony	Generic	Cannon	220 V	1024 x 768
Case 19	Compaq	512 MB	Dell	Sony	NEC	Lexmark	110 V	800 x 600
Case 20	Gateway	256 MB	Generic	Generic	NEC	Cannon	220 V	800 x 600
Case 21	Micronics	512 MB	Generic	Teac	NEC	HP	220 V	800 x 600
Case 22	ASUS	256 MB	Generic	Generic	Sony	HP	110 V	800 x 600
Case 23	Dell	512 MB	Generic	Sony	Viewsonic	HP	220 V	1024 x 768
Case 24	Compaq	256 MB	Dell	Generic	KDS	Cannon	220 V	1024 x 768
Case 25	Dell	128 MB	Generic	Sony	Generic	HP	110 V	800 x 600

Example of Iterative Approach to Modeling and Simulation to Optimize Transmission Performance



Used with permission from John Deere

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Technologies Used on Transmission Project

- **High Throughput Testing (HTT)**

  To generate a minimal number of test cases; in this scenario, 11 combinations of 29 variables that would allow testing all two-way combinations on Easy 5. This made running the Easy 5 simulator much easier, without interruption.

- **DFSS Master: for Robust Design and Expected Value Analysis (Monte Carlo Techniques)**

- **Highly Orthogonal Latin Hypercube Sampling**

  To populate the design space with test cases which are highly orthogonal. Typically used with deterministic simulation to screen out the CDPs and also to use modeling DOEs on the simulator to generate transfer functions which characterize the simulator

- **DOE PRO: for Multiple Response Optimization**

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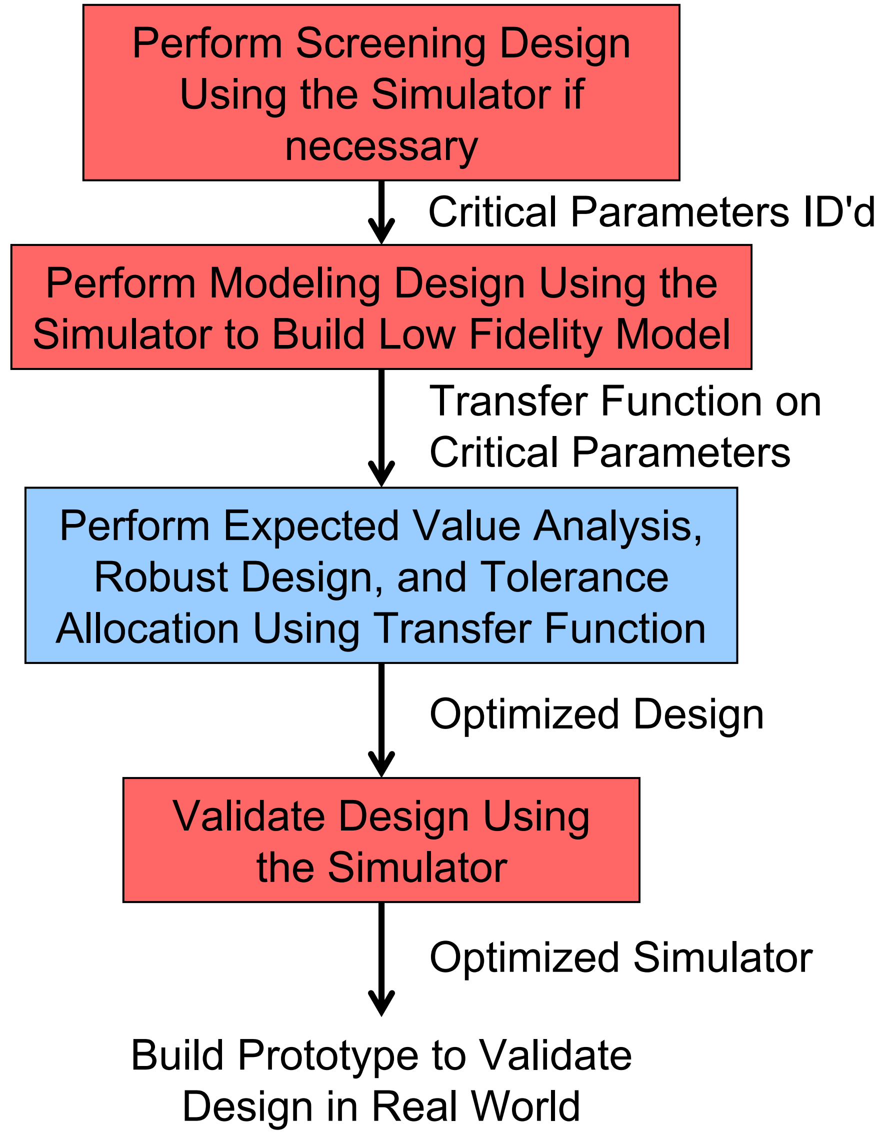
Summary of "Modeling the Simulator"



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Environments Where Simulation and Modeling Is Beneficial

- **A high number of design variables**
- **A substantial number of design subsystems and engineering disciplines**
- **Interdependency and interaction between the subsystems and variables**
- **Multiple response variables**
- **Need to characterize the system at a higher level of abstraction**
- **Time and/or space must be compressed**

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Key Take-Aways

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