Using Cost and Schedule Estimates Guided by Design of Experiments Process to Plan Schedule-Optimal or Cost-Optimal Test Designs for Integrated Development Testing and Operational Testing

National Defense Industrial Association Test and Evaluation Conference March 2, 2010

Dr. David Lee (Primary Author)

E. Andrew Long (Co-author, Briefer)

LMI

CLEARED FOR OPEN PUBLICATION, 09-S-1734

Work Sponsored By:

- Director of Operational Test and Evaluation
- Deputy Director, Assessments and Support, Systems and Software Engineering
- Deputy Under Secretary of Defense (Logistics and Materiel Readiness)

Although the sponsors' support is gratefully acknowledged, positions expressed are those of LMI, not official positions of the sponsors.

Overview

- Reliability Investment Model (RIM) Development and Calibration
- RIM handling of Design for Reliability (DFR) and DOE concepts
- RIM usage with classical reliability test design to make schedule- or cost-optimal plans for testing
- Testing efficiencies resulting from usage of POF data

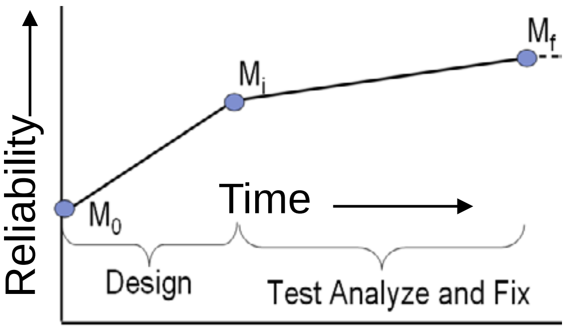
LMI

The Reliability Investment Model

Extends AMSAA's AMPM* to give cost and schedule

Definitions:

- A-mode failure: Will not be mitigated.
- B-mode failure: Will be mitigated.
- Reliability engineering process:



Assumptions:

- The cost of operating a period increases proportionally to the time in that period.
- Action taken to mitigate the i^{th} observed B-mode failure adds an incremental cost b_i (TAAF) or B_i (Design).
- Mitigation of the ith B-mode failure reduces the failure rate by a factor d_i (TAAF) or D_i (Design)
- Failure rates of B-modes are realizations of K independent random variables identically distributed with the Gamma distribution having scale, shape parameters α, β.
- Model takes a large-K limiting form, meaning the number of initial B-modes is assumed very large.

Concepts:

- Assuming the same premises as the *AMPM, LMI rederived the AMPM model, incorporating terms representing cost.
- The model distinguishes between improvements achieved during Design and TAAF periods and relates the cost of improvements to the rate of mitigating B-mode failures in each Period.

Limitation:

- The design-phase and TAAF-phase models are limited by the set of data on which they are calibrated.
- Data, particularly for the design phase, comprise relatively small sets of cases dominated by programs of one commodity class.

*Army Materiel Systems Analysis Activity (AMSAA) Maturity Projection Model

LMI

Reliability Improvement-Cost Model for the Test Analyze and Fix (TAAF) Period

Eq. 1: Expresses reliability

$$\frac{1}{M(\tau)} = \frac{1}{M_A} + \frac{1}{M_i}\left[(1-\mu_d) + \frac{\mu_d}{1+\tau}\right]$$

Eq. 2: Expresses cost

$$\gamma(\tau) = \frac{1}{cv^2}\left[gM_i\tau + \mu_b \ln(1+\tau)\right]$$

Eq. 1: Parameters

M_A :Mean time between A-mode failures

M_i :Mean time between B-mode failures at start of TAAF. Upper bound known; not considered an adjustable factor

μ_d :Average value of the reliability improvements made by corrective action; experience has developed typical values of μ_d, (0.7 to 0.8),so that this parameter also is often known a priori.

Eq. 2: Parameters

cv^2 : A large-K limiting form of coefficient of variation of the sum of the K failure rates giving the initial MTB B-mode failures.

g :Burn rate for operating the tests ($/time)

μ_b :The average value of the cost increments incurred by corrective action taken to mitigate identified B-modes ($).

