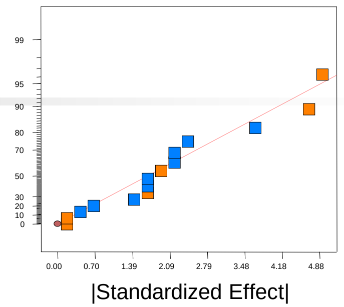
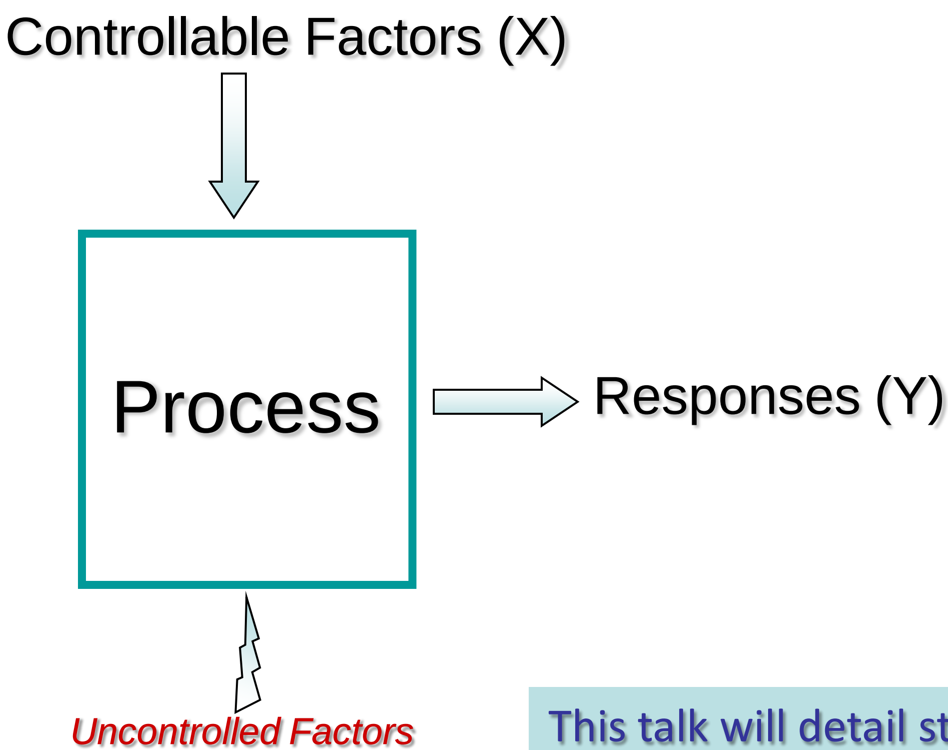


Optimal Verification Testing with Graphical Effects Analysis

Presented by Mark J. Anderson, PE, CQE, MBA
Stat-Ease, Inc., Minneapolis, Minnesota
mark@statease.com

Anecdote: Verifying tooth fairy

Statistical Design of Experiments (DOE)



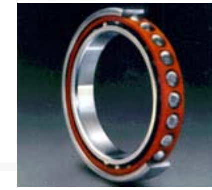
"A <u>series</u> of tests, in which <u>purposeful</u> changes are made to input factors,

to identify causes for <u>significant</u> changes in the output responses."

This talk will detail state-of-the-art tools for the design and analysis of verification tests that include both numeric and categoric factors.

Multi-Factor Testing (VS OFAT)

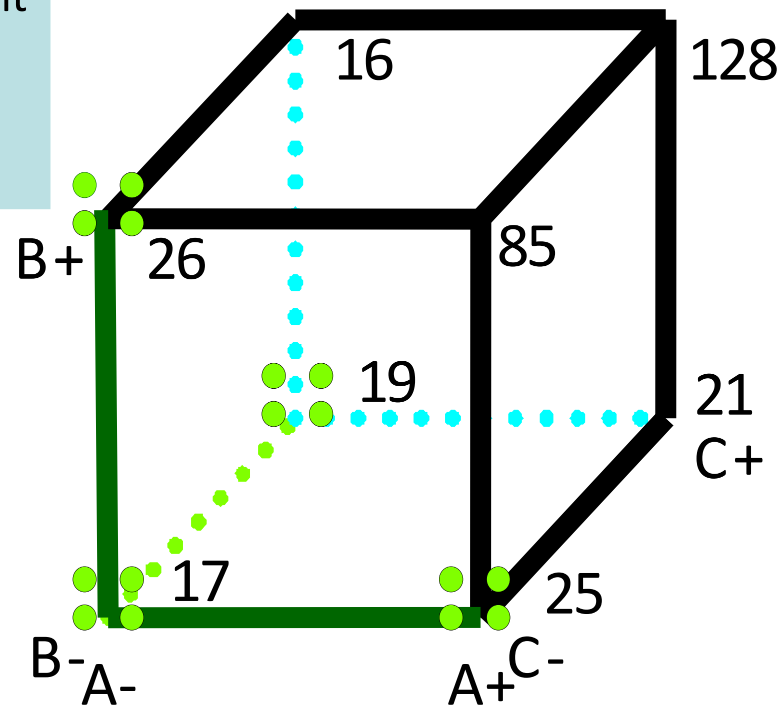
(Bearing life from accelerated test)



Factors:

