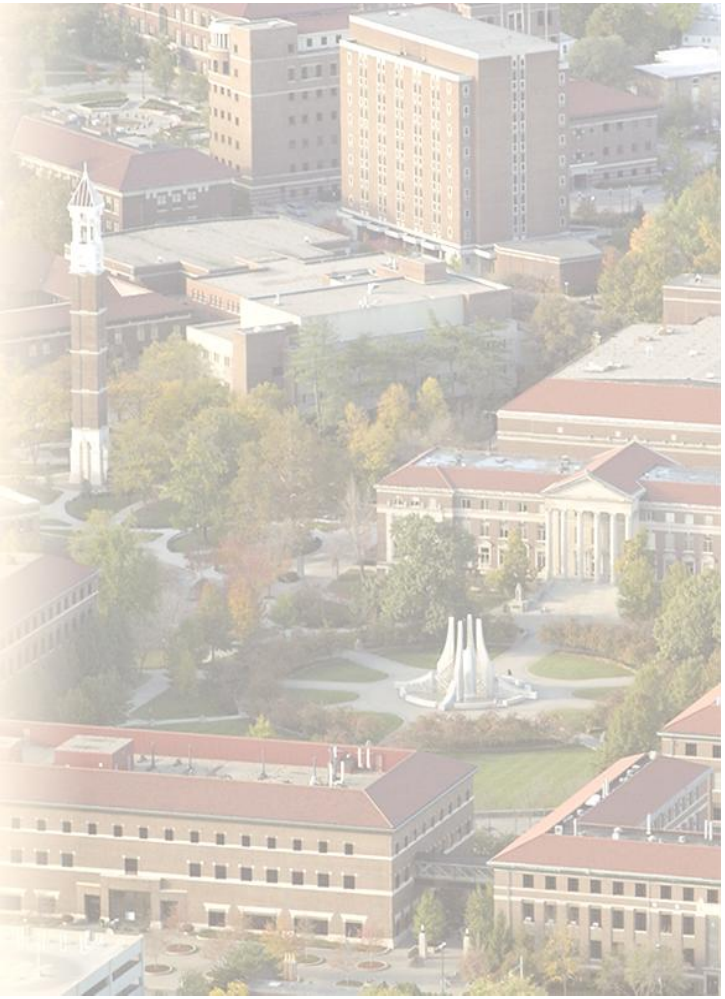
SYSTEMS ENGINEERING
Research Center

PURDUE AERONAUTICS & ASTRONAUTICS

A Portfolio Approach to System-of-Systems Acquisition and Architecture



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Dr. Daniel DeLaurentis
Dr. Navindran Davendralingam
davendra@purdue.edu
School of Aeronautics & Astronautics
Center for Integrated Systems in Aerospace
Purdue University

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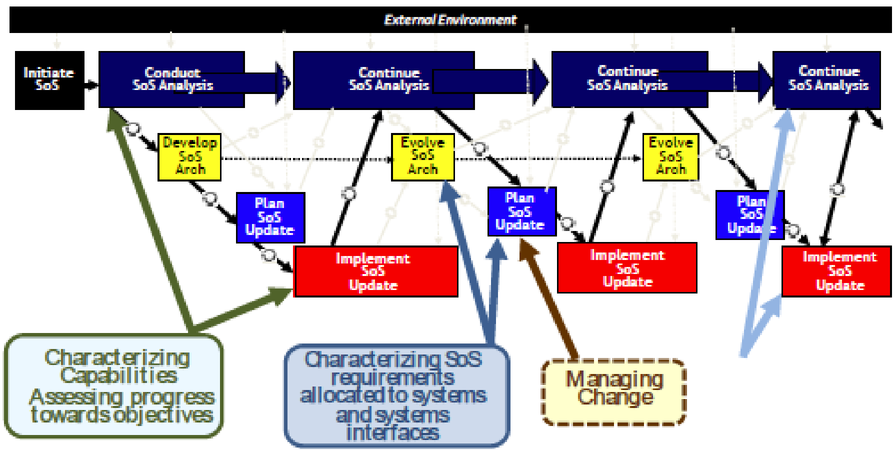
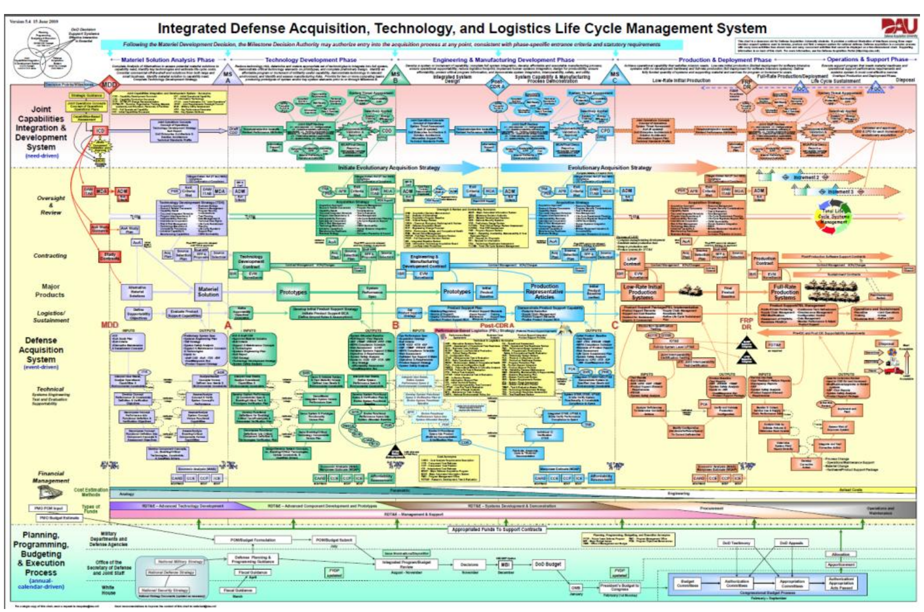
Presentation Outline