Design-phase model identical in form, parameter values differ

LMI

Current RIM Calibration Results

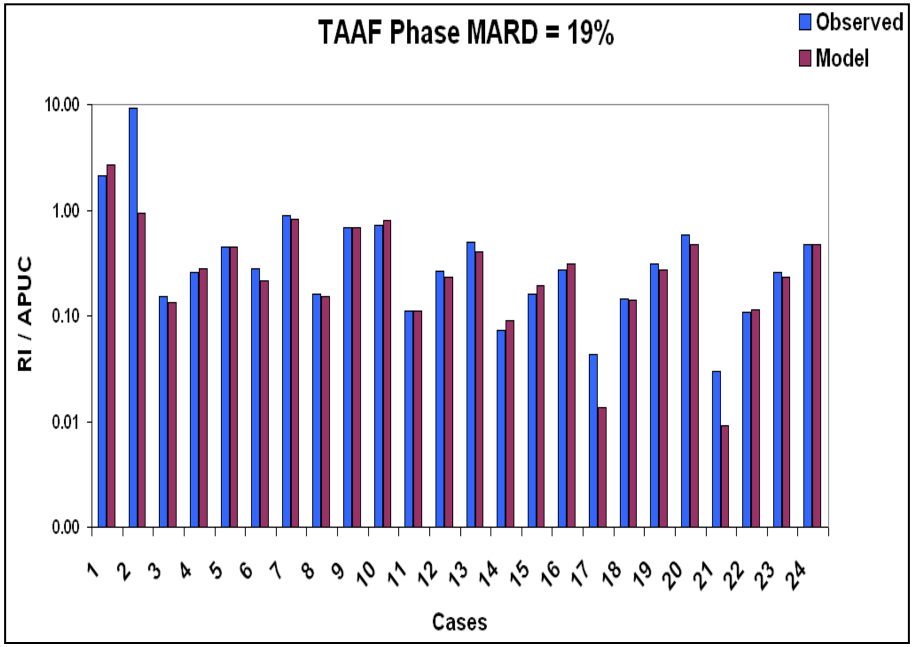
Data:

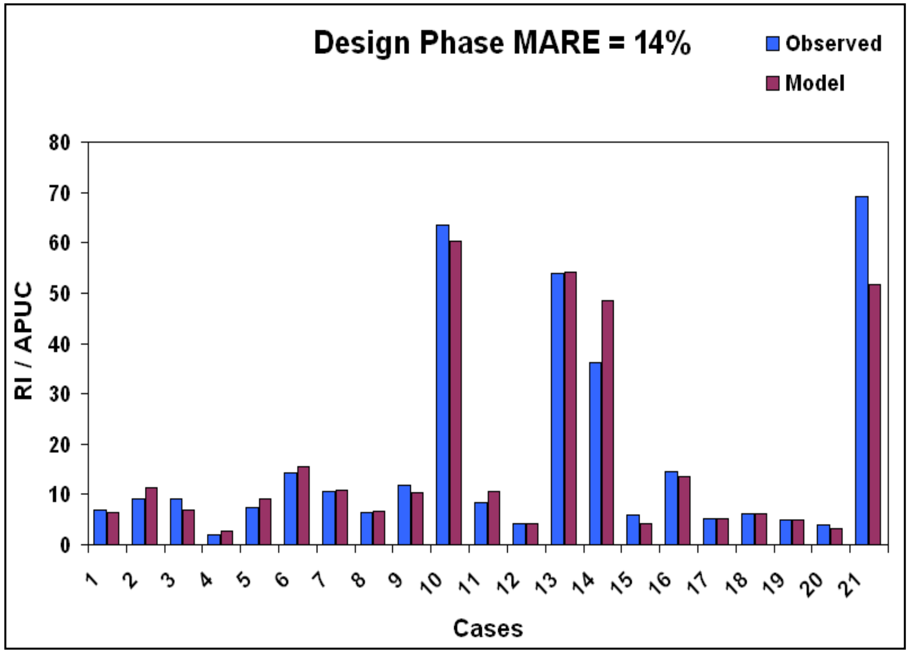
•Data for the reliability and cost of TAAF and Design periods are from 24 and 21 cases respectively involving ground and air platforms.

