Impact Switch Study

Modeling & Implications

Dr. Dave Frankman

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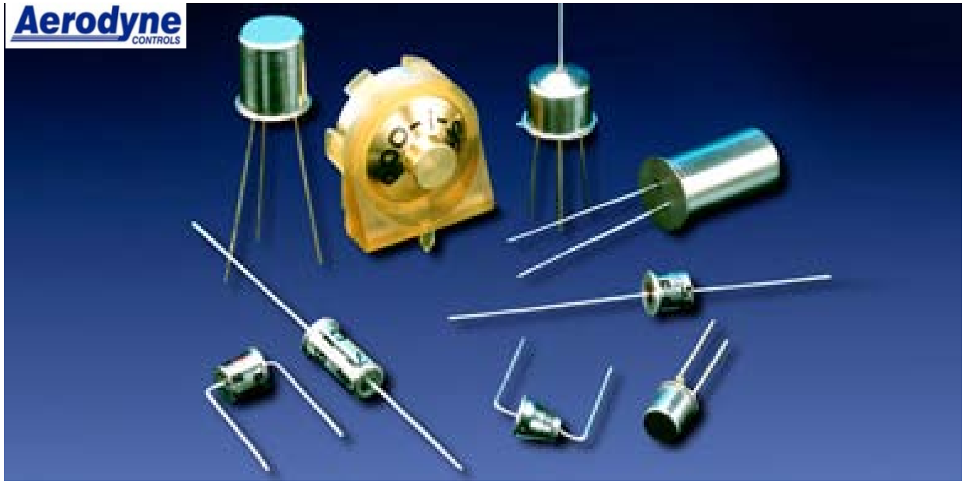
Agenda

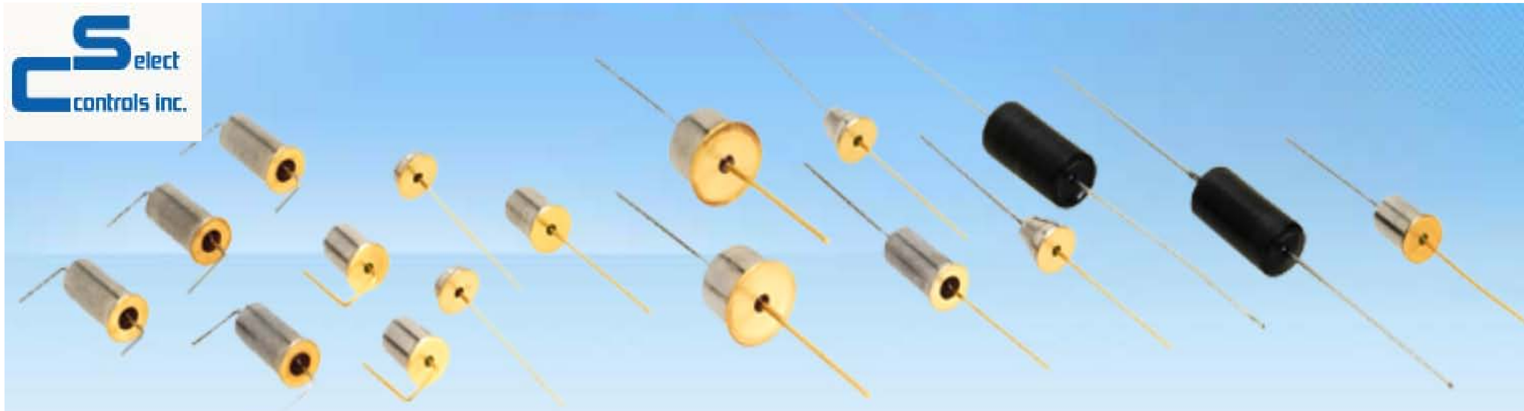
- Study motivation
- Introduction to spring/mass impact switches
- Derivation of spring/mass governing equations from first principles
- Results of study
- Derivation of mass/spring/damper system
- Results of parametric damping study
- Conclusions

