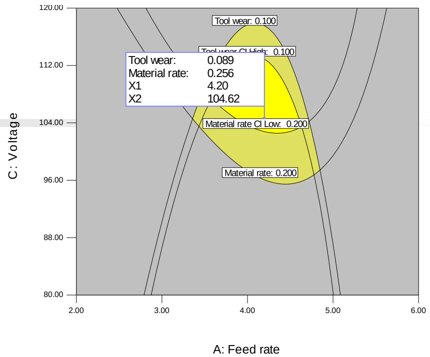

How to Frame a Robust Sweet Spot via Response Surface Methods (RSM)

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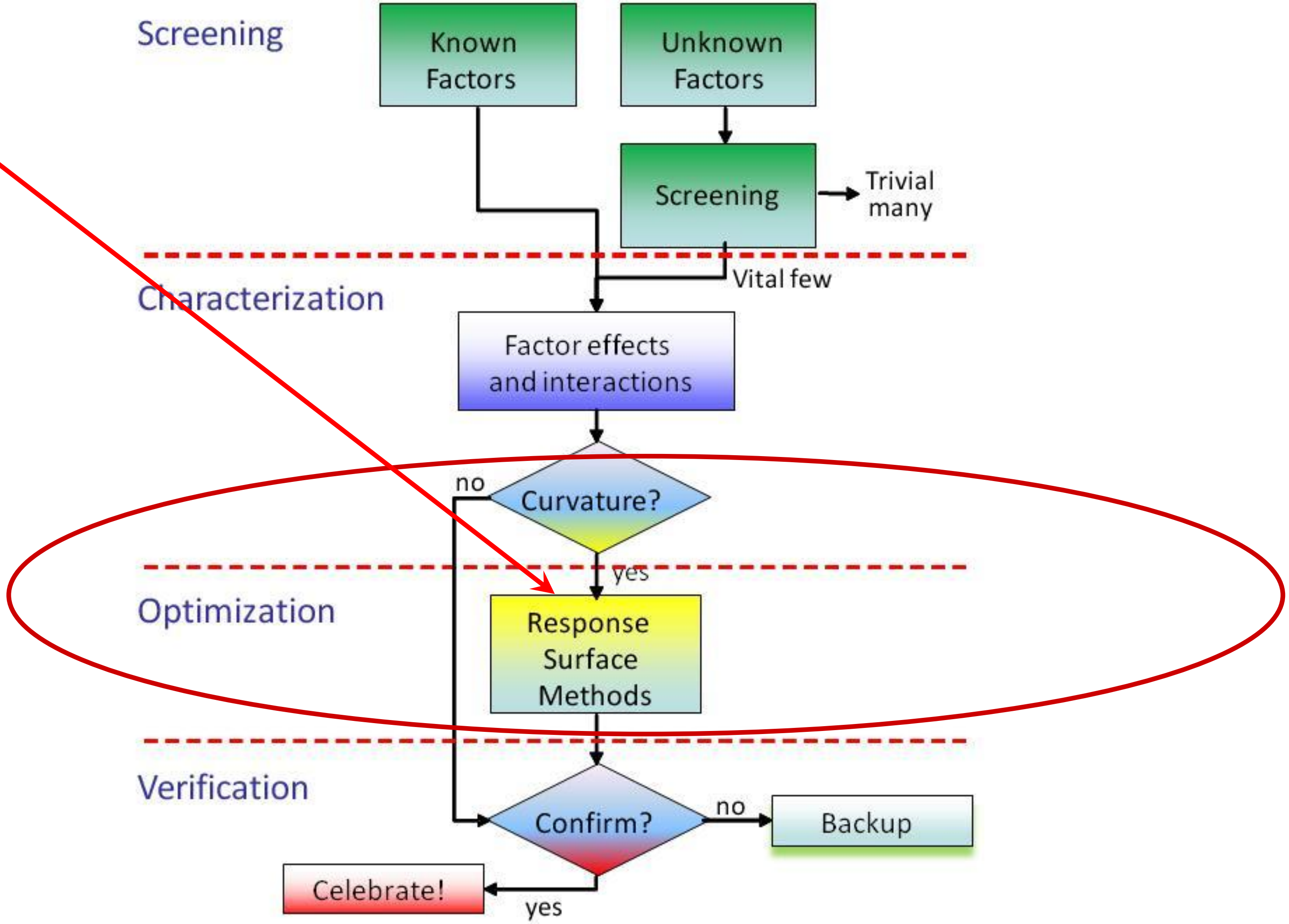
Strategy of Experimentation

RSM



Response Surface Methods (RSM)*

When to Apply It (Strategy of Experimentation)