Approach:

•Both models calibrated to produce minimum mean absolute relative deviation (MARD).

Results:



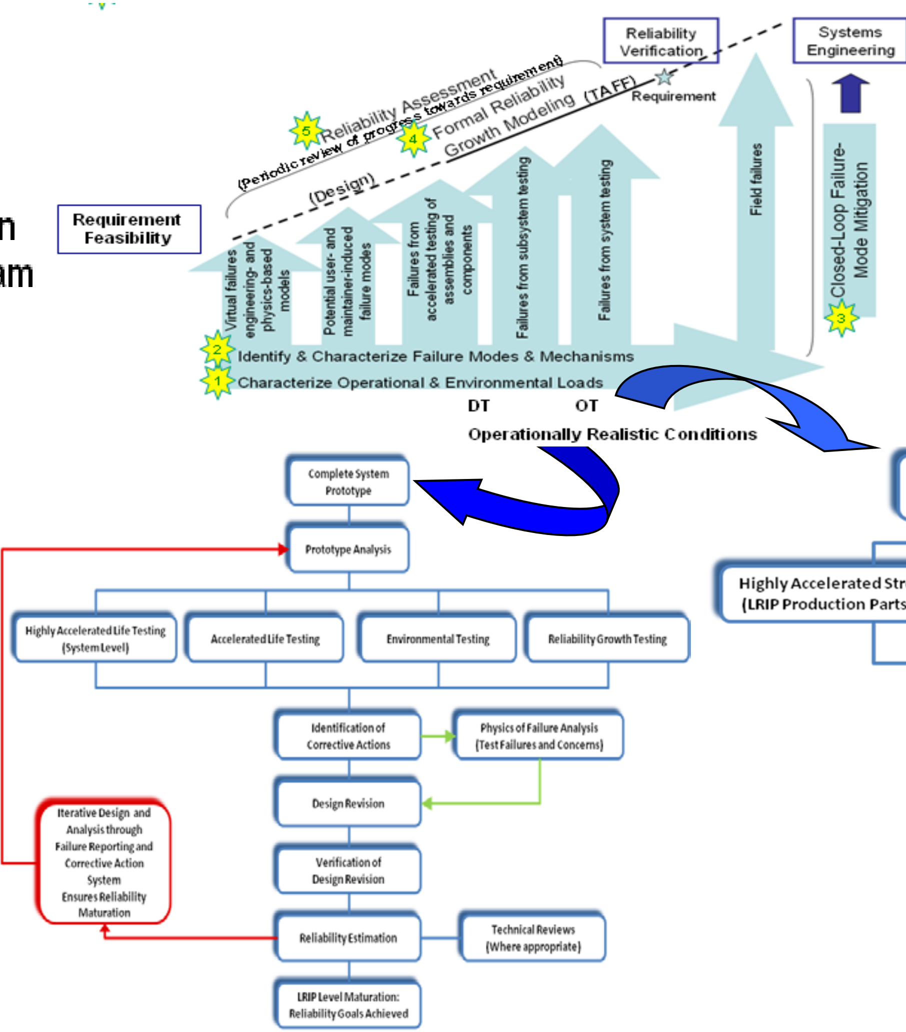


LMI

DoE, DFR, and RIM

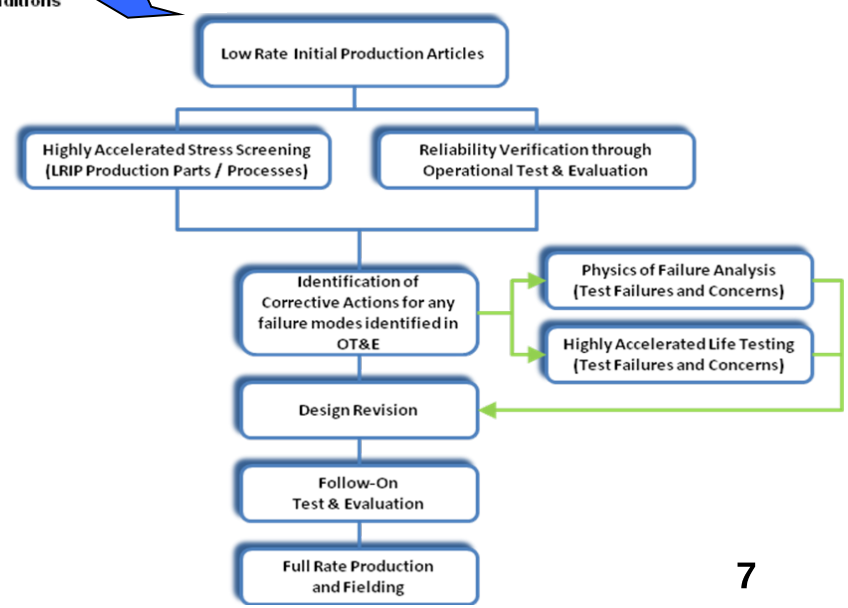
DOE Key Elements ⟷ **DFR Best Practices** ⟷ **RIM**

1. Start early
2. Form team of stakeholders
3. Develop master plan for complete program
4. Focus strategy
5. Iterative deign & analysis
6. Accumulate eviden[ce] across operational envelope
7. Apply DoE to design/redesign for reliability growth



$$\gamma(\tau) = \frac{1}{cv^2}\left[gM_i\tau + \mu_b \ln(1+\tau)\right]$$