- Motivation: Defense Acquisitions and Systems Engineering
- SoS Architecting and Acquisition: Wave Model context
- An Investment Portfolio Approach
  - Mean Variance Approach
  - Mean-Variance: A Robust Version
- Concept Problem: Simple Littoral Combat Ship (LCS)
  - Robust Portfolio application
  - Multiple risk measures
  - Operational Robustness using Bertsimas-Sim method
- Future Work

Motivation: Acquisitions and Systems Engineering





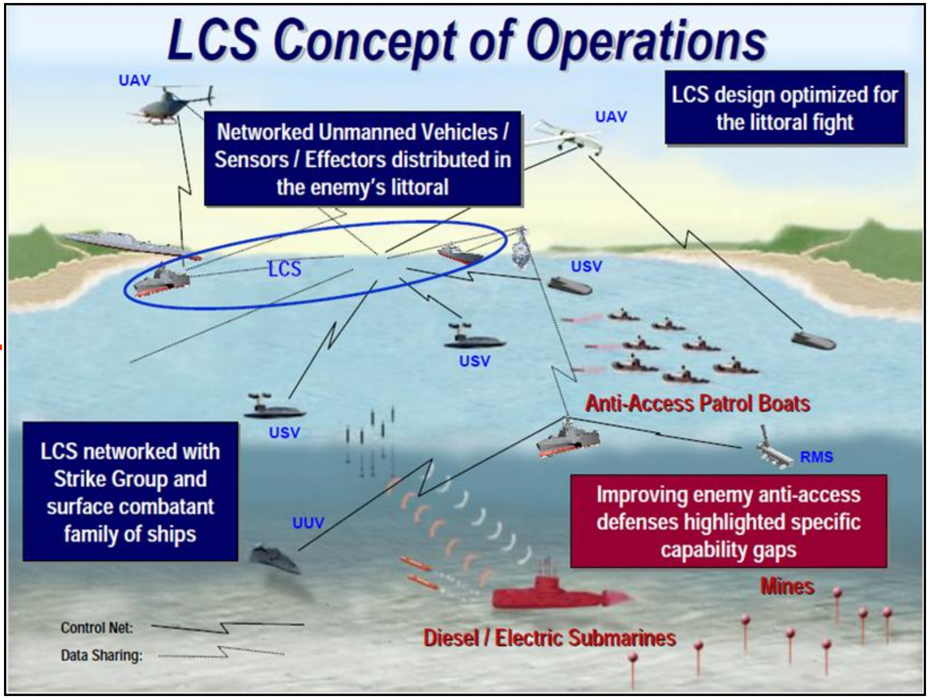


Image from: Presentation slides by RDML Vic Guillory of OPNAV at Mine Warfare Association Conference (titled "Littoral Combat Ship", 08-May-07)

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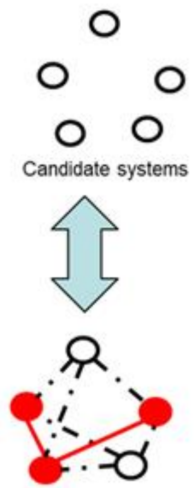
The Big Picture

SoS Selection of Systems (Operational Layer)

Applicable Method(s)
- Petri Nets (myopic)
- (N+S) Redundancy (myopic)
- Portfolio Method (strategic)

Inputs :
- Available systems (capability, requirements)
- Interdependencies

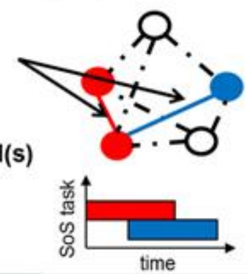
Outputs :
- Performance characteristics
- Cost
- Schedule



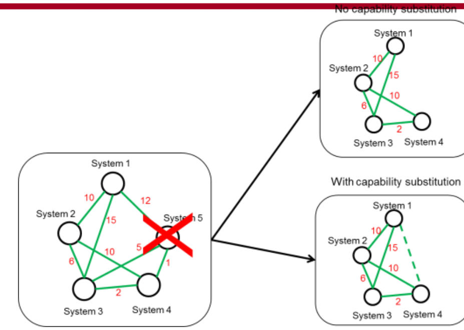
Schedule

SoS Change Management (Developmental Layer)

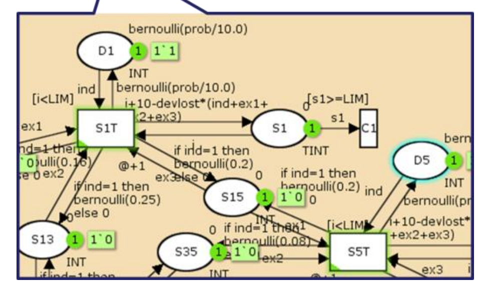
Applicable Method(s)
- FDNA-PERT
- Bayesian



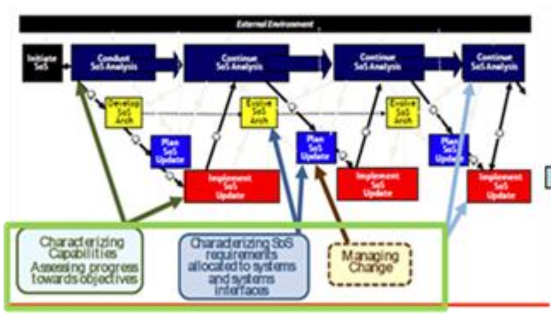
E [Cost, Capability, Performance]