A. Heat Treatment
B. Osculation
C. Material*

*Categorical



Relative efficiency = 16/8

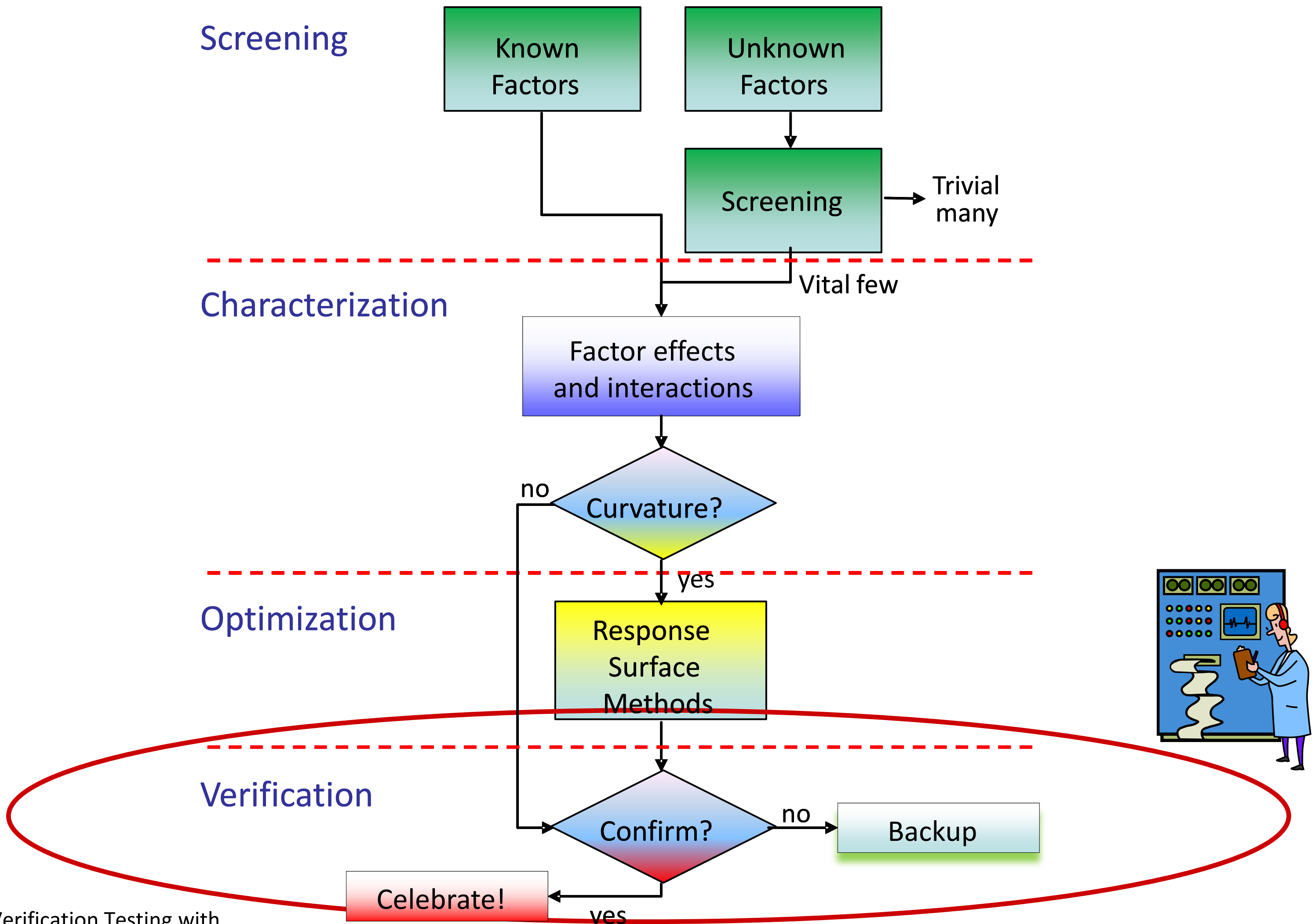
2 to 1!

"To make knowledge work productive will be the great management task of this century."

-- Peter Drucker

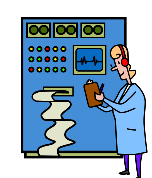
Strategy of Experimentation

Focus on Verification



Factorial Design Planning Process

1. State objective in terms of measurable responses. For each:
   a. Define the effect (response difference Δy) that is important to detect for each response.
   (This is the signal, at a minimum, you are listening for.)
   b. Estimate experimental error (response variation σ) for each response. *(The noise.)*
2. Select the input factors to study and establish their ranges.
   (Wider the better for creating effects exceeding Δy.)
3. Select a design and evaluate it for:
   - Resolution of effects (beware of aliasing).
   - Power based on its signal to noise ratio ($\Delta y/\sigma$).
     (For verification aim high: > 90 % for every response.)
   - Examine all runs for unsafe factor combinations.
     (Pre-test any that may not work and/or create hazards.)



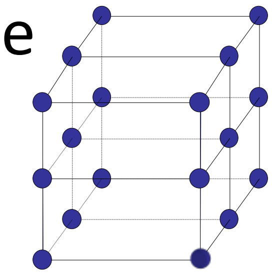
Requisites of a Good Test Design*

- ✓ The test design matches the test objectives
- ✓ All the important responses are measured
- ✓ Factor ranges are practical
- ✓ Replication – measures experimental error
- ✓ Randomization – counteracts lurking variables
- ✓ Blocking – filters systematic variation
- ✓ Everyone is involved (teamwork)

*(Adapted from "Proven Cost Savings Using Modern Design Of Experiments (MDOE)" presented by William B. Line, DOES (Design of Experiments) Institute, to the American Institute of Aeronautics and Astronautics Aerospace Sciences Meeting, Orlando, Florida January 4, 2010)

Verification Designs with Categorical Factors

Ideally these test matrices can accommodate any number of levels for any many factors, some or all of which might be categorical.



Typical problem*:

"We have 6 factors – 5 at 3 levels each and 1 at 2 levels. Each run is ~$600 (ouch). Our budget will likely support ~60 experimental runs, if needed – but I'd like to conduct this experiment in less runs and save costs."

