Operational Availability

Estimation, Modeling, and Analysis

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Overview

- Operational Availability (A_O) is conceptually simple
  - It is the portion of time that a system is either operating or capable of operating
- Estimating A_O, however, is not always so simple
- Several equations can be developed or found in literature that can be used to estimate A_O
  - Not all of the equations are mathematically equivalent
  - Some of the A_O equations apply to specific system usage scenarios
  - The equations are often incorrectly used interchangeably
- Key factors affecting A_O equation applicability are
  - Mission versus steady state availability
  - Intermittent system usage

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A$_O$ Defined

- Operational Availability indicates the percentage of time that a system or group of systems within a unit are operationally capable of performing an assigned mission. A$_O$ can be expressed as (uptime/(uptime + downtime)). *

$$A_O = \frac{uptime}{uptime + downtime}$$

- An alternative form of the A$_O$ equation is often used

$$A_O = \frac{MTBDE}{MTBDE + MDT}$$

MTBDE = Mean Time Between Downing Events
MDT = Mean Down Time

- The specific acronyms used in the alternate equation vary based on chosen terminology (MTBF, MTBCF, MTBOMF, MRT, MTTR) but the equation is valid as long as the first term is the average time between events causing the system to go down, and the second term is the average down time. MTBDE is the most general term since it can include any downing event including required scheduled maintenance downtime.

Department of Defense Reliability, Availability, Maintainability, and Cost Rationale Report Manual (paragraph 1.2)

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A$_O$ Example
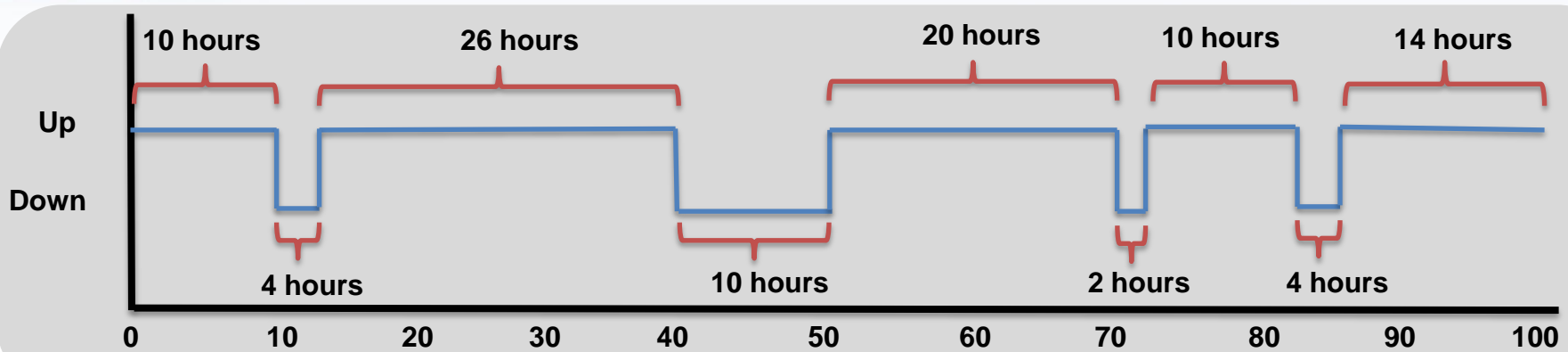
- A system operates for 100 hours as shown in the above timeline
    - Uptime = 10+26+20+10+14 = 80 hours
    - Downtime = 4+10+2+4 = 20 hours
    - A$_O$ = uptime/(uptime+downtime) = 80/(80+20) = 0.80

- Using the alternate method
    - MTBDE = (operating hours)/(number of downing events) = 80/4 = 20
    - MDT = (down hours)/(number of downing events) = 20/4 = 5
    - A$_O$ = MTBDE/(MTBDE+MDT) = 20/(20+5)=0.80

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Equation Comparison

$$A_O = \frac{MTBDE}{MTBDE + MDT} = \frac{\dfrac{Uptime}{Number\ of\ Downing\ Events}}{\dfrac{Uptime}{Number\ of\ Downing\ Events} + \dfrac{Downtime}{Number\ of\ Downing\ Events}} = \frac{Uptime}{Uptime + Downtime}$$

- Over a specified period of time, when MTBDE and MDT are calculated based on observed data, the alternative equation is exactly equivalent to the general equation