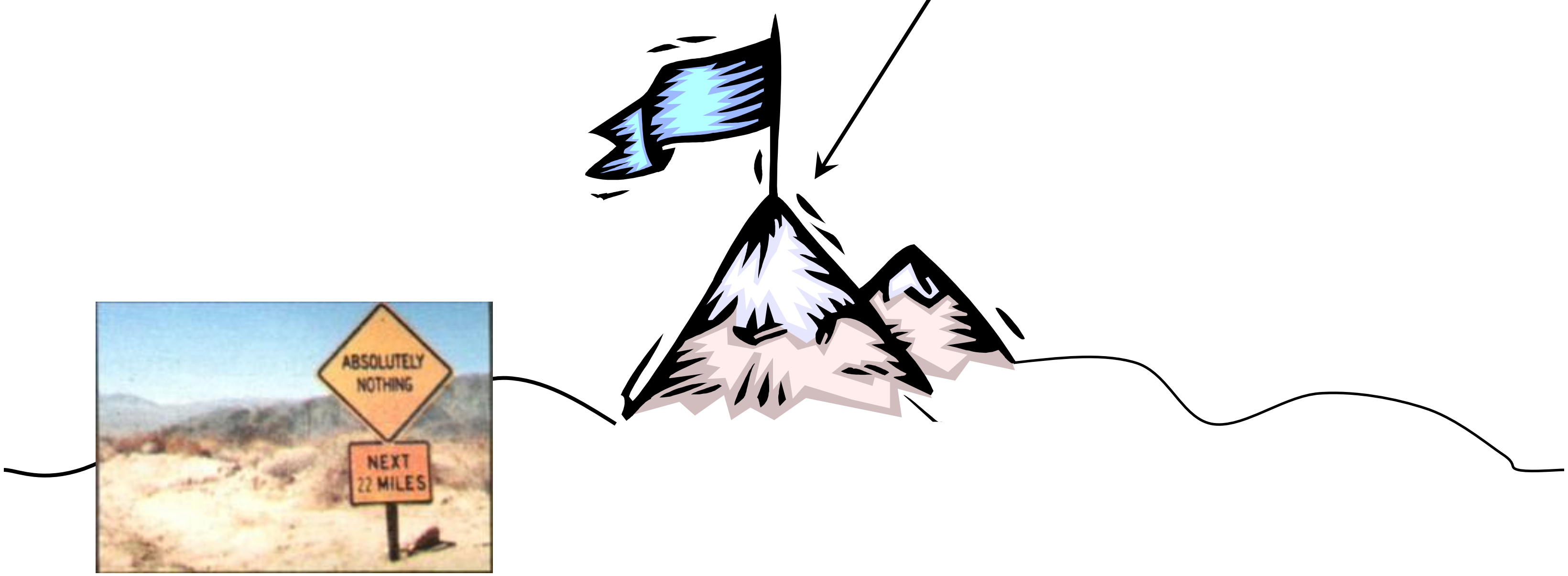
1. Fractional factorials for screening
2. High-resolution fractional or full factorial to understand interactions *(add center points at this stage to test for curvature)*
3. Response surface methods (RSM) to optimize.

Contour maps (2D) and 3D surfaces guide you to the peak.

RSM: When to Apply It

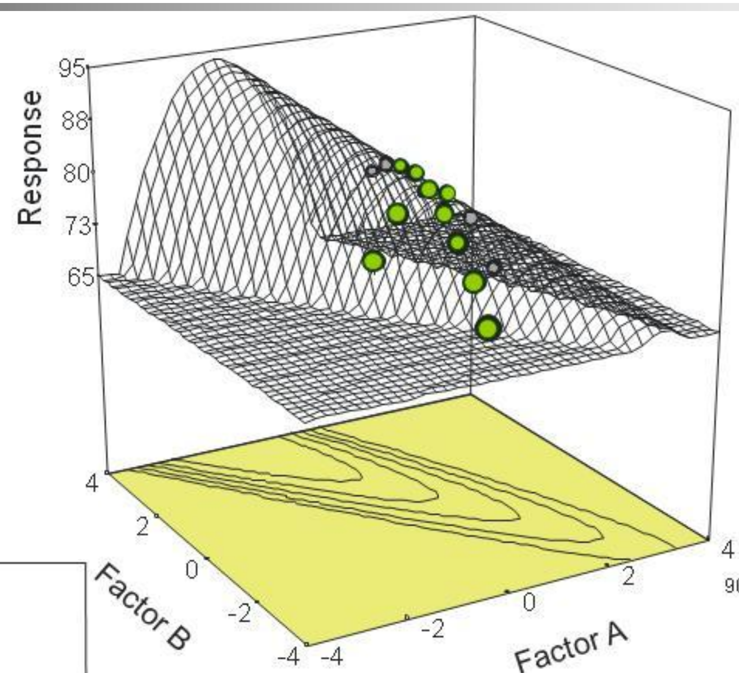
Use factorial design to get close to the peak. Then RSM to climb it.