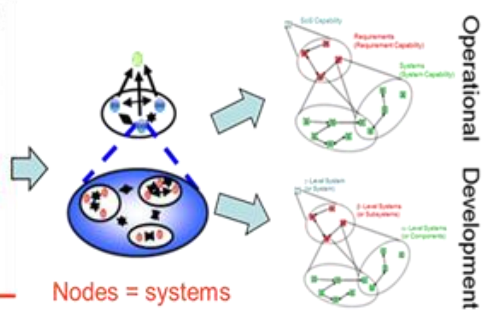


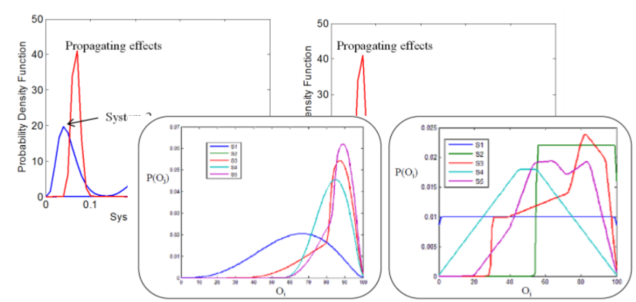
Stand-In Redundancy



Petri-Nets

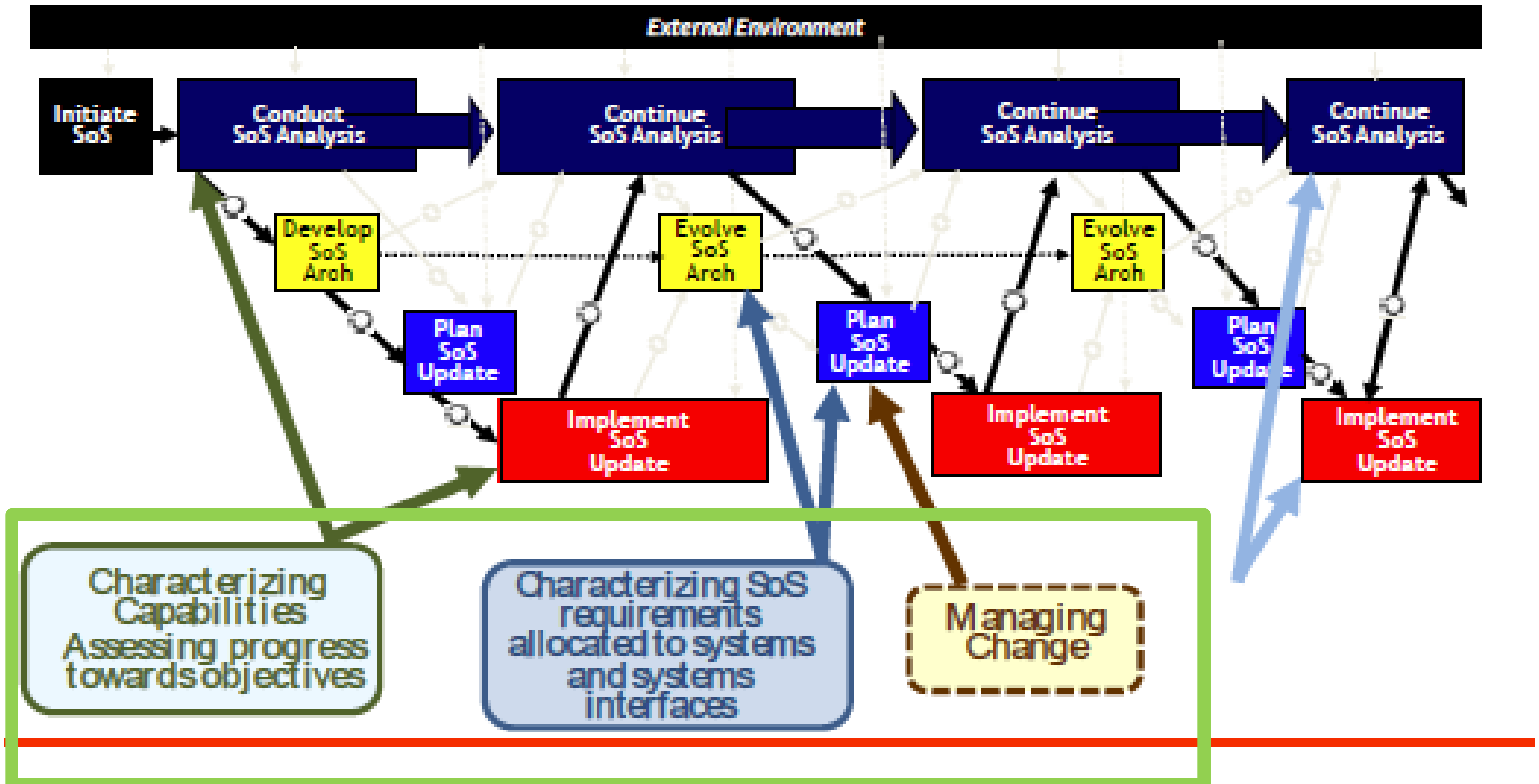






Bayesian & FDNA Approach

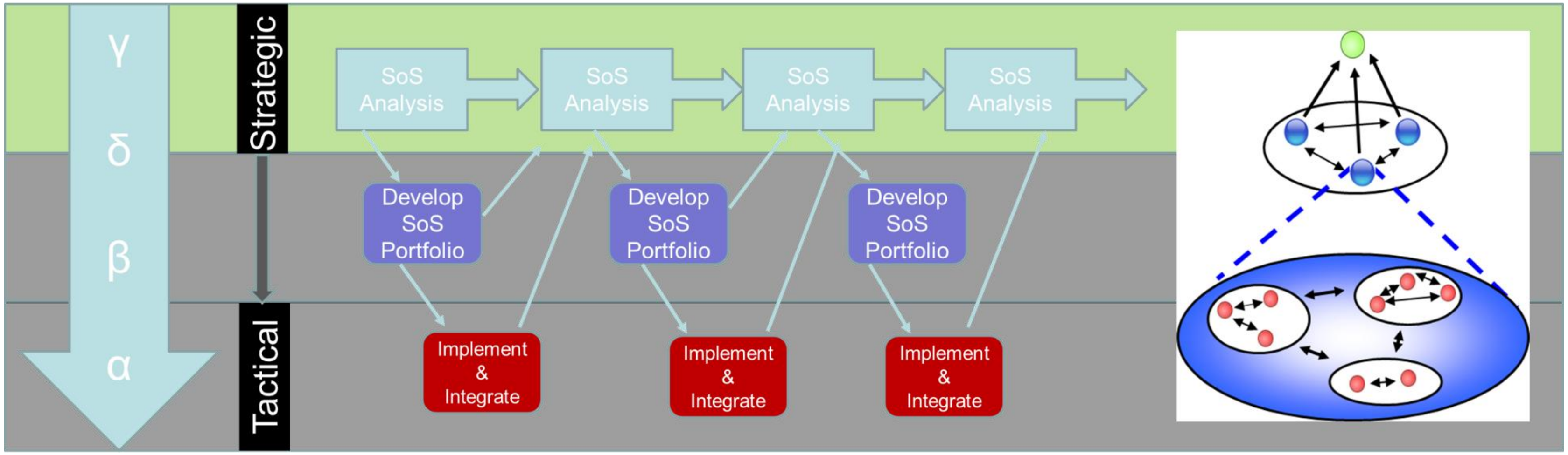
SoS Architecture Development



How do we support these actions for SoS acquisitions?

*adapted from Dahmann et. al, “Integrating Systems Engineering and Test & Evaluation in System of Systems Development” IEEE Vancouver, 2011

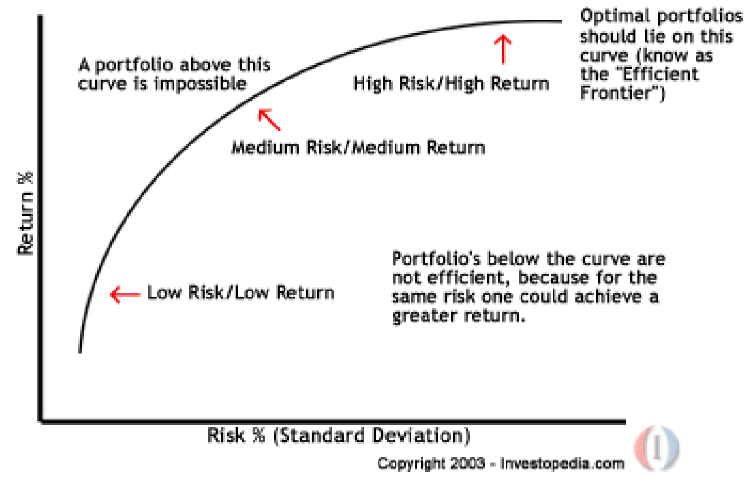
SoS Acquisition and Architecture

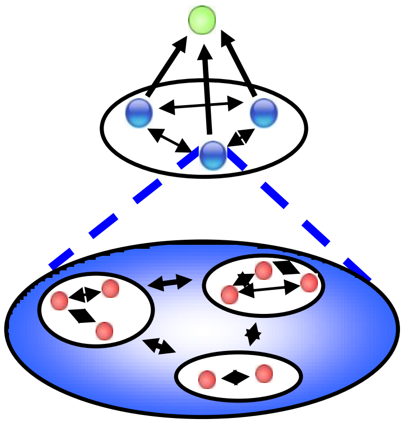


- How to leverage acquiring capabilities against associated risk?
- What about system interdependencies?
- What about performance/development uncertainty considerations?
- Can I exploit architectural connectivity for robustness?