There are 486 possible combinations, which would cost almost $300,000 to perform. Is there a way to run only a fraction?

*(Correspondence to author on 2/8/10)

Optimal Design to Minimize Runs

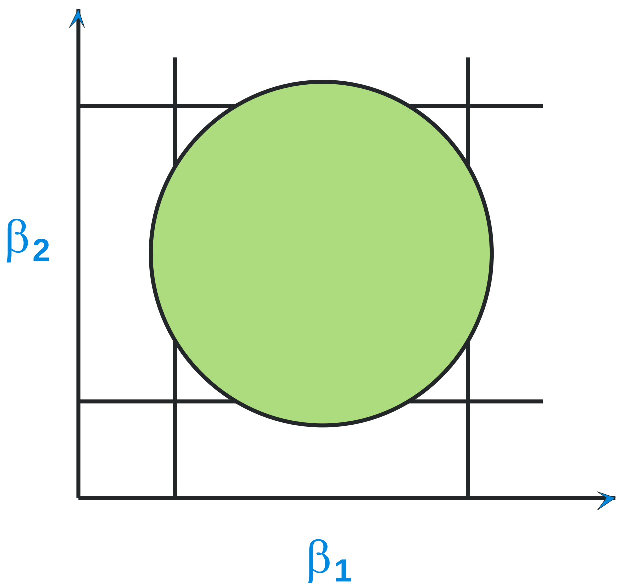
1. Specify a polynomial that you think is needed to get a decent approximation of the actual mechanism.
   - *Do not overlook two-factor interactions ("2FIs").*
2. Select minimal points to estimate all coefficients in your design-for model.
   (Computer-based exchange algorithm.)
3. Consider augmenting the design with points for:
   - Replicates: *To estimate pure error.*
   - Lack-of-fit: *To test how well the model represents actual behavior in our region of interest.*

D-optimal

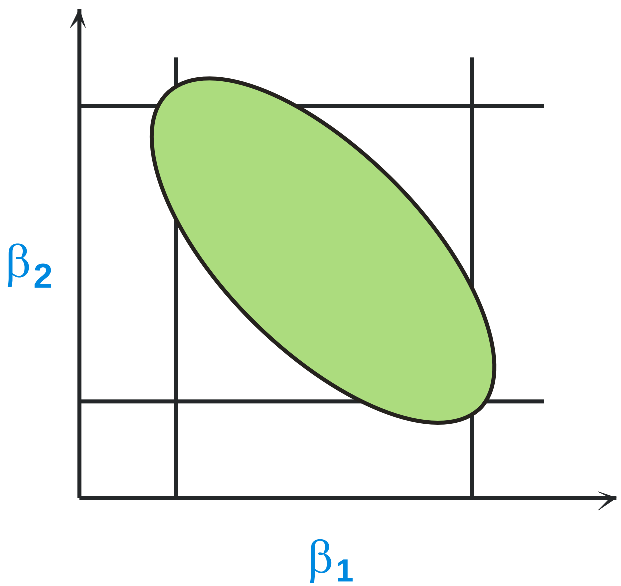
Statistical detail

Criterion: D-optimal design minimizes the determinant of the $(X'X)^{-1}$ matrix. This minimizes the volume of the confidence ellipsoid for the coefficients and maximizes information about the polynomial coefficients.

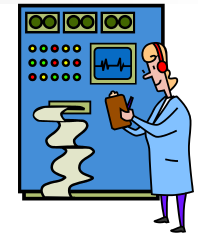
Uncorrelated Coefficients



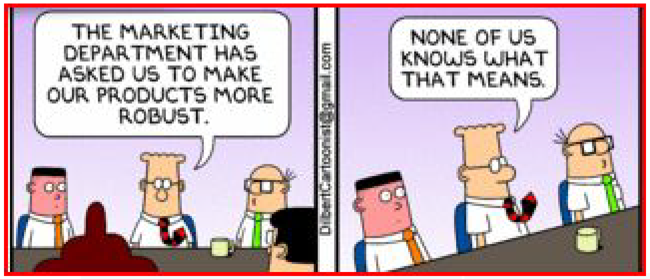
Correlated Coefficients



Importance vs Significance
Verification



Many are unclear on this difference!



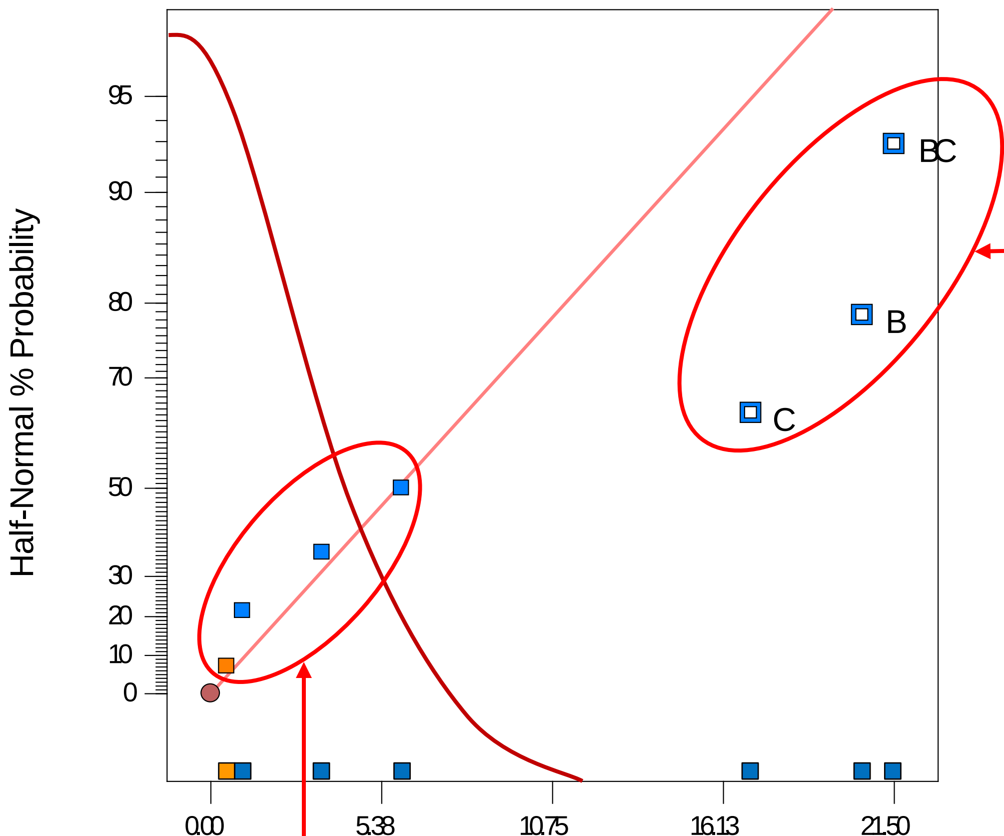
		Significant	
		No	**Yes**
Important	No	☺	😐
	Yes	? ☹	\!/ ☹