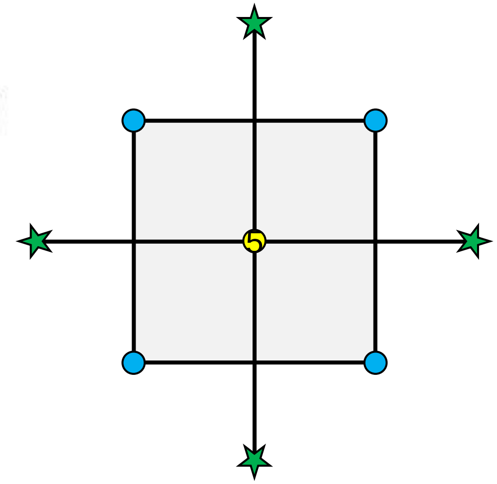
Region of Interest

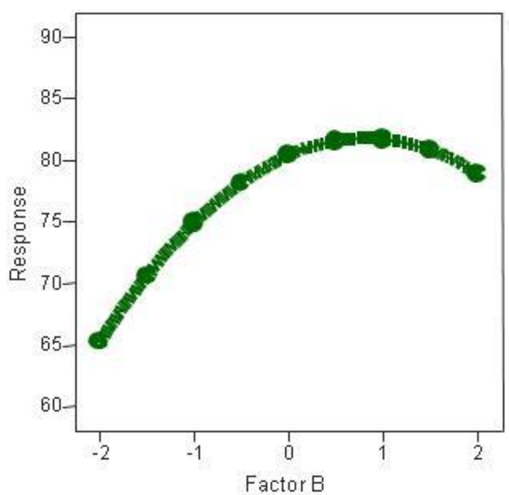


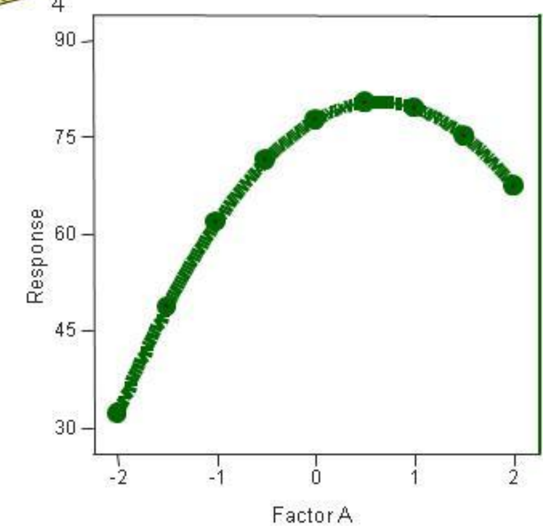
← Region of Operability →

RSM vs OFAT









Frame a Robust Sweet Spot

RSM: Process Flowchart

Subject Matter Knowledge
(Plus Factorial Screening)

↓

Vital Few Factors (x's)

↓

Process → Measured Response(s) (y(s))

↓

↓

Polynomial Model

↓

Response Surface

"All models are wrong, but some are useful." - George Box

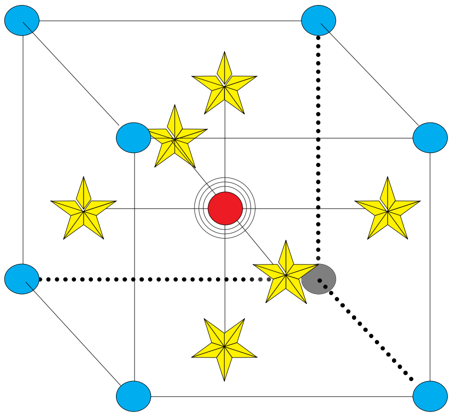
Case Study – RSM Design & Analysis

Aerospace Example*

Via a face-centered central composite design (FCD) aimed at minimizing weight of an active aeroelastic wing, aerospace engineers studied three vital structural factors:

A. Aspect ratio, 3–5.
B. Taper ratio, 0.2–0.4.
C. Thickness ratio, 0.03–0.06

"A designer knows he has achieved perfection <u>not</u> when there is nothing left to add, but when there is nothing left to take away."
- Antoine de Saint-Exupery