- Training and Development
- Reliability Analysis
- Reliability Testing
- Supply chain management
- Failure tracking and reporting
- Validation and verification
- Reliability improvements



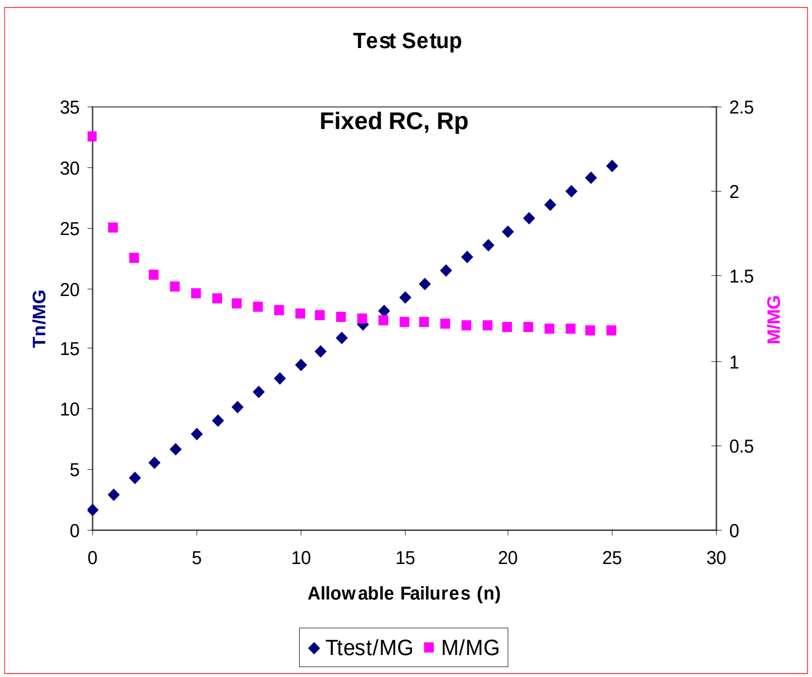
LMI

Classical Reliability Demonstration Test

(assumes exponentially distributed times-between failures)

Eq. 3: $$P(E_n \mid MG, T_n) = \exp\left(-\frac{T_n}{MG}\right)\sum_{j=0}^{n}\frac{1}{j!}\left(\frac{T_n}{MG}\right)^j = 1 - \text{P(pass test with } M < MG) = \frac{1}{2}\chi^2_{2(n+1),R_C} \Rightarrow \frac{T_n}{MG}$$