For example, let's look at a two-level factorial verification test on a system that must be must not exceed 35 units of response due to factors varied within ranges that may be encountered in the field.

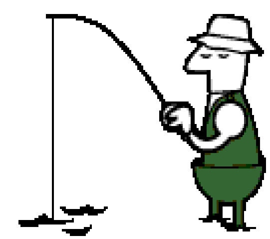
Half Normal Probability Paper

Sorting the vital few effects from the trivial many.

Before looking at the two-level factorial verification test case, this vital tool for analysis must be explained.



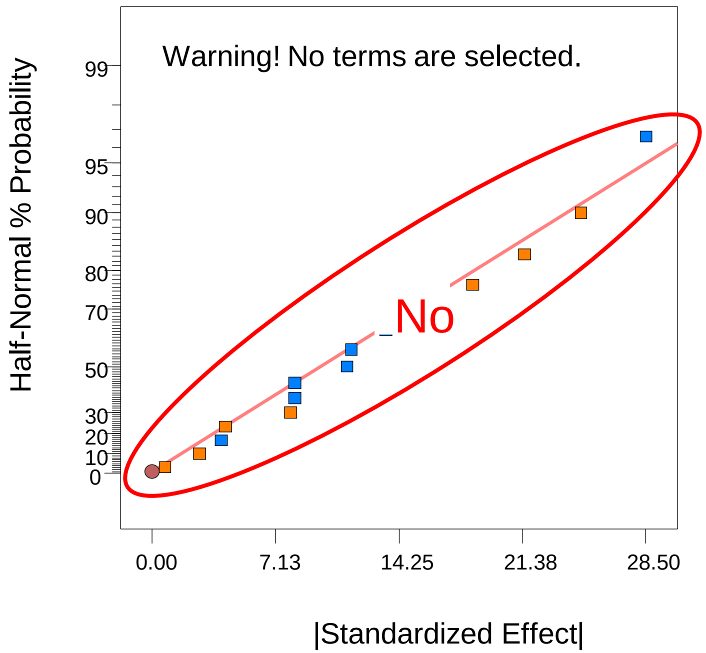
Significant effects:
The model terms!



Negligible effects: *The error estimate!*

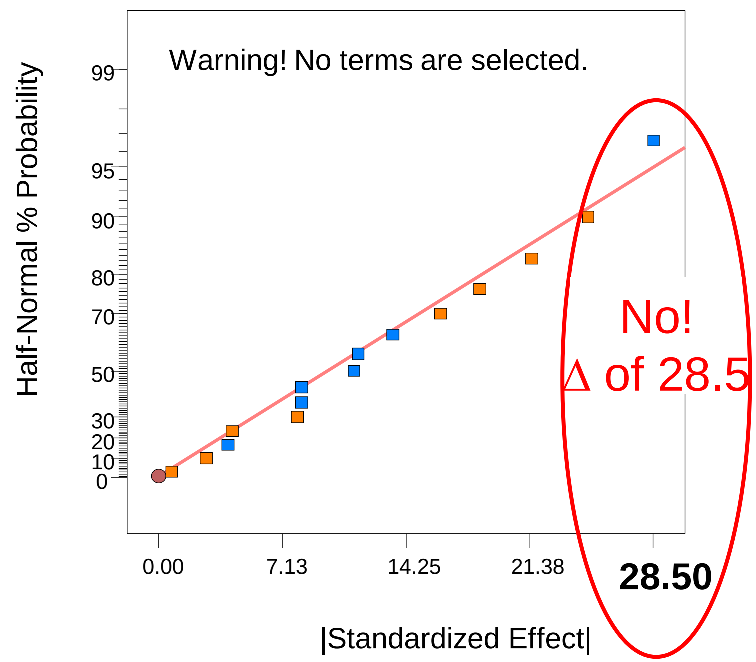
Verified ☺: *Not Significant, nor Important*

Is anything statistically significant?



Must not exceed 35 units of response due to factors varied within ranges that may be encountered in the field.

Could largest effect be important?