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Alternate A_O Equations

- The common A_O equations can be algebraically rearranged into several other forms

$$A_O = \frac{Uptime}{Uptime + Downtime} = \frac{(Total\ Time - Downtime)}{Total\ Time} = 1 - \frac{Downtime}{Total\ Time}$$

- The last equation can then be further broken down into less common forms that are useful for performing intermittent use system analysis

$$A_O = 1 - \frac{Downtime}{Total\ Time} = 1 - \frac{DT}{CT} = 1 - \frac{\frac{DT}{Failure}}{\frac{CT}{Failure}} = 1 - \frac{MDT}{MTBF + MDT}$$

DT = Down Time
CT = Calendar Time

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Steady State vs. Mission Availability

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Steady State Availability

- Often, various mean time between failure and mean repair time measures are calculated for a system through analysis
  - For availability purposes, the analysis should focus on failures that cause system downtime. The terms Mean Time Between Operational Mission Failures (MTBOMF) or Mean Time Between Critical Failures (MTBCF) are often used.
  - The corresponding average downtime for these failures is the MDT

- These calculated values are long term, steady state averages
  - Using them in the alternate A_O equation results in the steady state average for availability

$$\frac{MTBOMF}{MTBOMF + MDT} = Steady\ State\ Ao$$

  - This is the availability that would be observed if the systems were to run continuously for a long period of time (i.e., >> MTBOMF + MDT)

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Availability During Mission Time

- Often in military applications, the availability of a system over a limited deployed period (specified mission) is desired

- If the system starts in an "up" state, and the mission length is small relative to the MTBDE, then transients are important

- As a general rule, the availability will start approaching the steady state availability after a time period of approximately four times the average time to failure (Reliability Hotwire, Issue 79, September 2007)*

http://www.weibull.com/hotwire/issue79/relbasics79.htm

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Instantaneous Availability

- The instantaneous availability of a system is the probability that the system is "up" at a specific time (t)

- If the system starts in an "up" condition, its instantaneous availability begins at 1.0 and approaches the steady state availability after a few failure/repair cycles

- For systems that operate continuously, once the system passes an initial start-up period, the instantaneous availability equals its steady state availability

- In many analysis cases, the period of interest is such that the startup transient is negligible and is ignored

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Two State Markov Analysis

- Consider a two state model in which a system is either "up" represented by a 1, or "down" represented by 0
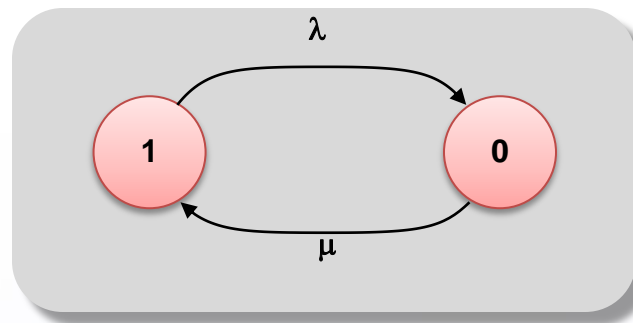
- The system moves from state 1 to state 0 at a rate λ and from state 0 to 1 with rate μ

- $\lambda = 1/\text{MTBDE}$ and $\mu = 1/\text{MDT}$

- The instantaneous availability can be calculated as a function of time. It is the probability of being in state 1 at time t.

$$P_1(t) = \frac{\mu}{\mu + \lambda} + \frac{\lambda}{\mu + \lambda} e^{-(\lambda + \mu)t}$$

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Two State Markov (continued)

- The average availability is the uptime percentage through time t and can be calculated as

$$A_O(t) = \frac{1}{t}\int_0^t P_1(u)\,du$$

$$= \frac{1}{t}\int_0^t \left[\left(\frac{\mu}{\mu + \lambda}\right) + \left(\frac{\lambda}{\mu + \lambda}\right)e^{-(\lambda+\mu)u}\right]du$$

$$= \frac{\mu}{\mu + \lambda} + \frac{-\lambda}{t(\mu + \lambda)^2}\left(e^{-(\lambda+\mu)t} - 1\right)$$

- The limit of $A_O(t)$ as t approaches infinity is $\dfrac{\mu}{\mu + \lambda}$

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Markov Example

- Consider a system with MTBDE = 75 and MDT =18.75
- The system will be deployed for 280 hours
    - Its instantaneous availability is plotted below
    - This system has a steady state availability of 75/(75+18.75)=0.8