Eq. 4: $$\text{Ppass} = \exp\left(-\frac{T_n}{MG}\frac{MG}{M}\right)\sum_{j=0}^{n}\frac{1}{j!}\left(\frac{T_n}{MG}\frac{MG}{M}\right)^j = \text{1-P(fail test with } M > MG) = \frac{\chi^2_{2(n+1),R_C}}{\chi^2_{2(n+1),1-R_P}} \Rightarrow \frac{M}{MG}$$

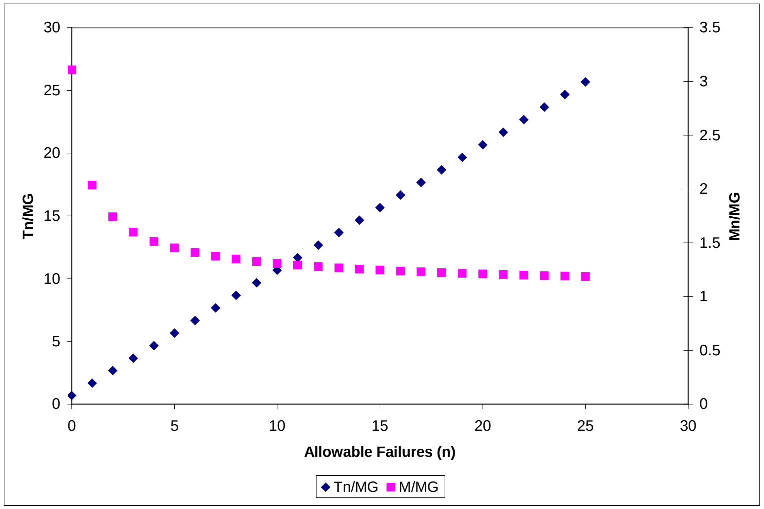


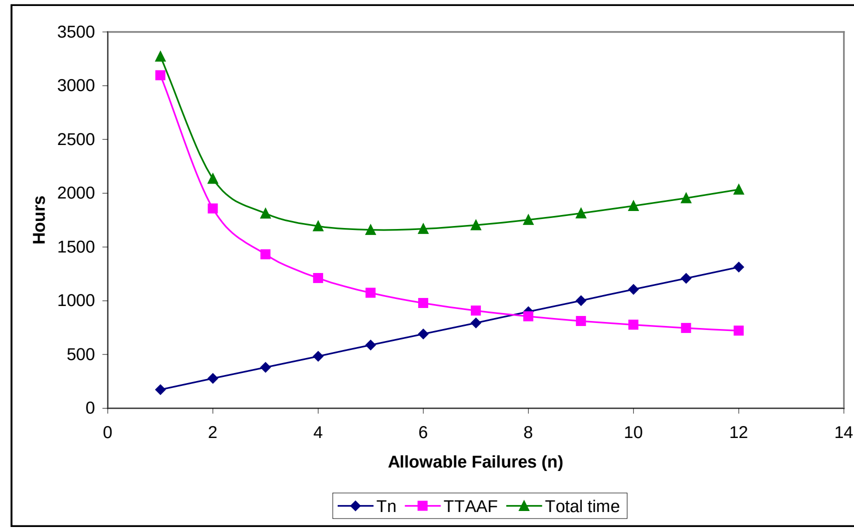
OR:

For fixed R_C, Rp, increasing n increases test time, decreases M/MG

LMI

Schedule-optimal and Cost-optimal Integrated Reliability Improvement and Testing





$$\frac{T_n(R_C)}{MG} = \frac{1}{2}\chi^2_{2(n+1),R_C}$$

$$\frac{M_n(R_C,R_P)}{MG} = \frac{\chi^2_{2(n+1),R_C}}{\chi^2_{2(n+1),1-R_P}}$$

$$\tau = \frac{M_i\left[\frac{1}{M_n}-\frac{1}{M_A}\right]-1}{1-\mu_d-M_i\left[\frac{1}{M_n}-\frac{1}{M_A}\right]}; \quad T_{TAAF} = \frac{1}{\beta}\tau \qquad (*)$$