A Portfolio Approach: Background

- Classical **Mean-Variance optimization** among techniques adopted by financial engineering and operations research.
- Balance **expected profit (performance) against risk (variance)** in investments
- Generates efficiency frontier of optimal portfolios given investor risk averseness
- Systems (nodes) can be modeled as potential investment assets→ how do we invest?



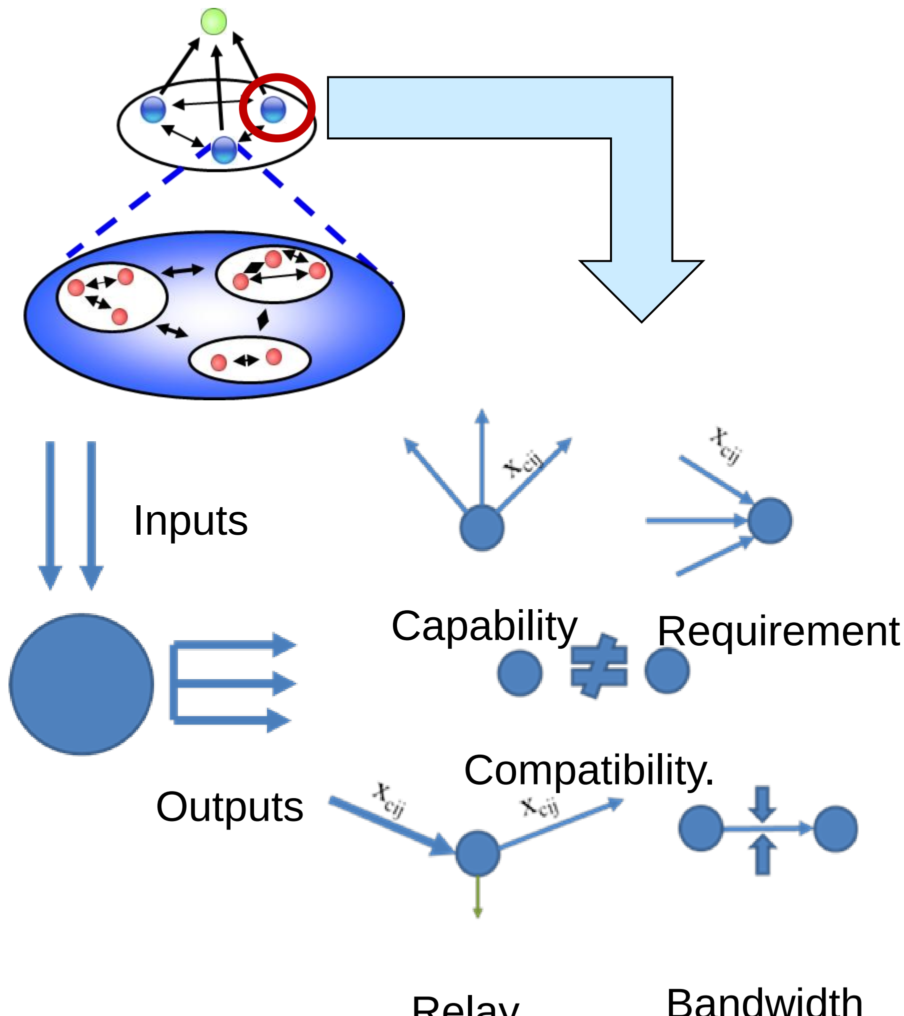


Nodes = systems

Portfolio Approach: SoS Modelling Additions

- Model individual system as 'nodes'
  - Functional & Physical representation

- Rules for node connectivity
  - Compatibility between nodes
  - Bandwidth of linkages
  - Supply (Capability)
  - Demand (Requirements)
  - Relay capability