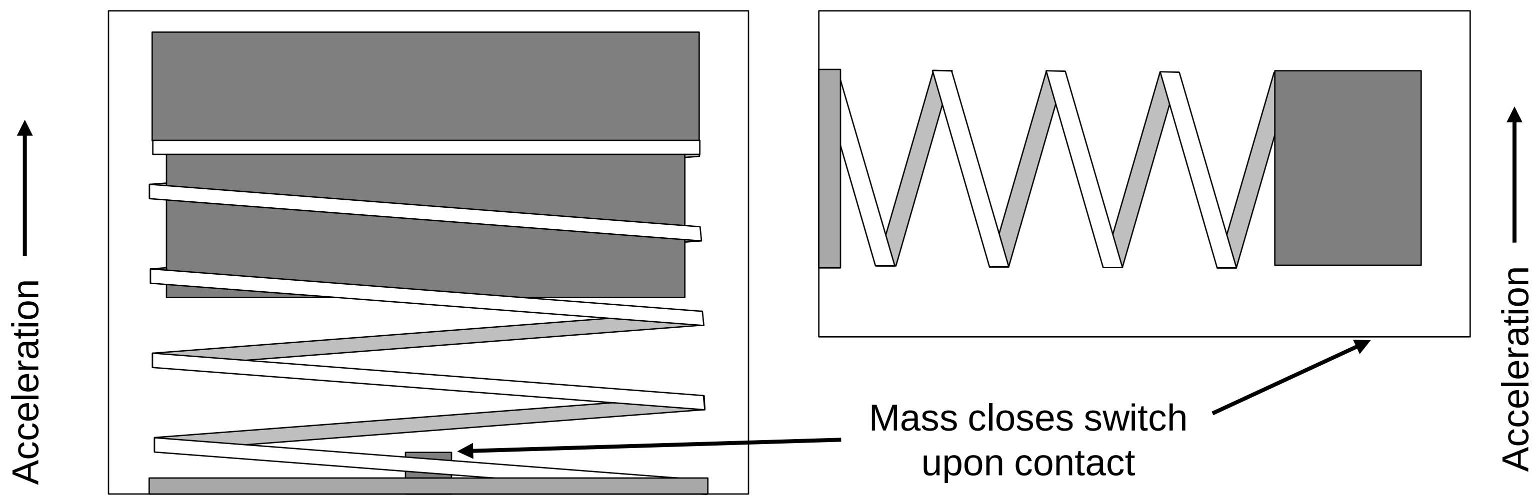
Motivation

- Dynamic/static behavior revealed
  - Switch closure is dependent on the amplitude and duration of shock
- Evaluate current testing practices
- Enable characterization of switch behavior analytically rather than empirically



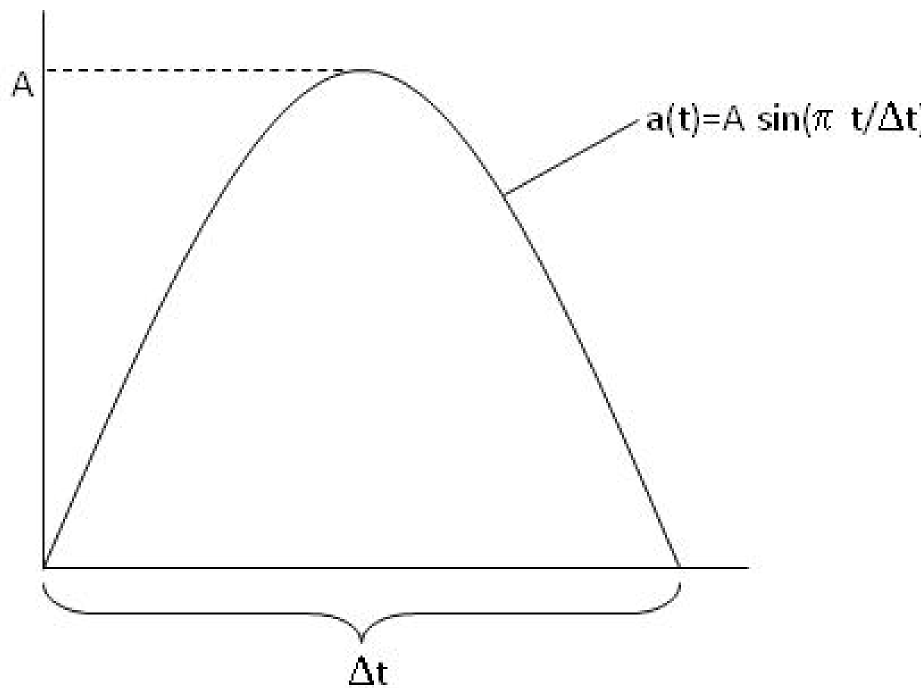
Impact Switches are Spring/Mass Systems





Spring/Mass Motion Derived from First Principles

- The governing inhomogeneous Ordinary Differential Equation (ODE) is derived from Newton's second law ($\Sigma F = ma$)
  - The spring mass system has a natural frequency of $\omega_o = \sqrt{(k/m)}$
  - A half sine acceleration pulse is applied to the switch