$P_1(t)$
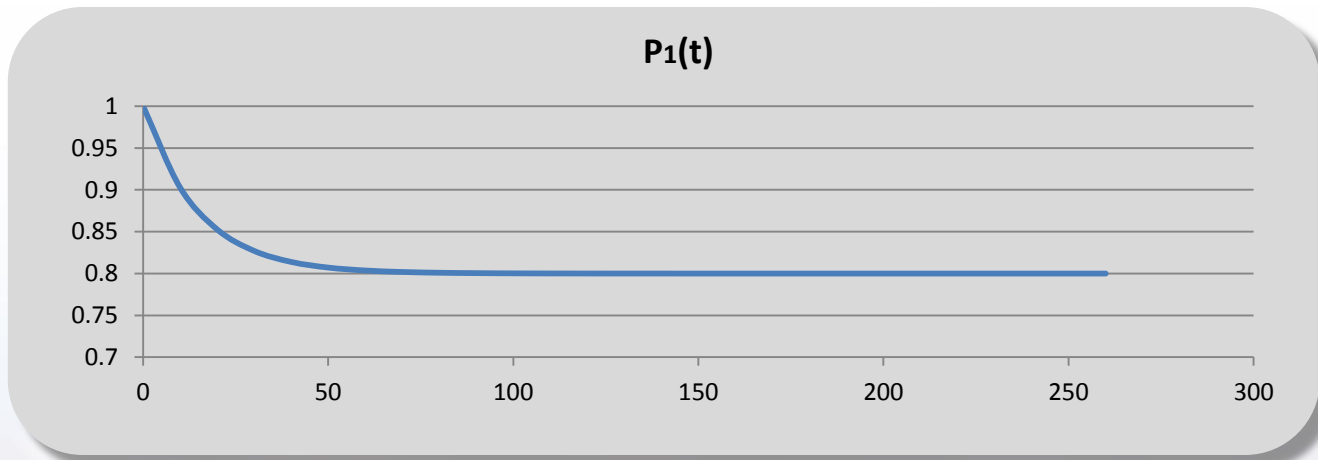
- The system *instantaneous* availability settles very quickly to 0.8
    - The instantaneous availability at t=50 is 0.81

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Markov Example

- The average availability at time t includes affects up to that time and approaches steady state more slowly

- If this system were to be deployed for 50 hours, the expected availability over the period from 0-50 would be about 0.86

  – The steady state approximation result (0.80) is significantly off

$A_O(t)$
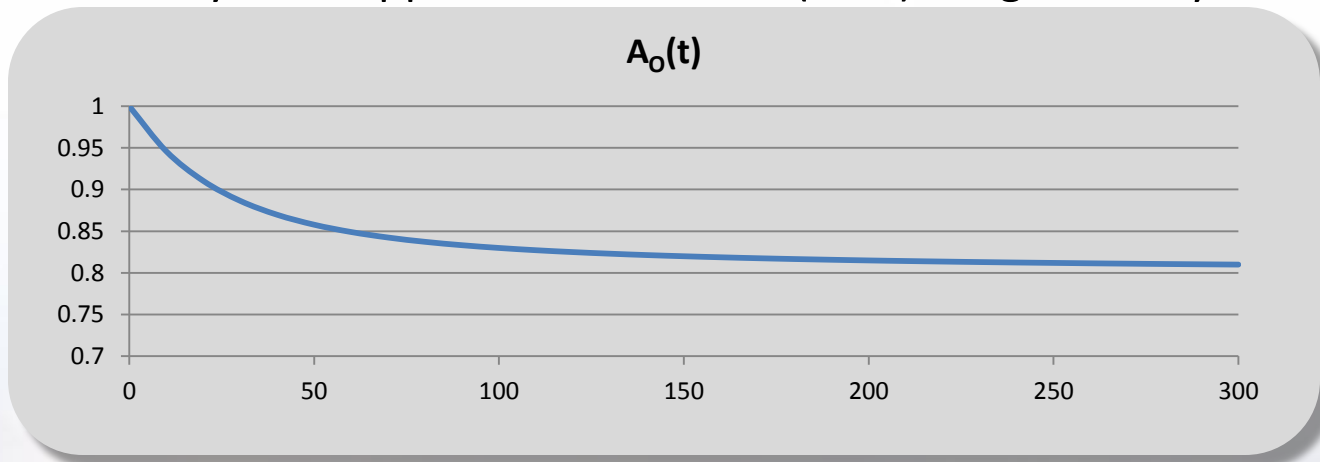
- Using the developed A_O equation, $A_O(280)=0.8107$