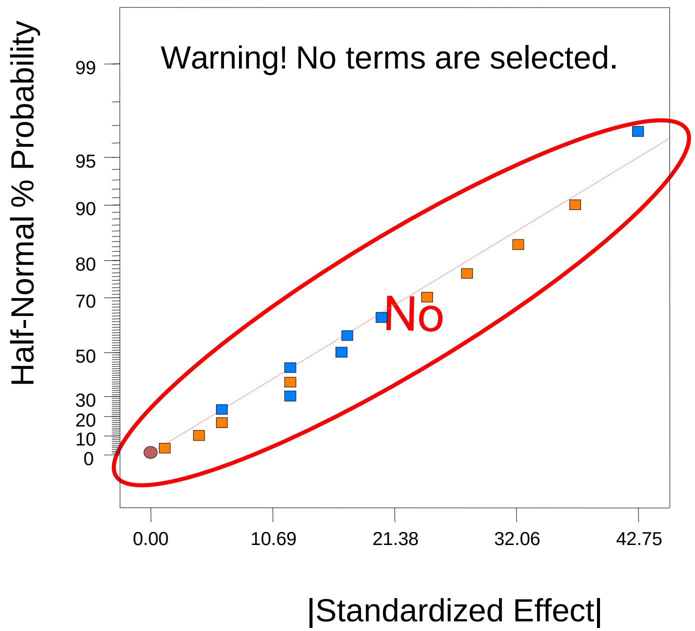


Verification Failed ☹:

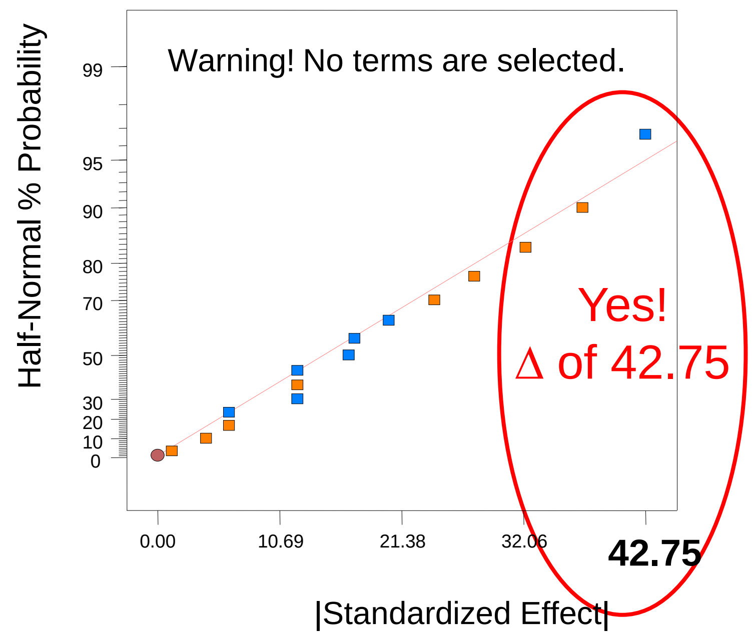
Not Significant, but Important (?)

Is anything statistically significant?

Must not exceed 35 units



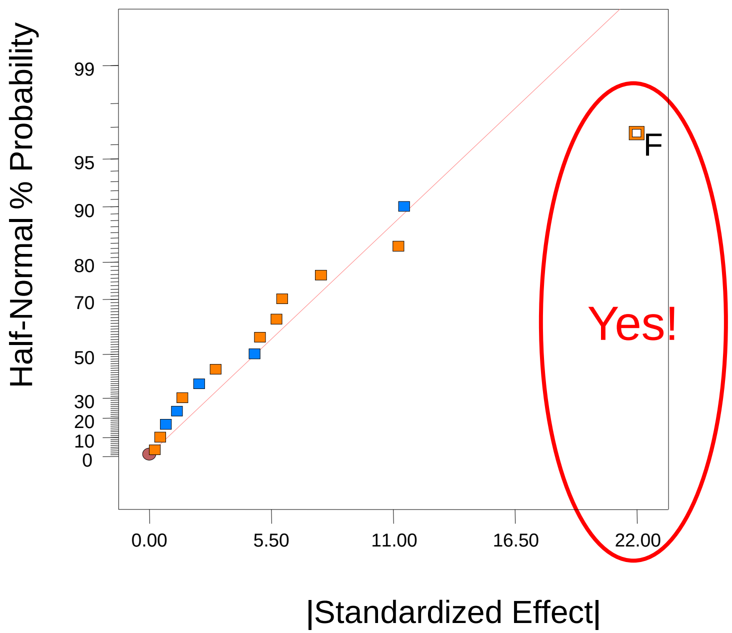
Could largest effect be important?



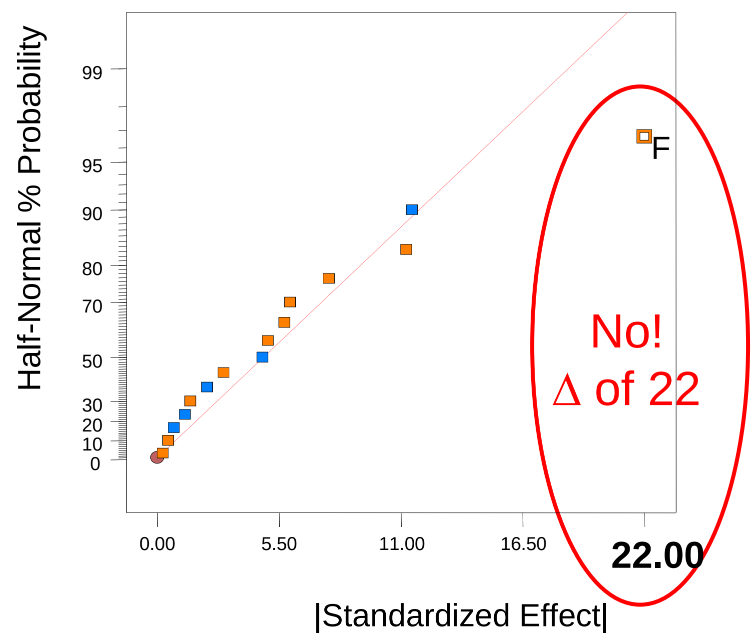
Verified (Qualified) ☺:

Significant, but not Important

Is anything statistically significant?