$$\sum F = m\ddot{x} = -kx + m\,a(t)$$

$$\Downarrow$$

$$\ddot{x} + \omega_o^2 x = a(t)$$

$$x(0) = 0$$

$$\dot{x}(0) = 0$$

ODE Solved via. Method of Undetermined Coef's

$$\ddot{x} + \omega_o^2 x = A\sin\left(\pi \frac{t}{\Delta t}\right) \qquad x(0) = 0 \qquad \dot{x}(0) = 0$$

$$x_h(t) = \frac{A\,\pi/\Delta t}{\omega_o\left(\pi^2/\Delta t^2 - \omega_o^2\right)}\sin(\omega_o t)$$

$$x_p(t) = \frac{-A}{\left(\pi^2/\Delta t^2 - \omega_o^2\right)}\sin\left(\pi t/\Delta t\right)$$

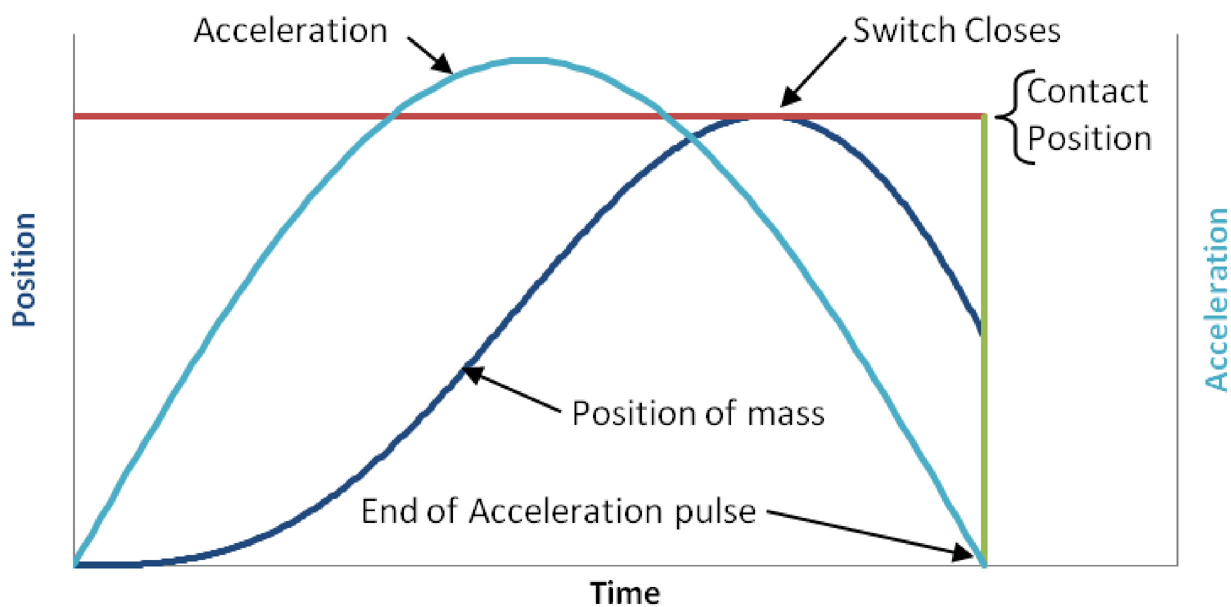
Homogeneous and particular solution are combined to form solution ($y=y_p+y_h$)

$$x(t) = \frac{A}{\omega_o\left(\pi^2/\Delta t^2 - \omega_o^2\right)}\left[\frac{\pi}{\Delta t}\sin(\omega_o t) - \omega_o \sin\left(\pi t/\Delta t\right)\right]$$