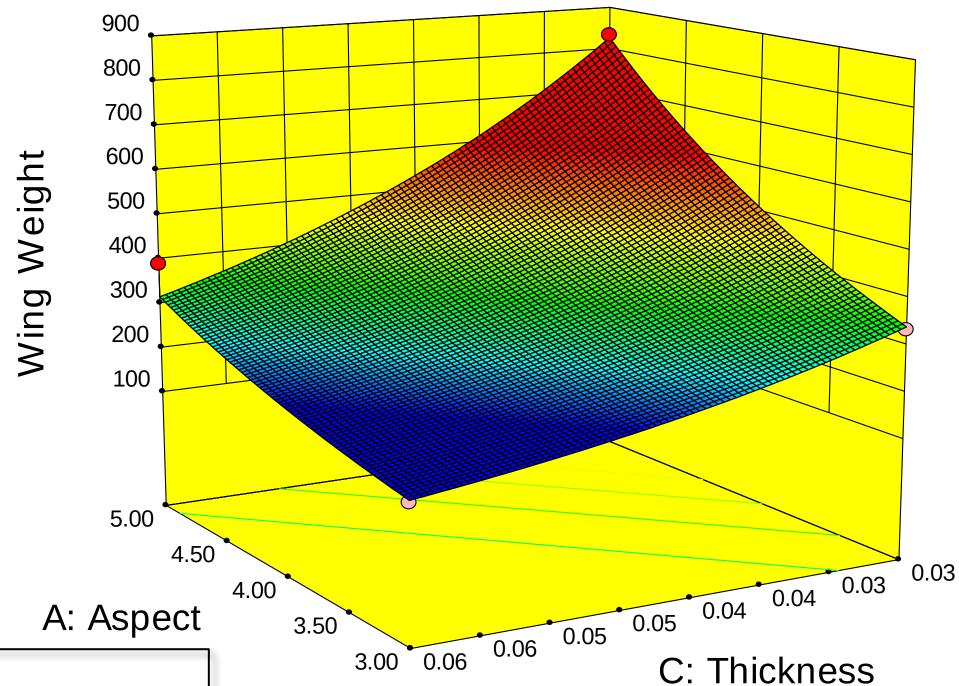


FCD

*(*RSM Simplified: Optimizing Processes Using Response Surface Methods for Design of Experiments*, Mark J. Anderson & Patrick J. Whitcomb, Productivity Press, NY, NY (2007) Chapter 10, pp: 224–228.)

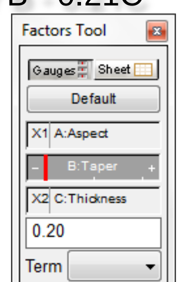
Response Surface Map for Wing Weight

The picture tells the story. It's generated by the fitted -equation (math model), which also provides a "transfer function" for numerical prediction and optimization.



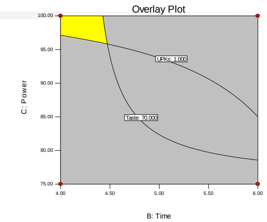
Equation in Terms of Coded Factors:
Log10(Wing Weight) = +2.56 + 0.19 A + 0.037 B - 0.21C

Equation in Terms of Actual Factors:
Log10(Wing Weight) =
+2.29660
+0.19251 * Aspect
+0.37457 * Taper
-13.86641 * Thickness



Data file: Wing weight

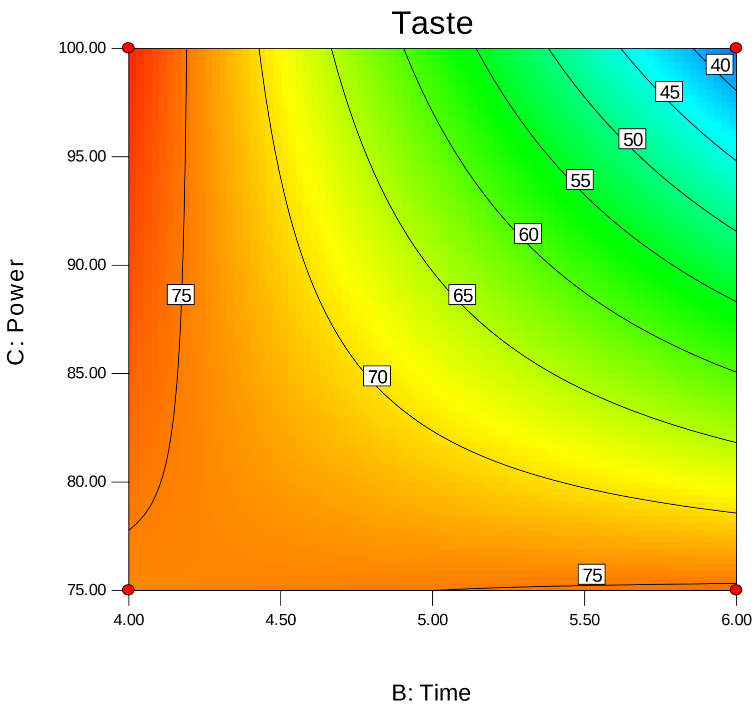
Graphical Optimization of Multiple Responses to Generate Design Space

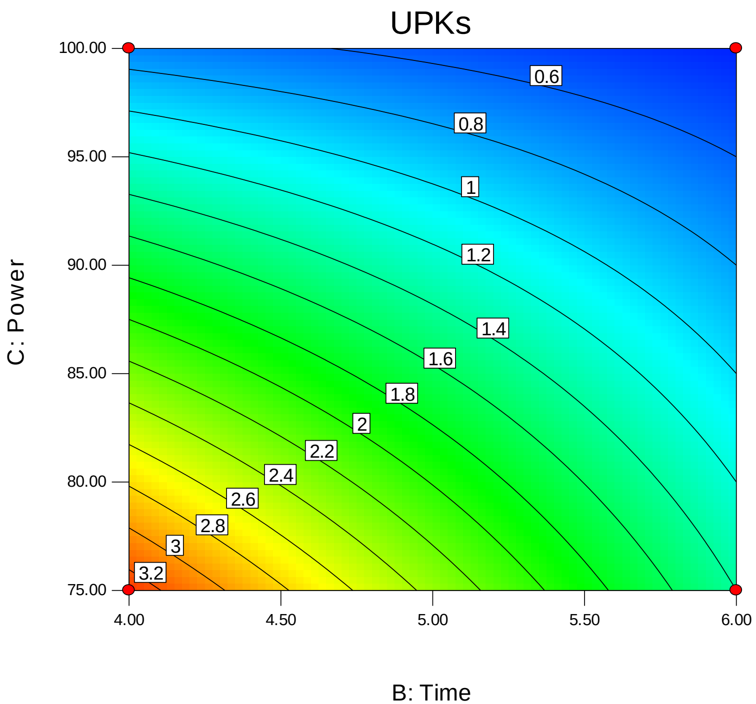


By overlaying contour plots for multiple responses – shading out regions out of spec, one can view the <u>design space</u> (aka “operating window” or “sweet spot”). The FDA defines “design space” as the *“multidimensional combination and interaction of material attributes and process parameters that have demonstrated to provide assurance of quality.”* This is a key element for their quality by design (QbD) initiative. <u>It merits attention for test and evaluation</u>.