Mean-Variance Portfolio Approach

Objective

Maximize Performance Index

Capability | Risk | Cost

$$\max \left(\sum_q \left(\frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right) - \lambda \left(X_q^F \right)^T \Sigma_{ij} X_q^F - \sum_q \left(C_q X_q^B \right) \right)$$

Constraints

Portfolio Fraction

$$X_q^F = \frac{X_q^B C_q}{\text{Budget}} \text{ (Portfolio Fractions)}$$

Portfolio Total Budget

$$\sum_q C_q X_q^B + \varepsilon = \text{Budget (Budget Constraint)}$$

Requirements Satisfaction

$$\sum_q S_{qC} X_q^B \ge \sum_q S_{qR} X_q^B \text{ (Satisfy All System Requirements)}$$

Selection Rules (Compatibility)

$$X_1^B + X_1^B + X_1^B = 1 \text{ (ASW System Compatibility)}$$

$$X_4^B + X_5^B = 1 \text{ (MCM System Compatibility)}$$

$$X_6^B + X_7^B = 1 \text{ (SUW System Compatibility)}$$

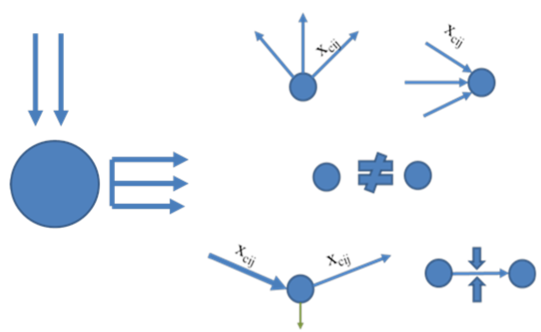
$$X_8^B + X_9^B + X_{10}^B = 1 \text{ (Package System Compatibility)}$$

Uncertainty in Covariance (Interdependencies)

$$\Sigma_{ij}^L \le \Sigma \le \Sigma_{ij}^U$$

Extension to SoS Interconnectivities

Maximize Capability Performance Index	$\max\left(\frac{\sum_i S_{ic}\cdot w\cdot X_i^B - R_c}{R_c}\right)$
	s.t.
Sufficient Capabilities Supplied	$\sum_i X_{cij} \ge X_j^B S_{rj}$
Individual System Requirements met	$\sum_i X_{cij} \ge X_j^B S_{rj}$
Connectivity Rules Obeyed (Big-M formulation)	$X_1 + \square + X_n = 0$ $\sum_c X_{cij} - X_{ij}M \le 0$ $M\sum_c X_{cij} - X_{ij} \ge 0$ $\sum_i X_{cij} - \sum_j X_{cij} - X_j^B S_{rj} = 0$
Risk Tolerance (per measure of risk)	$\sqrt{\left(X_i^B\right)^T \Sigma_{ij} X_i^B} \le \sigma_{critical}$
	$X_{cij} \le \text{Limit}_{cij}$
	$X_{cij} = 0 \quad c \in \text{capability}$
	$\Sigma_{ij}^L \le \Sigma_{ij} \le \Sigma_{ij}^U$
	$X_{cij}, X_j^B \in \text{binary}\ \{0,1\}$



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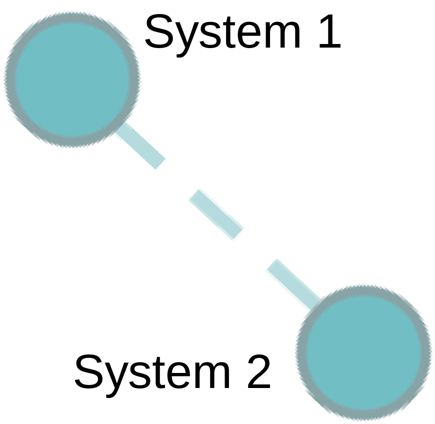
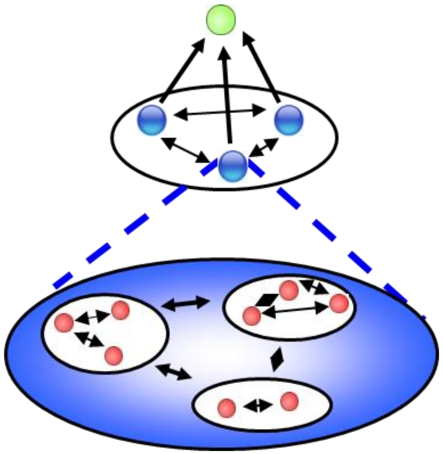
Portfolio Uncertainty



- Sources of uncertainty
  - —**System Capability**: Actual performance of system individually and as a whole SoS entity
  - —**System Interdependence**: Interdependency variances/covariances?
- Addressing uncertainty
  - —Operations Research/Financial Engineering Methods to address uncertainty measures
  - —Introduce uncertainty in interdependencies and individual asset performances
  - —Introduce SoS connectivity in portfolio space



Mean-Variance Portfolio: A Robust Approach

Capability | Risk | Cost

Objective

Maximize Performance Index

$$\max\left(\sum_q\left(\frac{S_{qc}-R_c}{R_c}\cdot w\cdot X_q^B\right)-\lambda\{\langle\bar{\Lambda}\bar{\Sigma}\rangle-\langle\underline{\Lambda}\underline{\Sigma}\rangle\}-\sum_q\left(C_q X_q^B\right)\right)$$