Equation governing position of mass

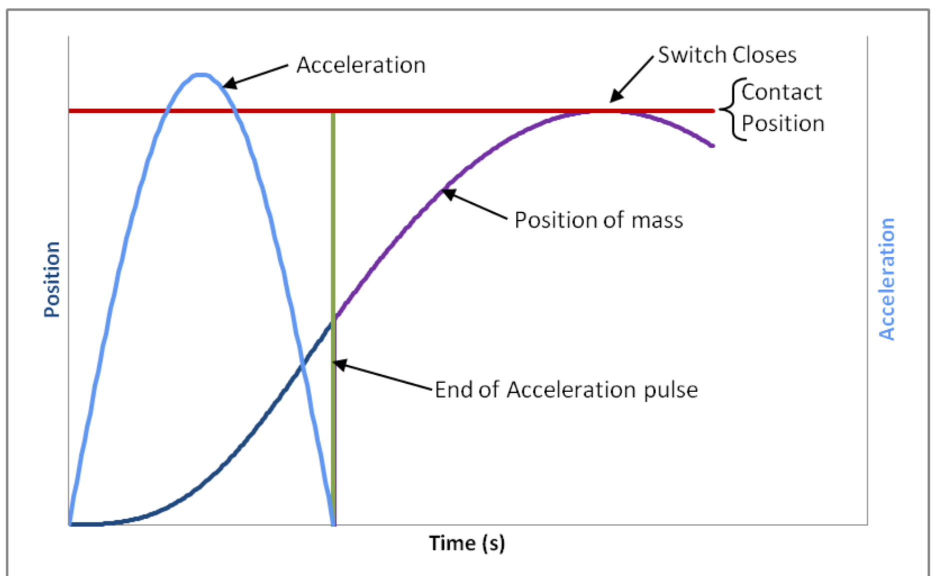
Switch Closure Before Pulse Ends

- Switch closes before acceleration pulse ends ($\Delta t<\pi/\omega_o$)
  - Mass moves at spring/mass natural frequency



Switch Closure After Pulse Ends

- Switch closes after acceleration pulse ends ($\Delta t > \pi/\omega_o$)
  - Mass has sufficient kinetic energy to close the switch after the acceleration pulse ends.
  - This scenario requires the solution of another ODE.



Motion of Mass After Pulse Requires Another ODE Solution

- Solution to the homogenous ODE is completed using the method of undetermined coefficients.

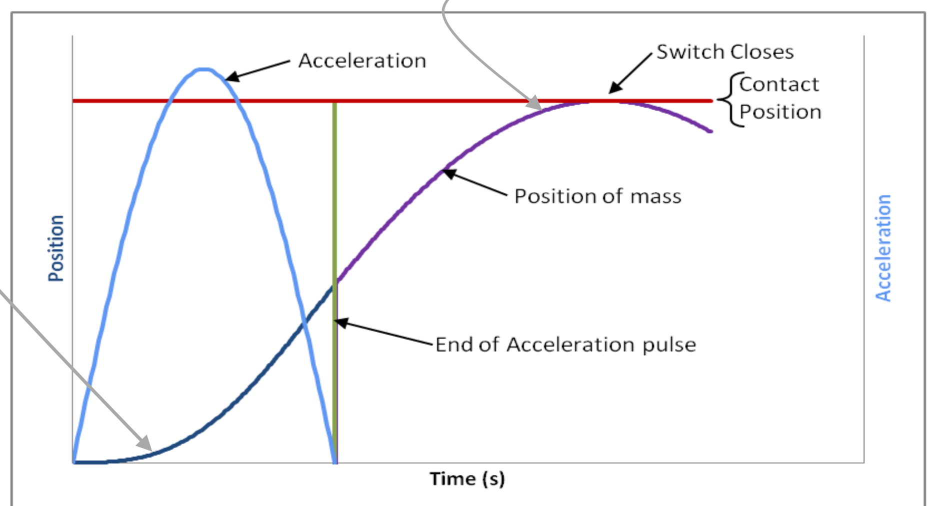
$$x(t) = \frac{A}{\omega_o\left(\pi^2/\Delta t^2 - \omega_o^2\right)}\left[\frac{\pi}{\Delta t}\sin(\omega_o t) - \omega_o \sin\left(\pi t/\Delta t\right)\right]$$