  – At mission length 280, A_O is approaching the steady state value but is still off by 0.01

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Simulation to Solve A_O

- The two-state Markov analysis is a close approximation of a deployed system on a short mission

- One significant difference is in the repair distribution
  - The Markov analysis assumes an exponential distribution for the time to repair, while in practice, a lognormal distribution is typically used

- Simulation allows for estimating the A_O without the need for the simplifying assumption

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Simulation of Example

- ## Replicating known solution case

  Failure Distribution: Exponential(mean=75)
  Repair Distribution: Exponential(mean=18.75)
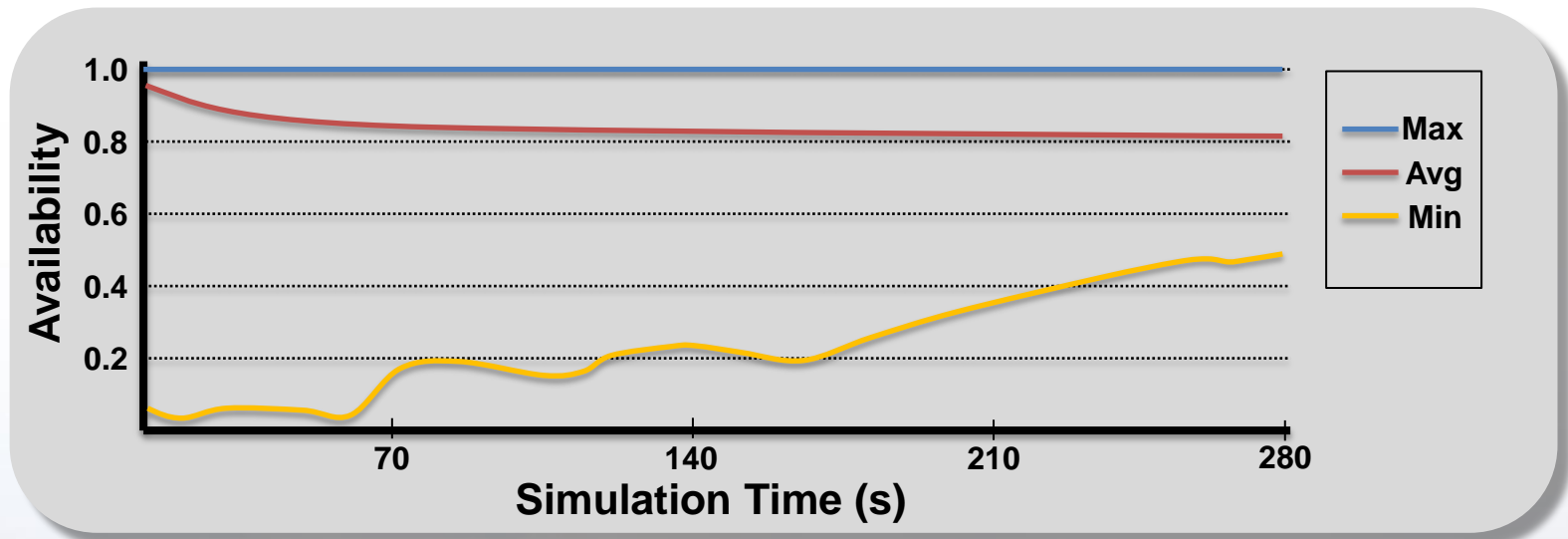
- ## Results from 1000 trials
  - $A_O(280) = 0.8097 +/- 0.0041$ (1 Standard Error of the Mean (SEM))
  - Known solution of 0.8107 is in range of simulation result (.8056 - .8138)
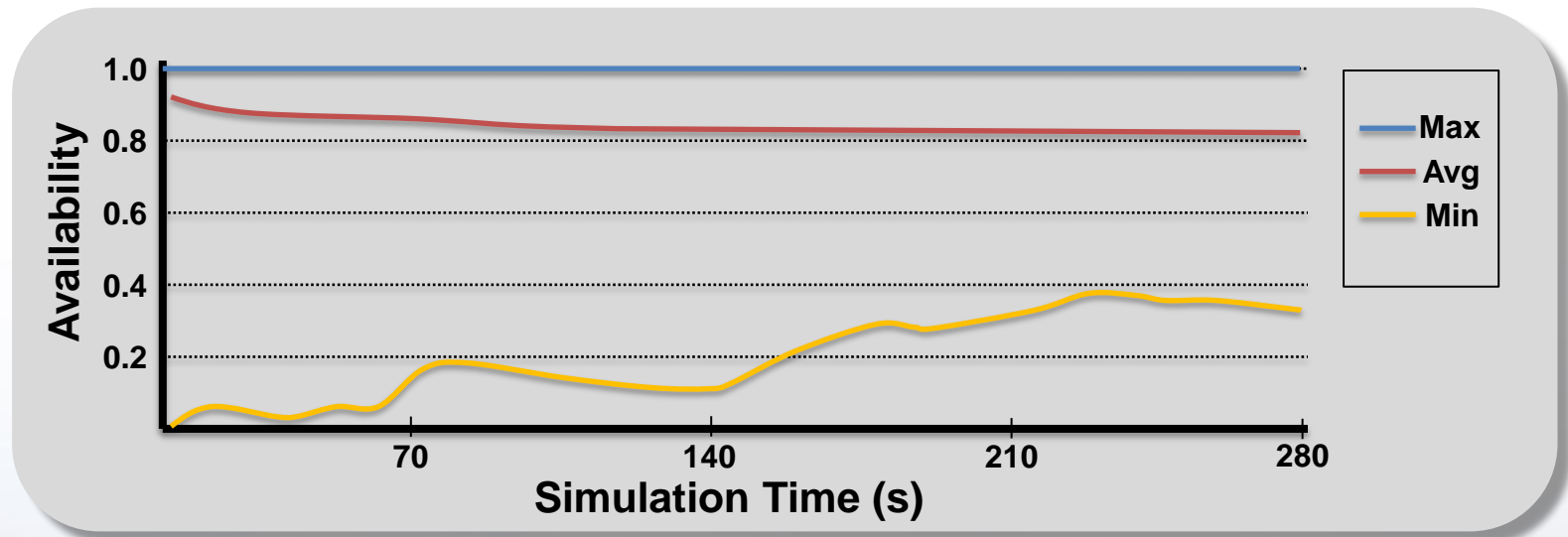 - Individual trials still reflect wide range of A_O results