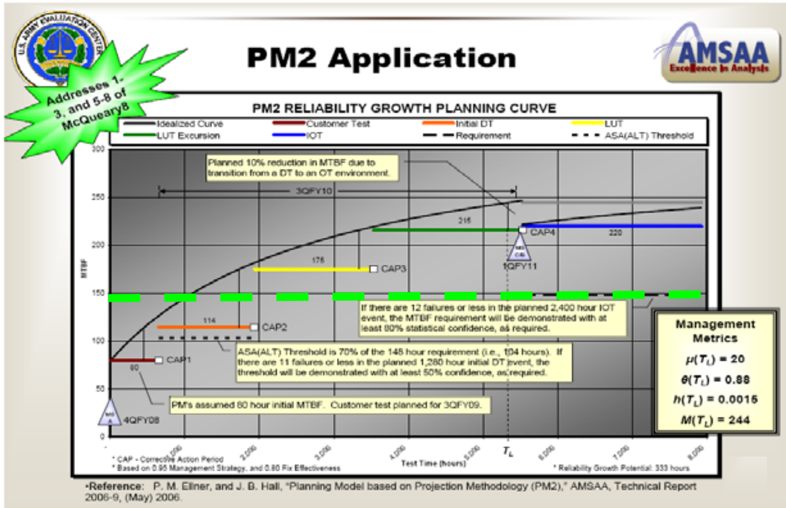
For cost-optimal plan, find τ as in (*), and use

$$\gamma(\tau) = \frac{1}{cv^2}\left[gM_0\tau + \mu_b \ln(1+\tau)\right] \qquad (**)$$

Total of TAAF and Test times may exhibit a minimum.

LMI

Sample Cost and Schedule Options for Army Threshold Requirement



ASA(ALT) threshold is 70% of the requirement demonstrated at 50% confidence. In this example, the system must demonstrate 103.6 hours at 50% confidence. Test time and allowable failures can be determined by referencing 50% confidence curves.

From the cost and schedule estimates of Eqs 1&2 with the test design information of Eqs 3 & 4, we obtain the following cost and schedule estimates:

Max. Fail.	Test Time.	TAAF Time	Total Time	TAAF Cost/APUC	Test Cost/APUC	Total Cost/APUC
1	174	3096	3270	11.7	0.02	11.7
3	380	1431	1811	7.8	0.04	7.8
5	**587**	**1073**	**1660**	**6.5**	**0.06**	**6.6**
6	691	978	1669	6.1	0.07	6.2

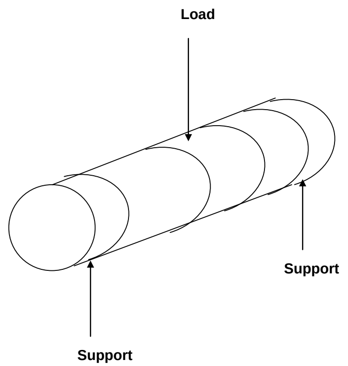
MG = 103.6 Hours, *RC = 50% and RP = 20%*, μd = 0.75, β = 0.00155 (hr) -1, and λ_A = 0.

LMI

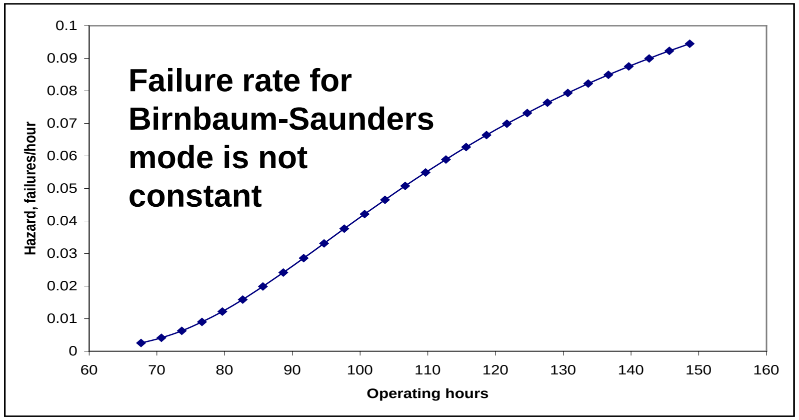
Integrating Prior Information to Improve Testing Efficiency