$$\ddot{x} + \frac{k}{m}x = 0$$

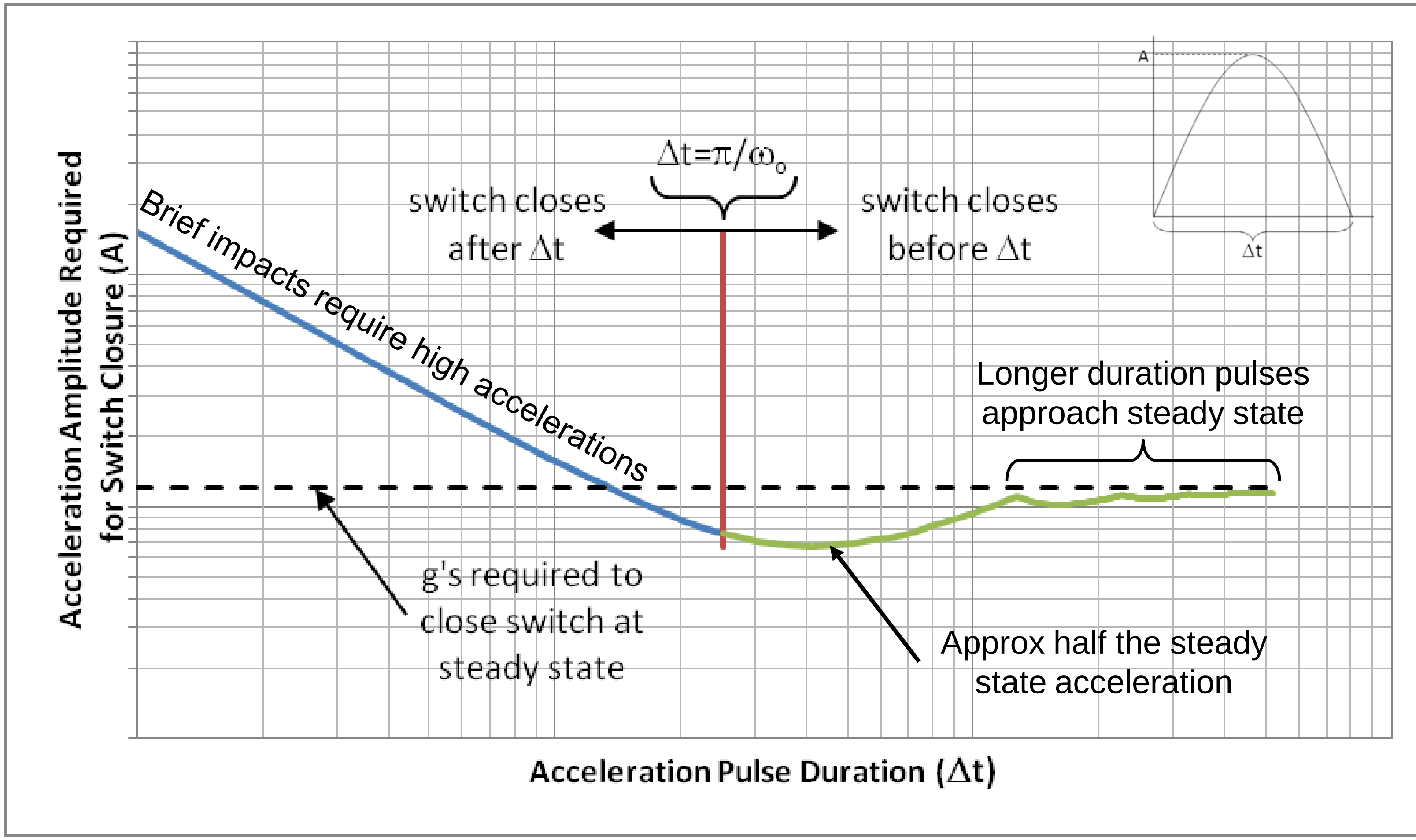
$$x(\Delta t) = x_i$$

$$\dot{x}(\Delta t) = V_i$$

$$x(t) = \frac{V_i}{\omega_o}\sin[\omega_o(t-\Delta t)] + x_i\cos[\omega_o(t-\Delta t)]$$