Constraints

Portfolio Fraction

$$X_q^F=\frac{X_q^B C_q}{\text{Budget}}\ \text{(Portfolio Fractions)}$$

Portfolio Total Budget

$$\sum_q C_q X_q^B+\varepsilon=\text{Budget (Budget Constraint)}$$

Requirements Satisfaction

$$\sum_q S_{qC} X_q^B \geq \sum_q S_{qR} X_q^B\ \text{(Satisfy All System Requirements)}$$

Selection Rules (Compatibility)

$$X_1^B+X_1^B+X_1^B=1\ \text{(ASW System Compatibility)}$$

$$X_4^B+X_5^B=1\ \text{(MCM System Compatibility)}$$

$$X_6^B+X_7^B=1\ \text{(SUW System Compatibility)}$$

$$X_8^B+X_9^B+X_{10}^B=1\ \text{(Package System Compatibility)}$$

Robust Formulation (Tutuncu & Koenig 2004)

$$\begin{bmatrix}\bar{\Lambda}-\underline{\Lambda} & X_q^F\\ X_q^{F\,\prime} & 1\end{bmatrix}\succeq 0\ \text{(Linear Matrix Inequality)}$$

$$X_q^B\in\{0,1\}\ \text{(binary)}$$

Robust Portfolio Case Study: Simple LCS Portfolio

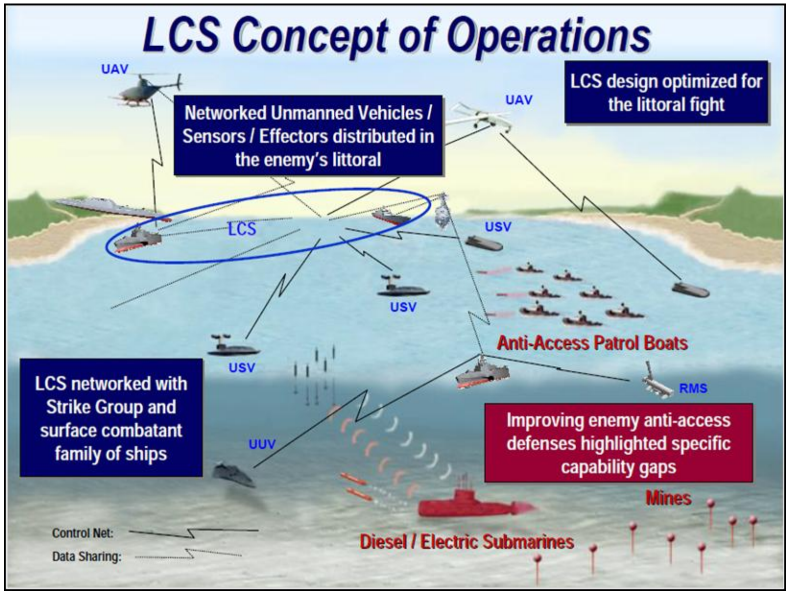


Table 2: System interdependency and development risk (covariance)

	able Depth	ti Fcn Tow	tweight tow	MCS II	MDS (MH-60)	OS Missiles	fin Missiles	kage System 1	kage System 2	kage System 3
Package System 2	0	0.1	0	0.2	0	0.1	0	0	0.3	0
Package System 3	0	0	0.2	0	0.3	0	0	0	0	0.2

Diagonal : System Variance

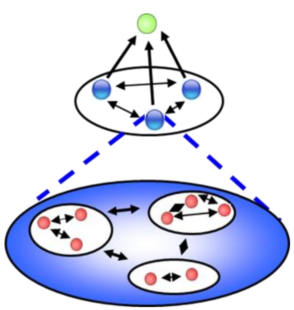
Off Diagonal : System Interdependency

Table 1: Individual system information

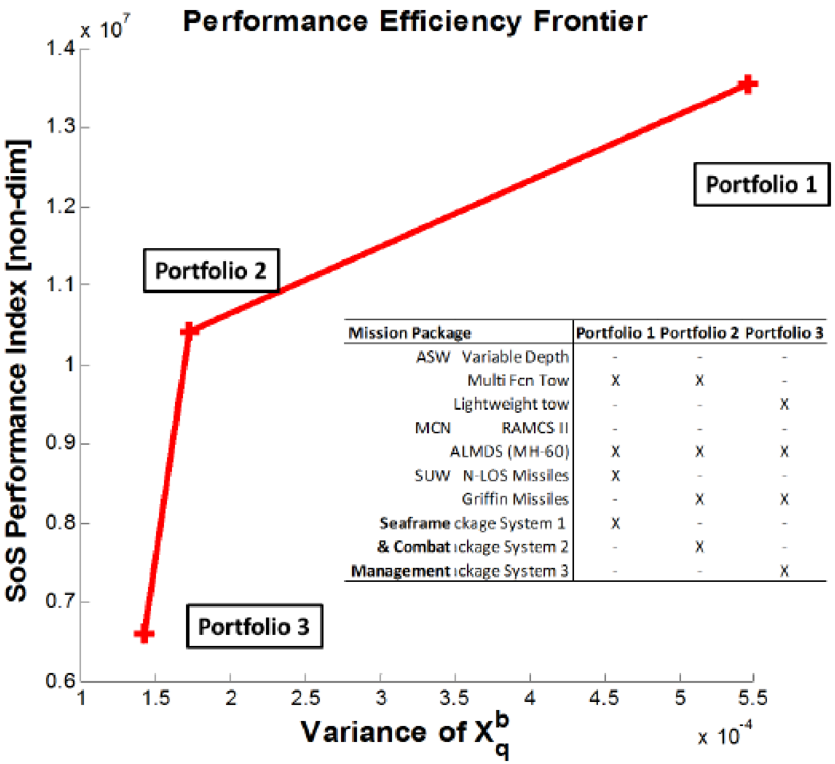
		System Capabilities					System Req		Develop. Time	Acq. Cost
		Weapon Strike Range	Threat Detection Range	Anti Mine Detection Speed	Comm. Capacity	Air/Sea State Capacity	Air/Sea State	Comm.	(Years)	($)
Package										
ASW	Variable Depth	0	50	0	0	0	0	250	3	3000000
	Multi Fcn Tow	0	40	0	0	0	0	150	2	2000000
	Lightweight tow	0	30	0	0	0	0	100	4	4000000
MCN	RAMCS II	0	0	40	0	0	3	200	1	1000000
	ALMDS (MH-60)	0	0	30	0	0	4	100	2	2000000
SUW	N-LOS Missiles	25	0	0	0	0	0	200	3	3000000
	Griffin Missiles	3	0	0	0	0	0	100	4	4000000
Seaframe	Package System 1	0	0	0	400	4	0	0	3	3000000
& Combat	Package System 2	0	0	0	300	4	0	0	4	4000000
Management	Package System 3	0	0	0	250	3	0	0	5	5000000