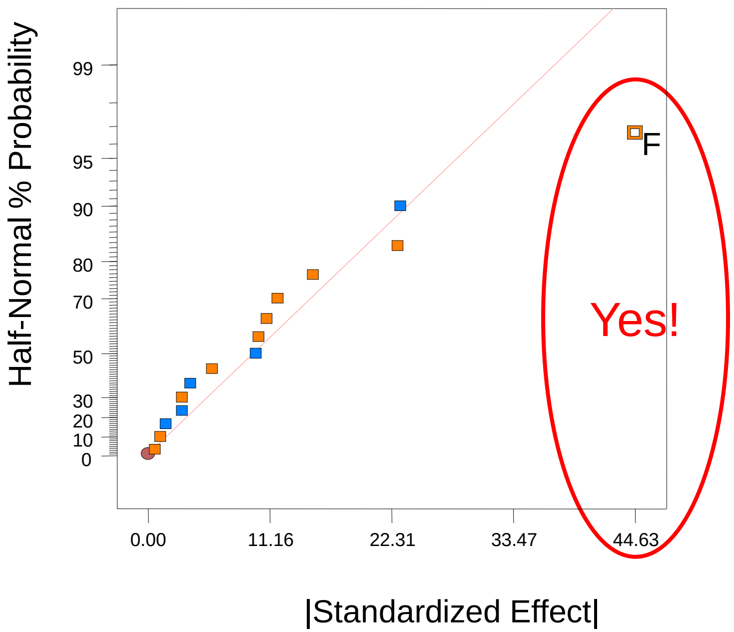
Could largest effect be important?



Verification Failed: *Significant and Important*

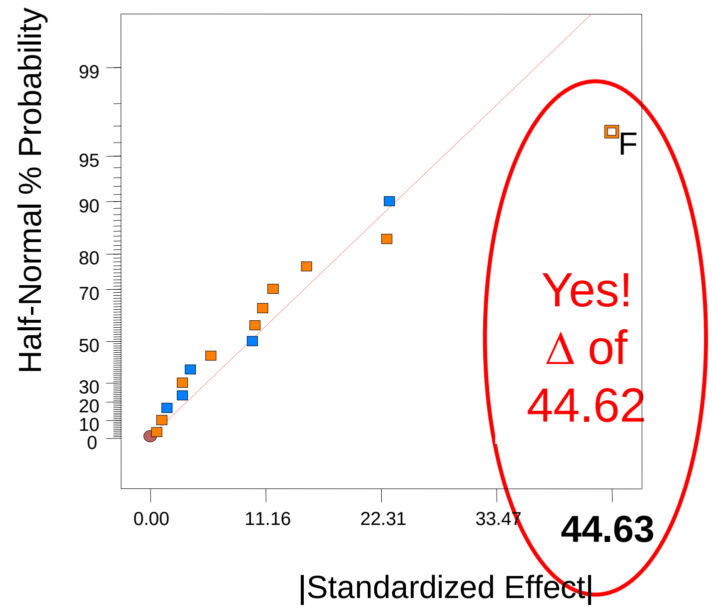
\!/ ☹

Is anything statistically significant?



Must not exceed 35 units

Could largest effect be important?



Case Study:
Hydraulic Gear Pump*

Hydraulic gear pumps are vital for many machines including vehicles and airplanes. However, they tend to lose efficiency due to internal leakage. OEM engineers must verify that one such device will remain within a specified range of performance regardless of normal production variations. They settle on nine factors, primarily categorical -- shown at right with number of levels each.

A. Flange (3)
B. Cover (3)
C. Float (3)
D. Bearing (2)
E. Involute (2)
F. Lead edge (2)
G. Side Gap (2)
H. Pressure (3)
I. Speed (3)

*(Loosely based on "Experiments for derived factors with application to hydraulic gear pumps," by C. J. Sexton , S. M. Lewis & C. P. Please, *Journal of the Royal Statistical Society: Series C (Applied Statistics)*, 2002, V50, Part 2, pp 155-170)

Specifying the Experiment Design

- Based on subject-matter knowledge, the engineers are most concerned about interactions involving the float (factor C).
- Customized pumps are costly, so a minimal-run design is desired.
  - Choose a d-optimal design for a reduced 2FI model: intercept, 9 main effects and 8 two-factor interactions (those involving C).
    - ✓ *39 model points (builds) must be picked from the 3888 (3x3x3x2x2x2x2x3x3) possible combinations, in other words, approximately a 1/10th fraction.*
    - ✓ To estimate lack-of-fit pick 5 more unique combinations
    - ✓ From these 44 builds, select 4 to replicated for pure error.
      - ❑ 48 pumps will be built in total.

Differences in leak-back of 2 units are of interest.
It must not exceed 10 units overall.

Will the design provided proper power?

1. Define the change (Δy) that is important. *(Signal)*
2. Estimate experimental error (σ). *(Noise)*
3. From signal-to-noise ratio ($\Delta y/\sigma$) estimate power.

If runs suffice, the averaging provided by the matrix will cut the grass (noise) to reveal the snake (effect)!