Ammunition Transfer System

Mode	Failure Time
F	107.3
F	235.2
F	332.5
F	413.3
1	454.4
F	495.3
F	569.8
2	580.3
1	607.7
3	619.2
2	668.5
F	720.1
1	802.3
F	826.1
1	909.1



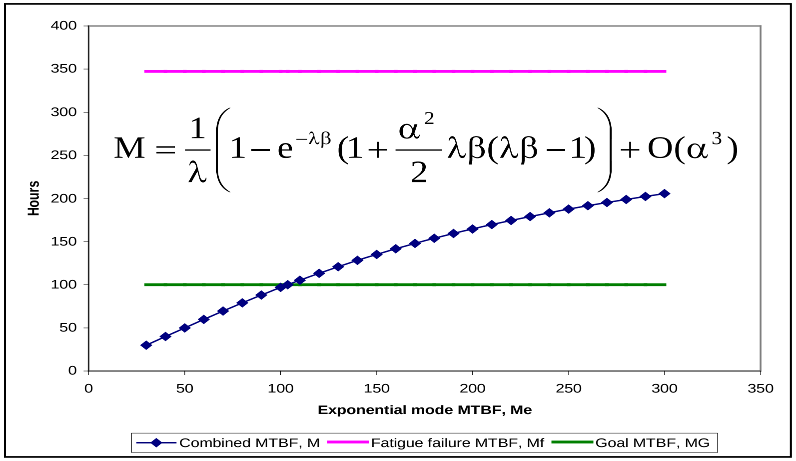
Mode F: crack propagation in rotating bending



Cumulative failure probability for crack propagation

(Birnbaum-Saunders Model)

$$F_f(t) = CN\left(\frac{1}{\alpha}\left[\sqrt{\frac{t}{\beta}} - \sqrt{\frac{\beta}{t}}\right]\right)$$



Analysis of combined MTBF, and available data on crack propagation, allow development and testing on exponential modes alone at considerable savings in test time compared with ab initio testing.

LMI

Summary and Conclusions

- With LMI Reliability Investment Model, managers can plan programs of integrated reliability improvement and demonstration test as designed experiments
- Methods of experimental design incorporating prior information give shorter, less expensive reliability demonstration tests for specified consumers risk and producers risk.

?s

References

Broemm, W.J., P.M. Ellner, and W. J. Woodworth, AMSAA Reliability Growth Guide, AMSAA TR-652, U.S. Army Materiel Systems Analysis Activity, Aberdeen Proving Ground, MD 21005, September 2000.

Drenick, R.F., "The Failure Law of Complex Equipment," Journal of the Society for the Industrial Applications of Mathematics, Vol. 8, No. 4, pp. 680–689.

Ellner, P.M., and J. Brian Hall, Planning Model Based on Projection Methodology (PM2), AMSAA TR-2006-9, May 2006.

Ellner, P.M., and Martin Wayne, "Reliability Investment Model Peer Review and Discussion," AMC, Aberdeen Proving Ground, April 28, 2009.

Forbes, J.A., E. Andrew Long, David A. Lee, William J. Esmann, and Lon C. Cross, Developing a Reliability Investment Model, LMI Report HPT80T1, July 2008.

Long, E.A., James A. Forbes, Jing Hees, and Virginia Stouffer, Empirical Relationships between Reliability Investment and Life Cycle Support Costs, LMI Report SA701T1, June 2007.

Meeker, William Q., "Renewal Processes and Superimposed Renewal Processes, Applications in Reliability," unpublished lecture notes, wqmeeker@iastate.edu.

NIST/SEMATECH e-Handbook of Statistical Methods, http://www.itl.nist.gov/div898/handbook, Chapter 8, "Reliability," National Institute of Standards and Technology, Gaithersburg, MD, 2006.

Press, W.H., Saul A. Teukolsky, William T. Vetterling, and Brian P. Flannery, Numerical Recipes in C, Second Edition, pp. 691 et seq., Cambridge University Press, fifth printing, 1996.

Bone yard