Capability

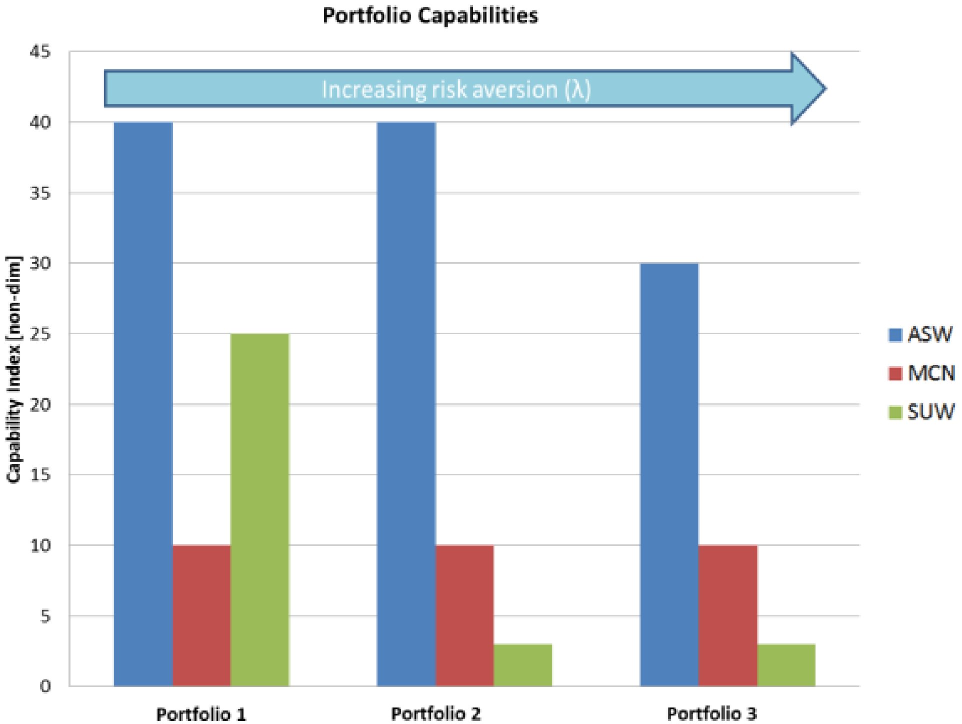
Requirement



Robust Portfolio Case Study: Simple LCS Portfolio

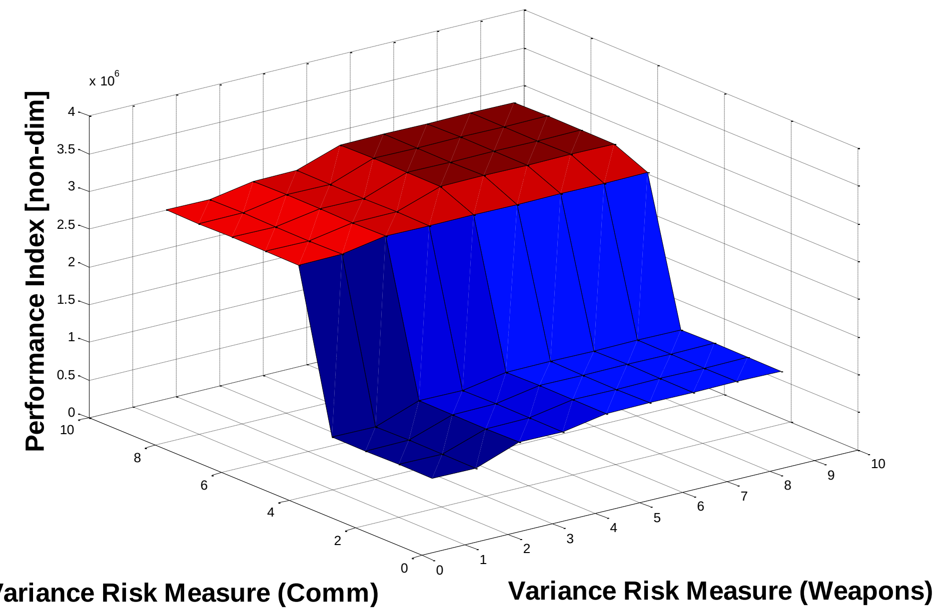


Mission Package		Portfolio 1	Portfolio 2	Portfolio 3
ASW	Variable Depth	-	-	-
	Multi Fcn Tow	X	X	-
	Lightweight tow	-	-	X
MCN	RAMCS II	-	-	-
	ALMDS (MH-60)	X	X	X
SUW	N-LOS Missiles	X	-	-
	Griffin Missiles	-	X	X
Seaframe	ckage System 1	X	-	-
& Combat	ickage System 2	-	X	-
Management	ickage System 3	-	-	X