Making Microwave Popcorn *(1/2)*

Try this experiment at home! *Where is the "sweet spot" for making popcorn?* (Hint: Want low unpopped kernels – UPK – and high taste rating.)

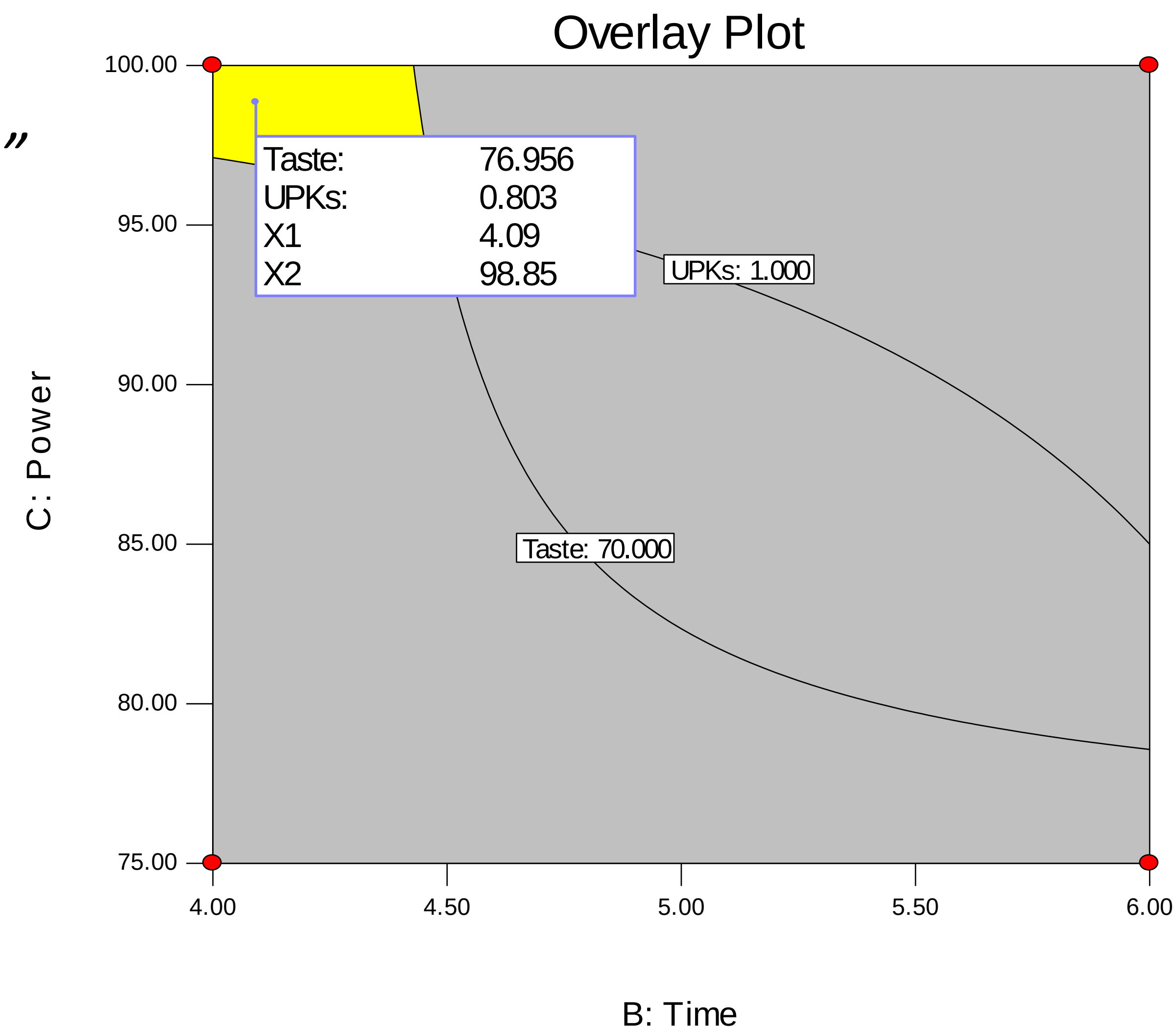




Data file: Popcorn

Simple Example of Design Space
Making Microwave Popcorn *(1/2)*

This is the "sweet spot" for making popcorn.



Case Study – Design Space

Aerospace Example*

Via an optimal RSM design aimed at characterizing a freejet nozzle's exit profile, aerospace engineers studied two vital factors:

A. Temperature, low to high.
B. Pressure, low to high.

Over an area of interest that required a linear constraint to cut off the region where both factors hit their high levels. The actual levels tested remain confidential. However, facility support testing at temperatures up to 4,700 degrees Rankine and pressures up to 2,800 psia.