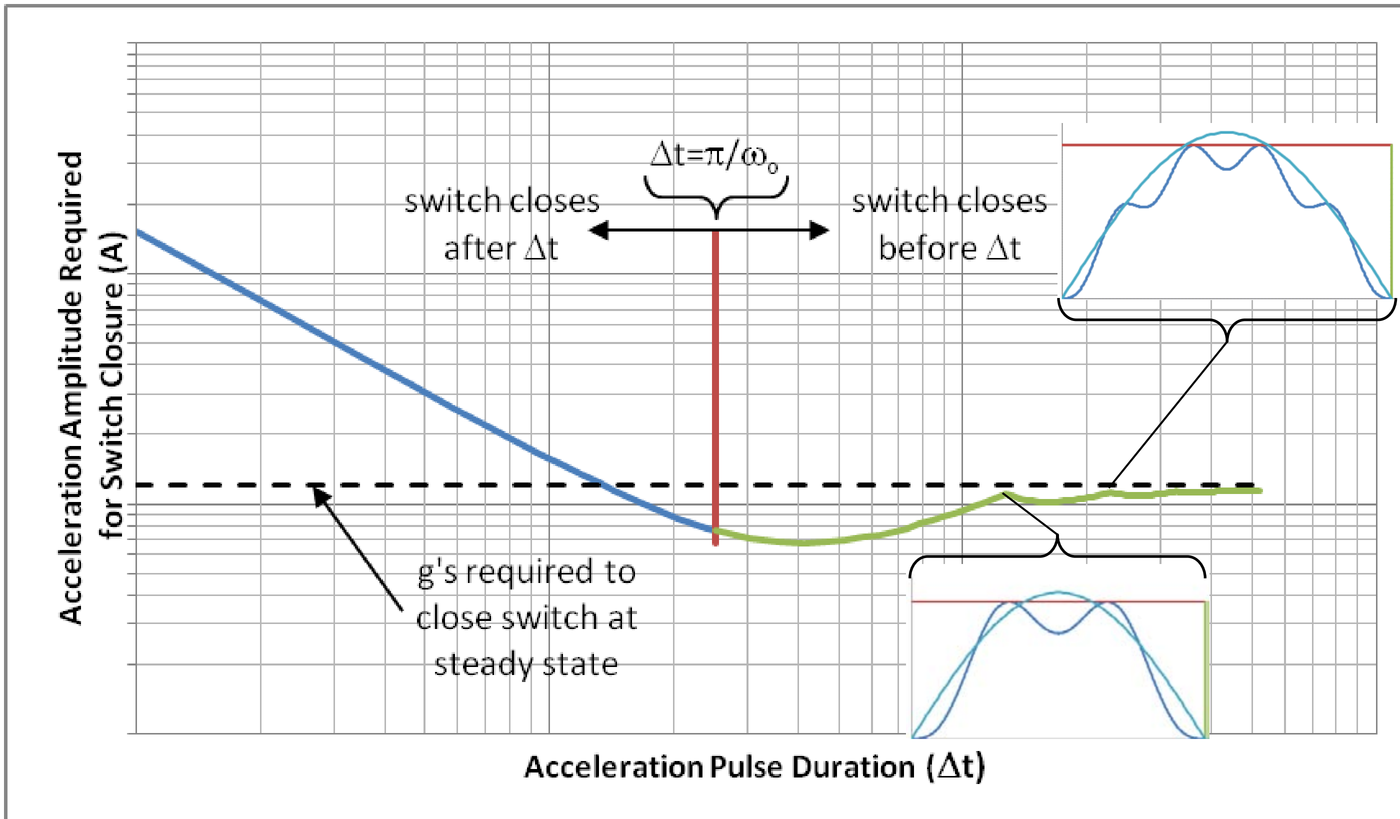


Switch Closes at Various Acceleration Levels



Unusual Behavior of Spring/Mass is Explained

- If the mass has zero net displacement and at rest at the end of the pulse, the solution approaches the steady state solution