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Simulation of Example (2)

- ## Simulation using lognormal repair (exact solution unknown)

Failure Distribution: Exponential(mean=75),
Repair Distribution: Lognormal(mean=18.75, St. dev. = 18.75)

- ## Results from 1000 trials
    - $A_O(280) = 0.8141 +/- 0.0039$ (1 Standard Error of the Mean (SEM))
    - Result is very close to known solution for exponential repair case of 0.8107
    - Graph – Transient region very similar to exponential case
    - Conclusion: Modeling repair with a lognormal distribution instead of an exponential distribution does not significantly affect the time for system availability to approach steady state values

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Intermittent System Usage

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System Usage

- Continuous-use systems: Systems that are (nearly) always in use during operations of their host platforms
  - Examples are search radars, computer systems, communications equipment, and power systems
- Intermittent-use (non-continuous or on-demand) systems: Systems that have relatively long periods of standby or inactivity between uses
  - Examples are radio transmitters, helicopters, airplanes, tanks, and special purpose equipment

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Utilization Rates

- A significant source of confusion regarding analysis of intermittent usage systems relates to interpretation of the terms describing the utilization
    - For aircraft usage, Utilization Rate or "Ute rate", describes the percentage of clock/calendar time that the aircraft is used. It is sometimes expressed in hours/day and represents the ratio of (operating time)/(total time).
    - The term "Utilization" is also commonly used to describe system usage. It expresses the percentage of the available uptime that the system is used. In this case, utilization refers to (operating time)/(uptime).