*("Developing, Optimizing and Executing Improved Test Matrices," presented by Dusty Vaughn and Doug Garrard to the U.S. Air Force T&E Days 2009, approved by U. S. Government for public release via the American Institute of Aeronautics and Astronautics.)

Defining the Operating Constraints

This is a “burnt pudding” problem – too much temperature and time overcooks the food. DOE software makes it easy to avoid these unwanted combinations. The experimenter need only identify the constraint points.

Here, after entering dummy values for each factor, a constraint point is set for the level of temperature that cannot be exceeded when the system is at high pressure.

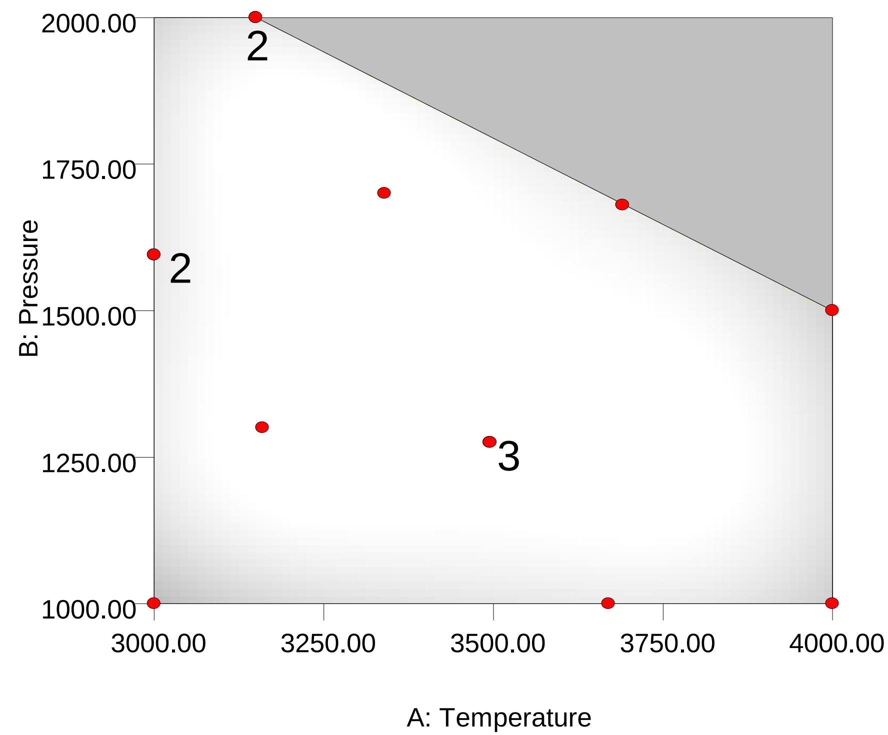
Conversely, a second constraint point is set for the maximum pressure level when temperature is at its highest level.

	Name	Low Actual	High Actual	Vertex	< > skip	Constraint Point
A:	Temperature	3000	4000	4000	A <	3150
B:	Pressure	1000	2000	2000	B <	1500

Laying Out an Optimal Design

Due to the demands of cost and schedule, the experimenters chose a minimum-run design of 6 points to fit the standard second-order (quadratic) RSM model. One point was replicated.

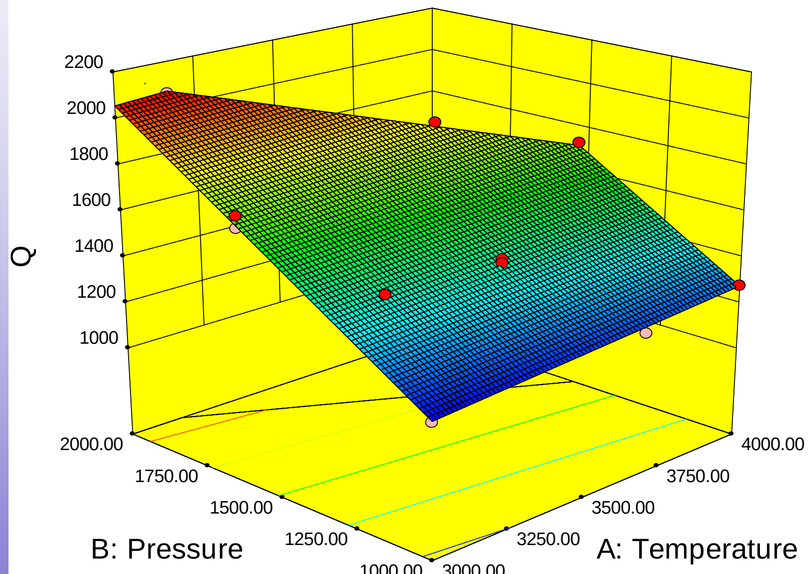
However, for expository purposes, here is a stouter design* with 4 additional test points to assess lack-of-fit and 4 points replicated for a stronger estimate of pure error. Also, the optimality criterion for this design is IV – now favored for RSM designs, not D-optimal as done by the experimenters.