Damping Was Also Studied

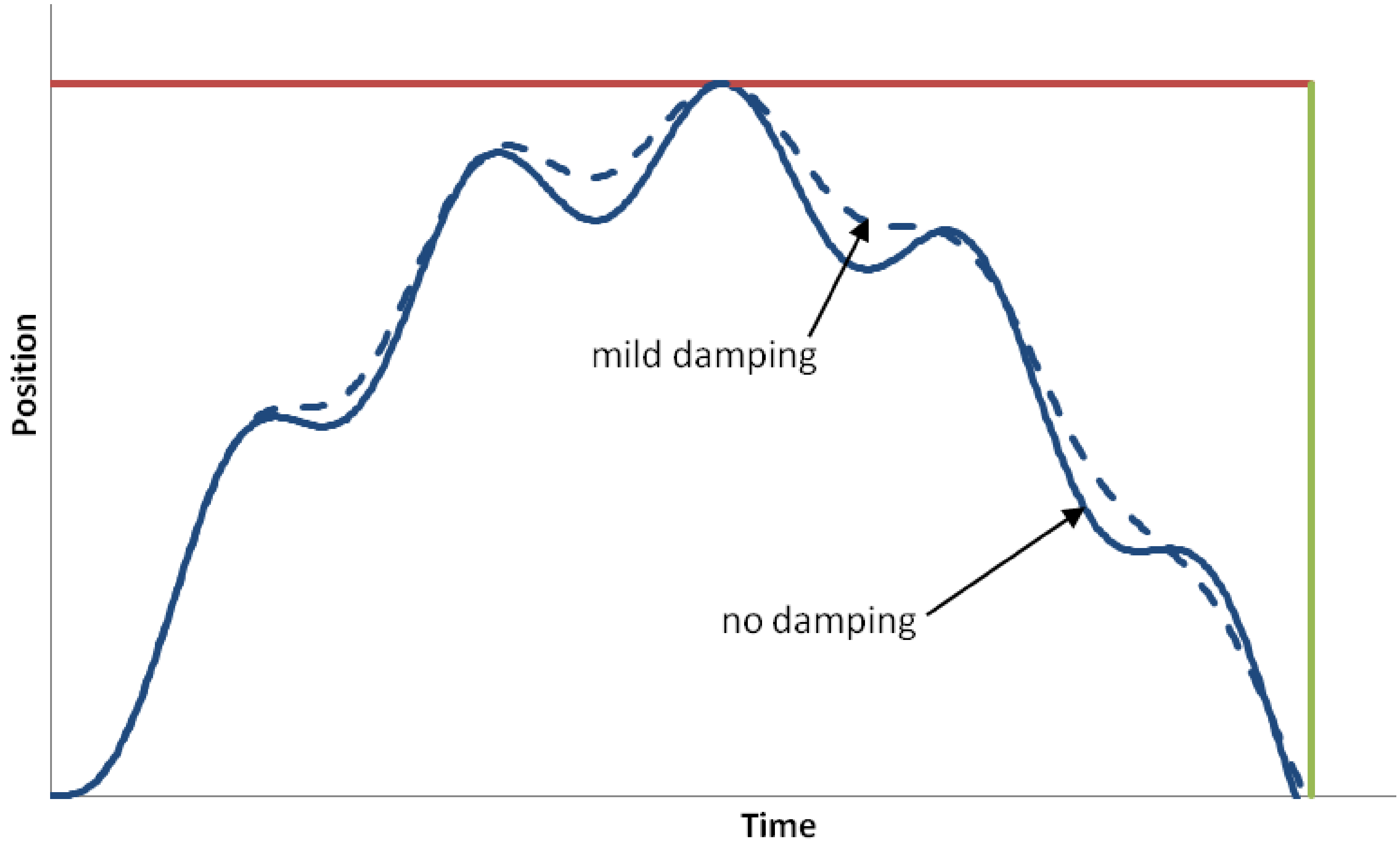
- Damping ratio was parametrically studied ($0 \le \zeta < 1$)

$$\sum F = m\ddot{x} = -kx + m\,a(t) - c\dot{x} \rightarrow \ddot{x} + 2\zeta\omega_o\dot{x} + \omega_o^2 x = A\sin\left(\pi \, t/\Delta t\right)$$