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Aircraft Intermittent Use

- Both the Air Force and Navy use Utilization Rate or "Ute rate" to describe the intermittent use of aircraft
  - If an aircraft is scheduled to fly 12 hours per day its Ute Rate is 12 hours /24 hours = 0.5
  - The Ute rate is a ratio of OT/CT (operating time)/(calendar time)
  - For analysis purposes the Navy also uses a term called K' which is the inverse of Ute rate and expresses CT/OT
- A_O for aircraft intermittent use is calculated as

$$A_O = 1 - \frac{MDT}{\frac{1}{UR} * MTBF} = 1 - \frac{MDT}{K' * MTBF}$$

- This equation can be verified with substitution for K' and MTBF based on their definitions

$$1 - \frac{MDT}{K'*MTBF} = 1 - \frac{\frac{DT}{Failure}}{\frac{CT}{OT} * \frac{OT}{Failure}} = 1 - \frac{\frac{DT}{Failure}}{\frac{CT}{Failure}}$$

This was previously shown to be a valid form of A_O

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Aircraft Intermittent Use – Example

- Aircraft will be used to transport cargo for 1000 hours
- Planned utilization is 12 hours per day: Ute Rate = 0.5, K' = 2
- MTBF = 75, MDT =25
- Using the formula

$$A_O = 1 - \frac{MDT}{K' * MTBF} = 1 - \frac{25}{2*75} = 0.833$$

- Since the goal is to maintain the 12 hours a day flying schedule (half of all calendar time), 500 mission hours will be flown
- There are 25 down hours for each 75 operating hours
- The remaining time is ramp time
- Calculating A_O from data

Time Category	Calculation	Hours
Operating Hours (OH)	"Ute" * 1000	500
Down Hours	OH * 25/75	166.7
Ramp Hours (RH)	1000-500-166.7	333.3
A_O	(OH + RH)/Total	0.833

- Note: The equation assumes the proposed Ute rate is possible
  - The Ute rate cannot exceed the full time usage A_O of MTBF/(MTBF+MDT)

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Non-Aircraft Intermittent Use

- Non-aircraft systems are often used on an intermittent basis that is *not* defined as a fixed amount of operating time per calendar time.
- Utilization for these systems means that when the system is not broken, it is used a portion of time
    - The utilization ratio is OT/UT (operating time/up time)
    - For analysis purposes the Navy also uses a term called K'' which is the inverse of this utilization and expresses UT/OT
    - K'' is related to K'

$$K'' = K' - \frac{MTTR + MLDT}{MTBF}$$ *OPNAVINST 3000.12.A Sec 3.5*

- A_O for non-aircraft intermittent use systems is given by

$$A_O = \frac{K'' * MTBF}{K'' * MTBF + MDT}$$

$$A_O = \frac{\frac{UT}{OT} * \frac{OT}{Failure}}{\frac{UT}{OT} * \frac{OT}{Failure} + \frac{DT}{Failure}} = \frac{\frac{UT}{Failure}}{\frac{UT}{Failure} + \frac{DT}{Failure}} = \frac{\frac{UT}{Failure}}{\frac{UT + DT}{Failure}} = \frac{UT}{UT + DT}$$

- Equivalently, with utilization (U) described as above, A_O is given by

$$A_O = \frac{MTBF/U}{MTBF/U + MDT}$$

- This is the most commonly used equation for calculating A_O of intermittent use systems

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Non- Aircraft Intermittent Use – Example

- A system will be deployed on an operation for 1000 hours
- System will be actively engaged in a mission one half of the time it is available
- Utilization = 0.5, K'' = 2
- MTBF = 75, MDT = 25
- Using the formula

$$A_O = \frac{K'' * MTBF}{K'' * MTBF + MDT} = \frac{2 * 75}{2 * 75 + 25} = 0.857$$

Time Category	Calculation	Hours
Operating Hours (OH)	Ao*1000*U	428.57
Down Hours	OH*25/75	142.86
Ramp Hours (RH)	1000-428.57-142.86	428.57
A_O	(OH + RH)/Total	0.857

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Conclusions

- Steady State vs Mission A_O
  - The steady-state availability equation yields an A_O that can be significantly lower than what is actually observed during a short mission
  - As a good guideline, expected average system A_O is not approaching steady state behavior until operation through four failure cycles
    - If the mission length is less than 4*(MTBDE+MDT), be careful!
  - If applied to short missions that start in an operable state, steady-state A_O will underestimate mission availability
    - You could end up under-selling the performance of the system

- Intermittent Use Systems
  - Aircraft utilization rate specifically refers to the ratio of operating time to calendar time
    - Equations specifically referencing "Ute Rate" most likely refer to this interpretation
  - For most non-aircraft systems, utilization refer to operating hours per "up" hour
  - Equations for availability that include utilization were developed based on a specific interpretation
  - Know what the utilization ratio you are using refers to!

- Use caution when applying A_O equations

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