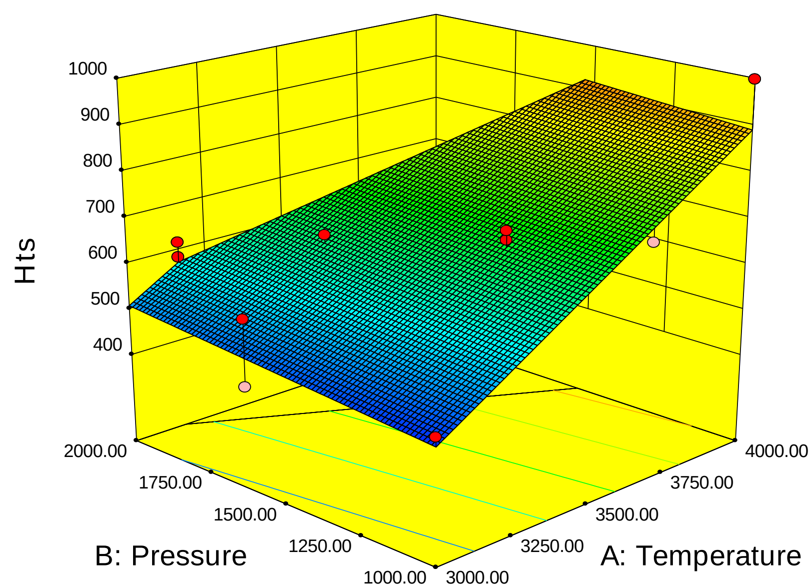


(How many test points will be needed is an issue of <u>power</u>, which goes beyond the scope of this talk. For details on design-sizing for RSM, see the Sept. '08 Stat-Teaser.)

Results

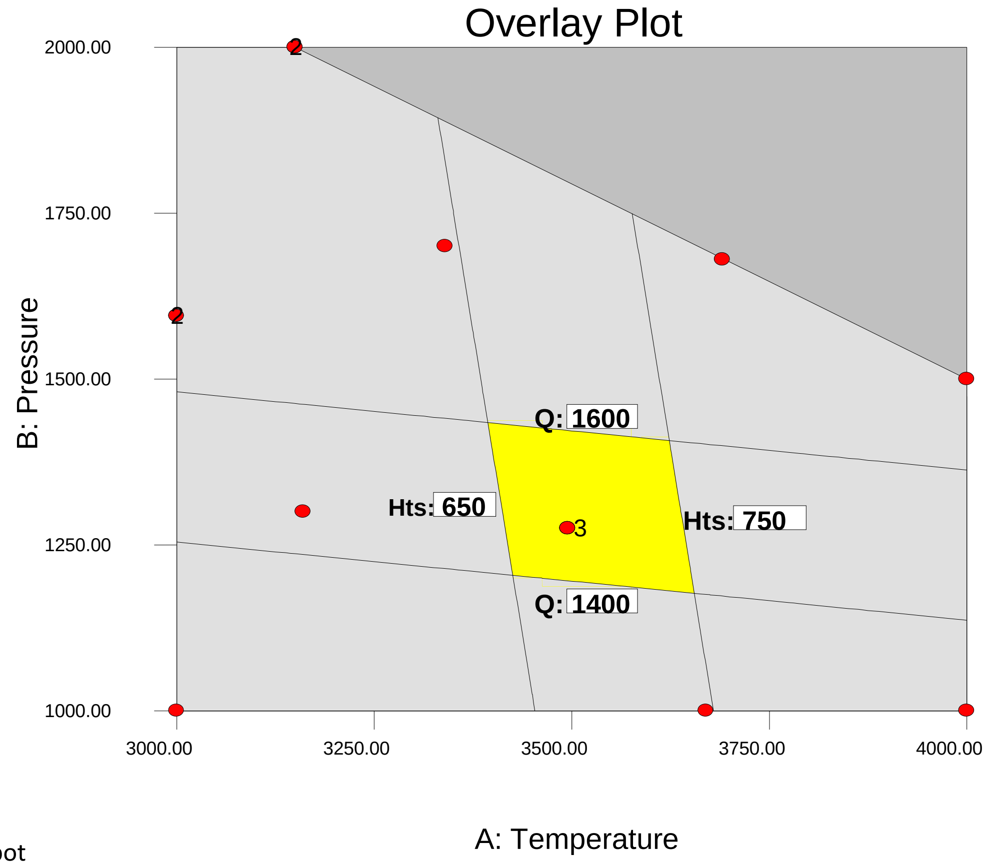
The following response surfaces were generated via re-simulation from predictive equations provided in coded form by the experimenters. The graphs closely resemble the published results for the key measures of dynamic pressure (Q) and total sensible enthalpy (Hts).