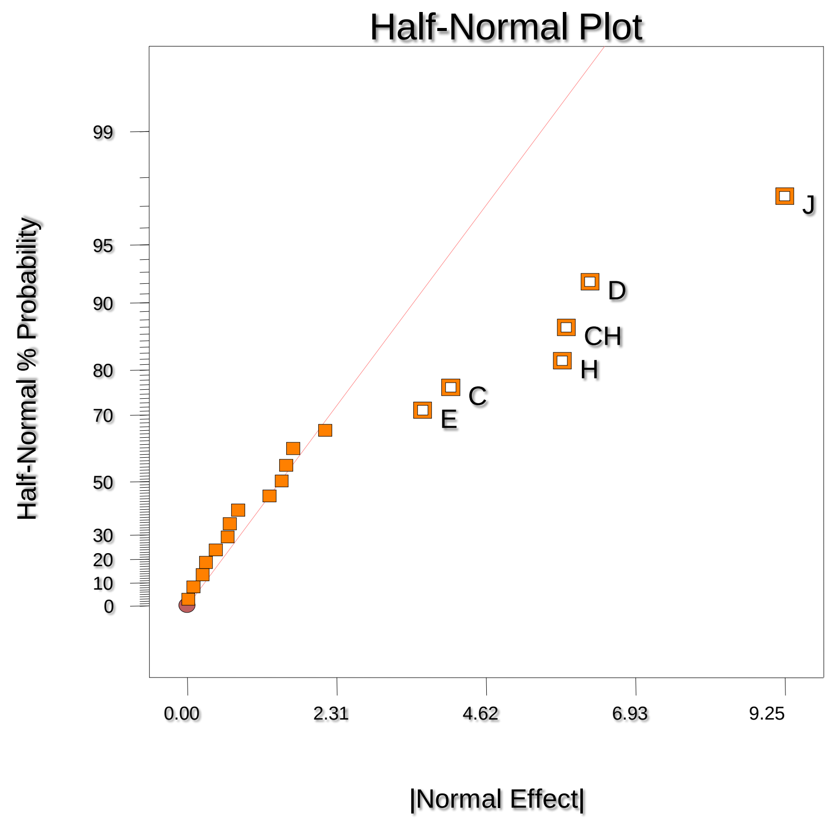
Signal (delta) = 2.00		Noise (sigma) = 1.00		Signal/Noise (delta/sigma) = 2.00				
A[1]	B[1]	C[1]	D	E	F	G	H[1]	J[1]
99.6 %	99.6 %	99.6 %	99.9 %	99.9 %	99.9 %	99.9 %	99.5 %	99.5 %

Yes, power at 5% risk to see signal exceeds the 90% guideline for verification testing.

Pump Case Results

Design-Expert® Software
Leak-back

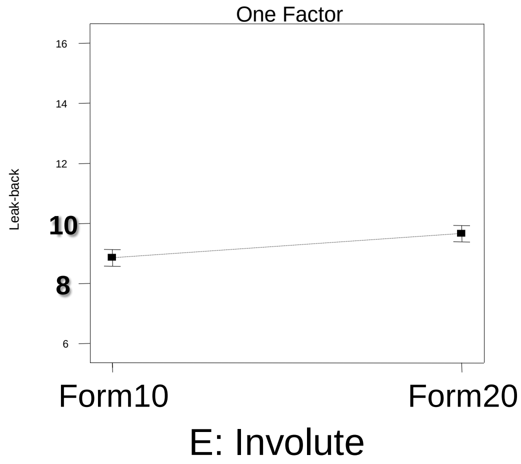
Shapiro-Wilk test
W-value = 0.702
p-value = 0.000
A: Flange
B: Cover
C: Float
D: Bearing
E: Involute
F: Lead edge
G: Side Gap
H: Pressure
J: Speed

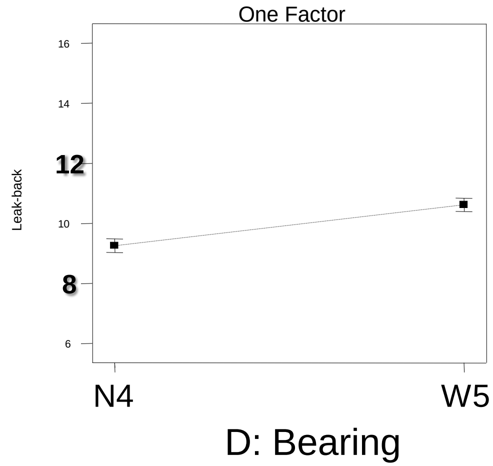


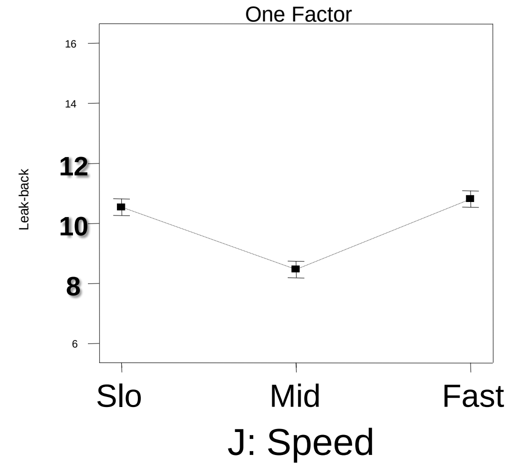
This is not a two-level factorial. Some trickery is required (see Ref. #2) to plot multiple contrasts on a comparative scale.

Significant effects, but is the largest less than 10 leak-back units? We cannot tell with this graph because the effects are in a "normal" scale similar to a Z score.