Portfolio Approach: LCS Multiple Risk Measures

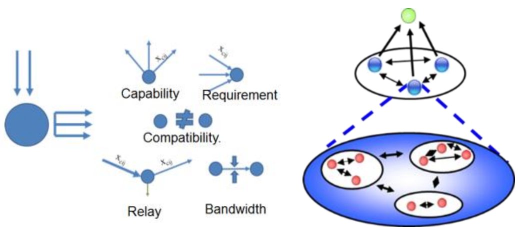
- Layered measure of risk (e.g. weapons vs. communications layer).
- Separate covariance for each measure of risk



Comm. Variance (Risk) Constraint → $\sqrt{(X_i^B)^T \Sigma_{ij}^{comm} X_i^B} \leq \sigma_{comm}$

Weapon Variance (Risk) Constraint → $\sqrt{(X_i^B)^T \Sigma_{ij}^{weapons} X_i^B} \leq \sigma_{weapon}$

Portfolio Robust Operational Constraints



Constraint Rules for Connectivity & Operations

$$\sum_i X_{cij} \ge X_j^B S_{rj}$$

$$\sum_i X_{cij} \ge X_j^B S_{rj}$$

$$X_1 + \cdots + X_n = 0$$

$$\sum_c X_{cij} - X_{ij} M \le 0$$

$$M \sum_c X_{cij} - X_{ij} \ge 0$$

$$\sum_i X_{cij} - \sum_j X_{cij} - X_j^B S_{rj} = 0$$

Bertsimas-Sim Method: Adjust conservatism Γ_i term to control probability of **constraint violation**

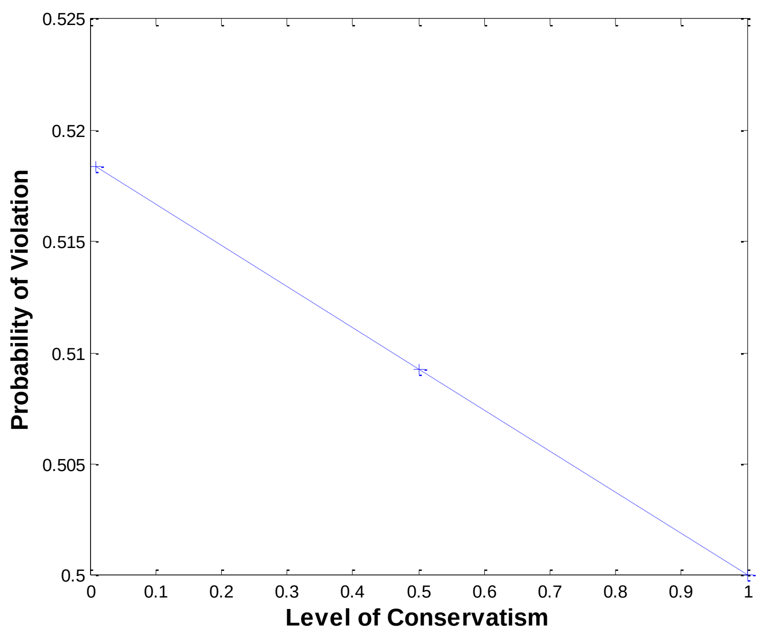
$$\sum_q S_{qC} X_q^B + \max\left\{\hat{S}_{qC} y_j + \left(\Gamma_i - \lfloor \Gamma_i \rfloor \hat{S}_{it_i} y_t\right)\right\} \le b_i$$

$$-y_j \le X_q^B \le y_j$$

$$y \ge 0$$

Conservatism Added
(This can be converted to an LP == easy to solve even for large problems)

Portfolio Robust Operational Constraint



Important Operational Constraint (e.g.)

Package		Bandwith Req
ASW	Variable Depth	3.54
	Multi Fcn Tow	60
	Lightweight tow	33.99
MCN	RAMCS II	24.25
	ALMDS (MH-60)	76.8
SUW	N-LOS Missiles	11.07
	Griffin Missiles	42.68
Seaframe	Package System 1	91.54
& Combat	Package System 2	55.06
Management	Package System 3	63.85

Subject to some uncertainty (+/-)

Future Work: Portfolio Approach

- Semi Definite Programming (SDP) – can be hard to solve/implement
  - Conic and Linear Programming versions → well developed open solvers

- Extend to multi-period portfolio → dynamic programming

$$\max \underbrace{\left(\sum_q \left(\frac{S_{qc} - R_c}{R_c} \cdot w \cdot X_q^B \right) - \lambda \left(X_q^F \right)^T \Sigma_{ij} X_q^F - \sum_q \left(C_q X_q^B \right) \right)_t}_{A} + E\left(A_{t+1} \mid w_{t+1}, \Sigma_{t+1}, \lambda_{t+1} \right)$$

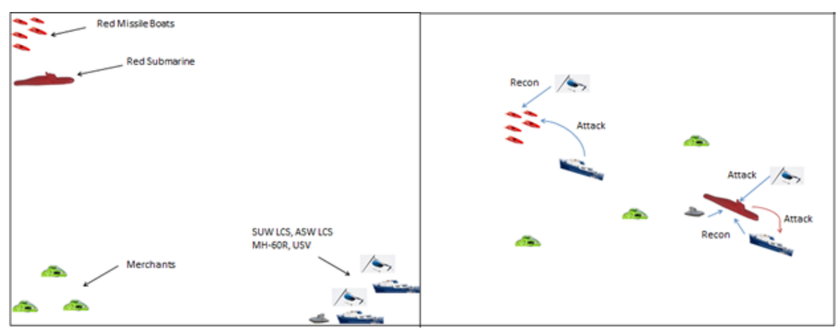
Capability vs. Risk now

Effect on Capability Later

- Agent-Based Simulation (e.g. for covariance estimation, CVaR)

$$\sqrt{\left(X_i^B\right)^T \Sigma_{ij}^{comm} X_i^B} \le \sigma_{comm}$$

$$\sqrt{\left(X_i^B\right)^T \Sigma_{ij}^{weapons} X_i^B} \le \sigma_{weapon}$$



Summary/Conclusion

- RMVO promising framework to leverage SoS performance against risk
- Considers uncertainty and system interdependencies explicitly in portfolio construction
- Develop further towards analytic workbench objectives