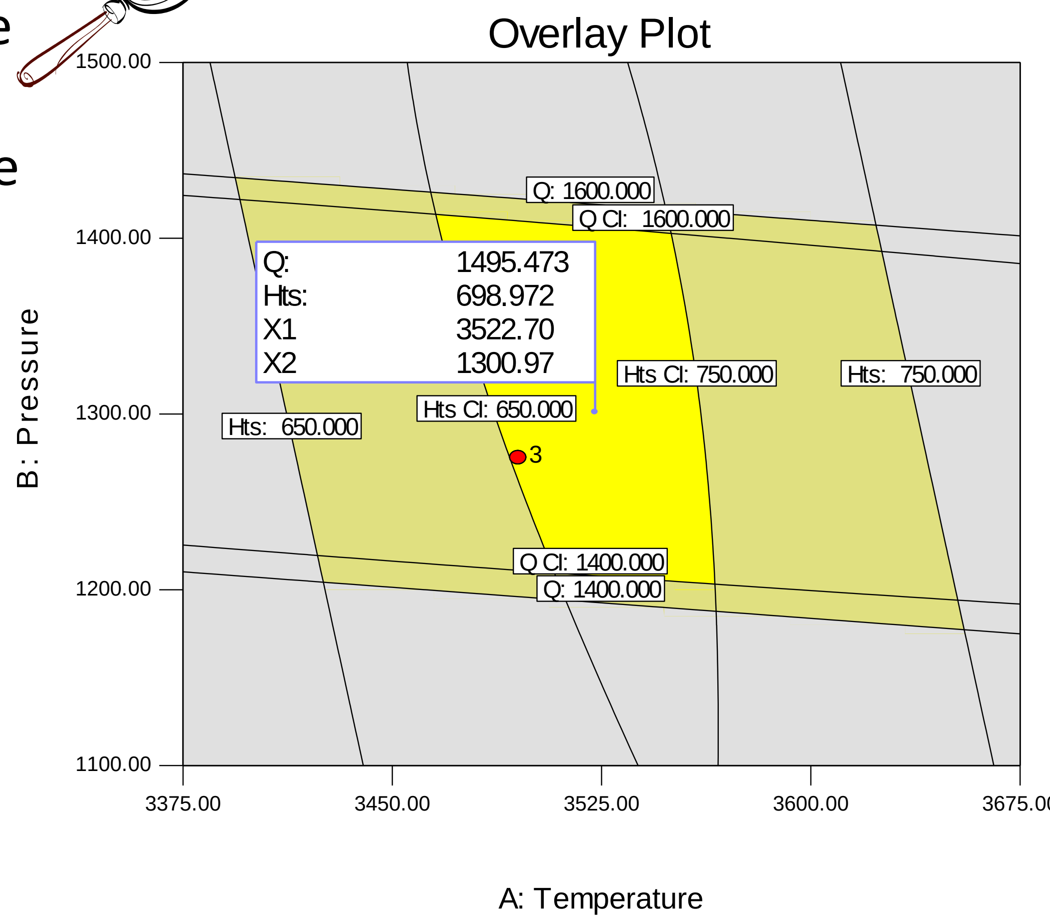
Sweet Spot (Hypothetical)

The customer requirements have not been revealed, but assume they are represented by the graphical overlay shown below.



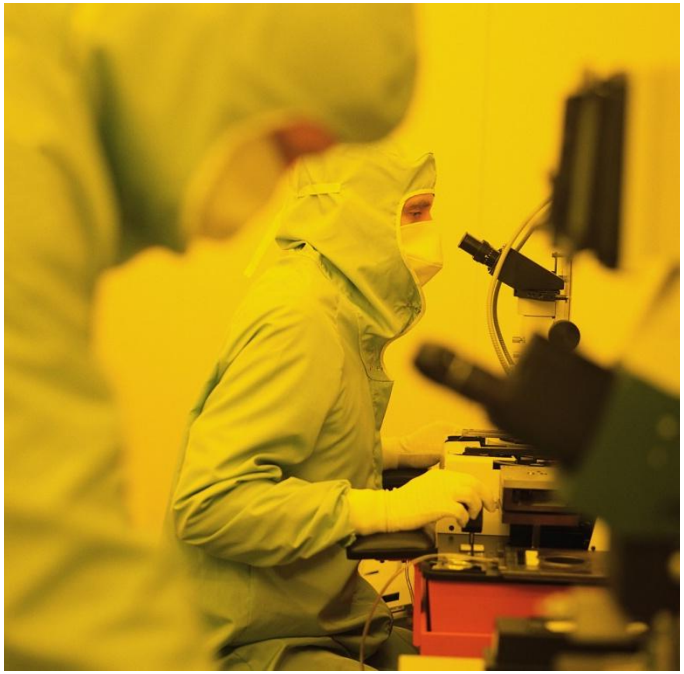
Robust Sweet Spot

To be more conservative (robust) in framing the sweet spot, superimpose the confidence intervals (CI) – a function of the underlying standard deviation (provided by the original publication) and the power of the experiment design (stronger in our re-simulation). <u>The flag in the center might mark a good place to operate</u>!



Conclusion

Via application of response surface methods (RSM) experimenters in the field of test and evaluation can frame an operating window (aka “sweet spot” or “design space”). To be more conservative (robust), shade out the regions that fall within the confidence intervals of the boundary lines.



Best of luck for your experimenting!

Thanks for listening!

-- Mark

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