Pump Case
Effects Plots

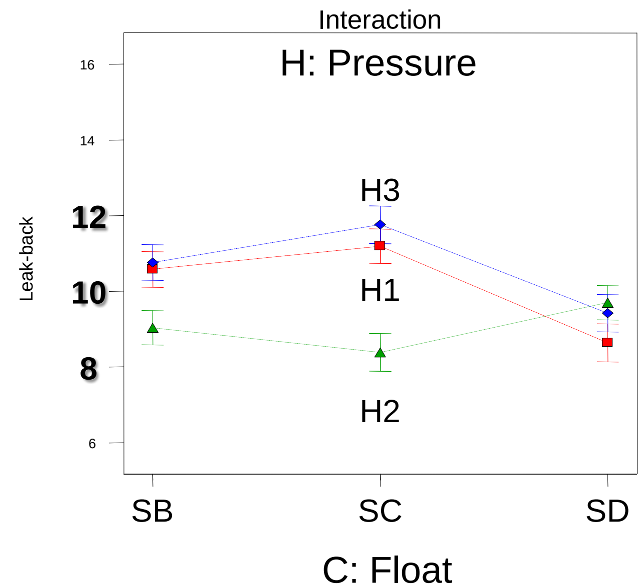




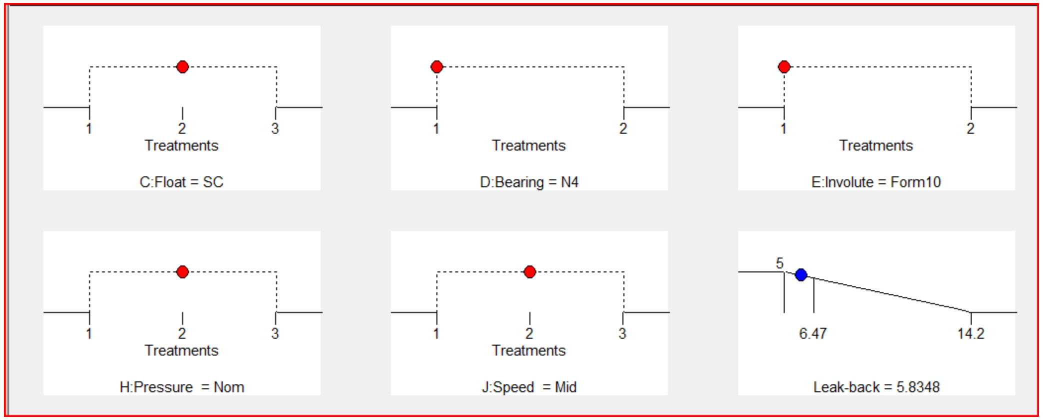


Three significant main effects (one-factor), plus one two-factor interaction. =>

Does the predicted response range more than 10 units? If so, it fails verification.

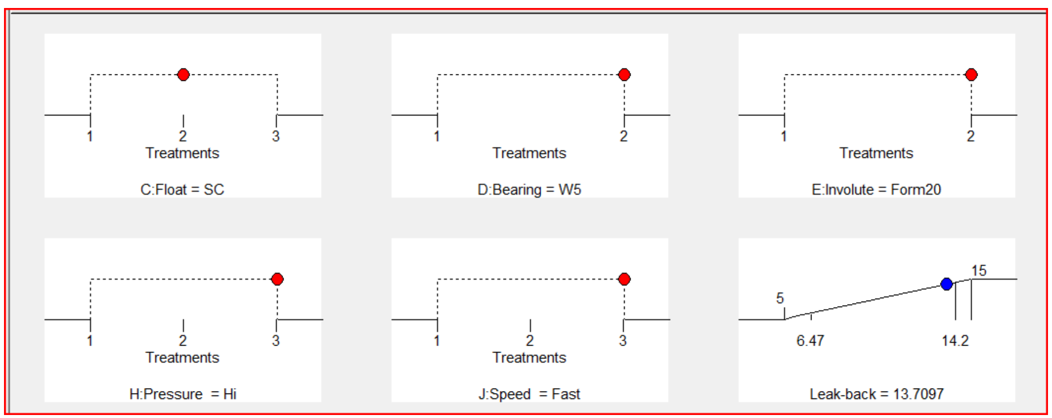


Pump Case
Numerical Optimization



Minimum Leak-Back

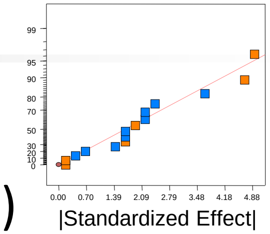
Difference < 10 units



Maximum Leak-Back

Predicted response ranges less than 10 units, so it passes verification.

Take-Home Messages

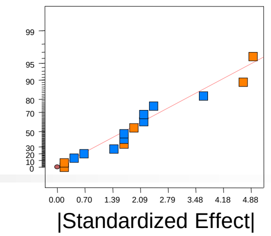


By way of a case study on verification testing of an hydraulic gear pump, this presentation on design of experiments (DOE) provided insights into statistically-optimal designs involving many categorical factors at multiple levels.

Upon collection of the response data, an innovative new graphical approach to assessing effects – the half-normal plot – revealed at-a- glance the likely significance and, for two-level factorials the importance of the signal generated by the experiment. *(General factorials require special scaling per Ref #2 shown on following slide.)*

Multifactorial DOEs like this are more efficient than traditional one-factor-at-a-time (OFAT) testing, which would never reveal an interaction such as the one that came to light in this case.

Further Reading for More Detail on Methodology



1. Mark J. Anderson and Patrick J. Whitcomb, *DOE Simplified – Practical Tools for Effective Experimentation*, *2nd Edition*, Productivity Press, NY, NY, 2007.
2. Patrick J. Whitcomb and Gary W. Oehlert, "Graphical Selection of Effects in General Factorials," 2007 Fall Technical Conference of the American Society for Quality (ASQ) and the American Statistical Association (ASA).

Your thoughts welcomed

For a copy of updated "Optimal Verification Testing with Graphical Effects Analysis," e-mail Mark@StatEase.com. Feel free to provide comments and suggestions!

Thanks for listening!

"It may happen that small differences in the initial conditions produce great ones in the final phenomena."

- Henri Poincare