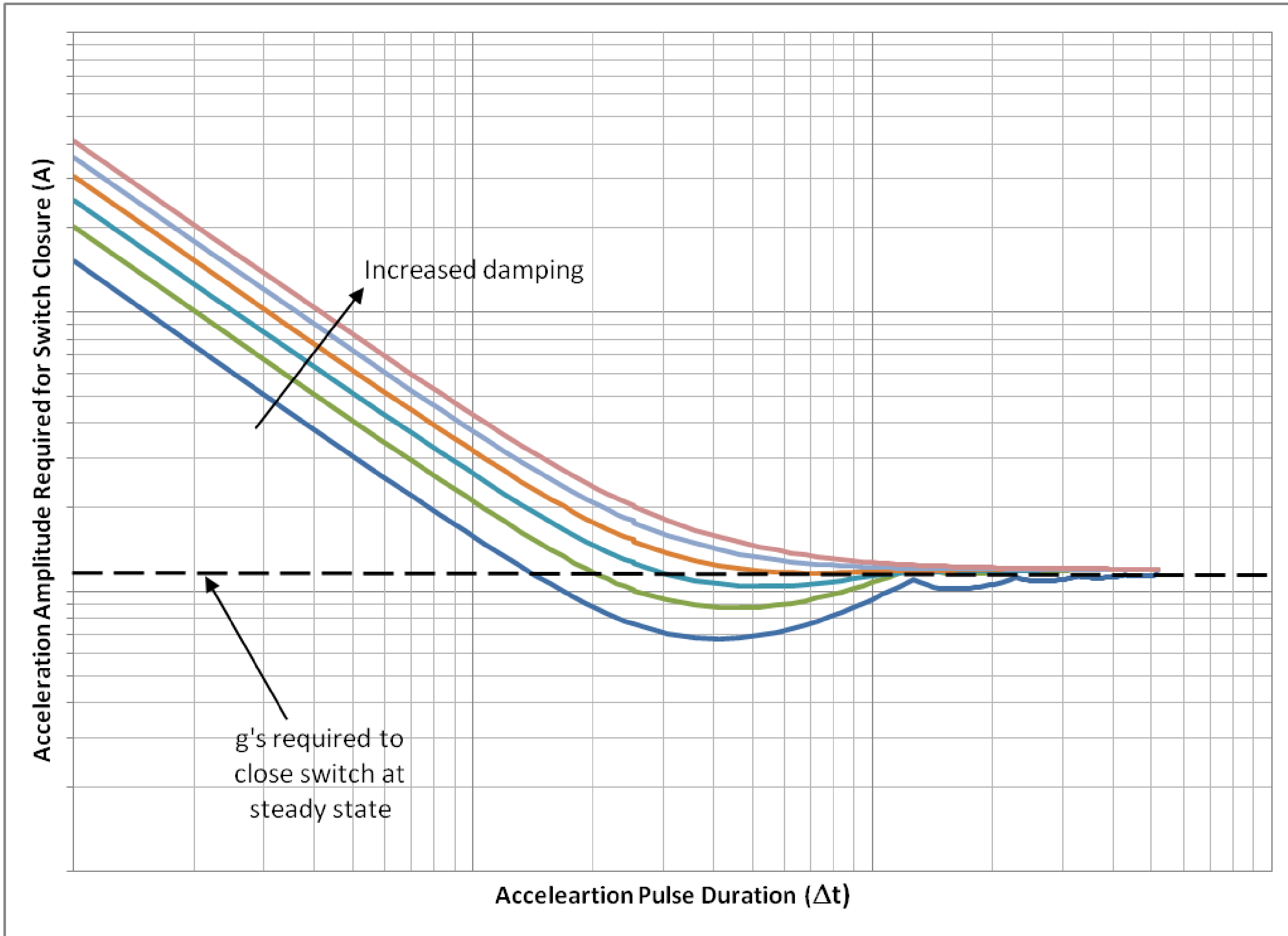
$$Where \quad \zeta = \frac{c}{2\sqrt{km}}$$

$$x(t) = \frac{A\left(\omega_o^2 - \pi^2/\Delta t^2\right)}{\omega_d\left(\left(2\zeta\omega_o \, \pi/\Delta t\right)^2 + \left(\omega_o^2 - \pi^2/\Delta t^2\right)^2\right)} \left[\begin{array}{l} \omega_d \sin\left(\pi \, t/\Delta t\right) - \pi/\Delta t \, e^{-\zeta\omega_o t}\sin(\omega_d t) \\ + \dfrac{2\zeta\omega_o \, \pi/\Delta t}{\omega_o^2 - \pi^2/\Delta t^2} \left\{ \begin{array}{l} \zeta\omega_o e^{-\zeta\omega_o t}\sin(\omega_d t) \\ + \omega_d e^{-\zeta\omega_o t}\cos(\omega_d t) \\ - \omega_d \cos\left(\pi \, t/\Delta t\right) \end{array} \right\} \end{array} \right]$$

Damping Mitigates Oscillations



Damping Suppresses the Spring/Mass Oscillations



Conclusions

- Impact switches will close at a variety of different acceleration levels
- Closure of the impact switch becomes independent of duration as the pulse is lengthened
- Damping increases the acceleration level required to close the switch
- Damping mitigates the switch natural frequency
- Predicting the behavior of the impact switch enables L-3 FOS to reduce development time