

NDIA 17th Annual Systems Engineering Conference

Abstract #16844

Using Game Theory for Optimizing
Trade Studies in Systems Engineering

Jason Dever, Jennifer Mills, Steven Stuban & Bereket Tanju

PhD in Systems Engineering
George Washington University

Introduction

- **Problem Statement**

While the outcome of trade studies depends on determining the best technical solutions to resolve system conflicts and constraints, a Game Theory based model that utilizes a linear programming technique can increase awareness of the current state and consequence (or reward) of the decision maker's actions to optimize stochastic trade study outcomes

- **Hypothesis**

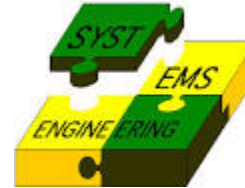
When compared to other traditional Systems Engineering (SE) trade study approaches, a Game Theory based model quantitatively optimizes stochastic outcomes

- **Significance**

This model produces a list of alternatives that its users can select and use to optimize the outcome of trade offs. The model is applicable and expandable at all levels, from the system level to each functional level, by focusing on the overall program impact (consequences) as a result of implementing trade offs (pay offs). Each alternative is generated based on the multivariate, key parameters (players) within the pre-determined rules / requirements.

Research Focus Area

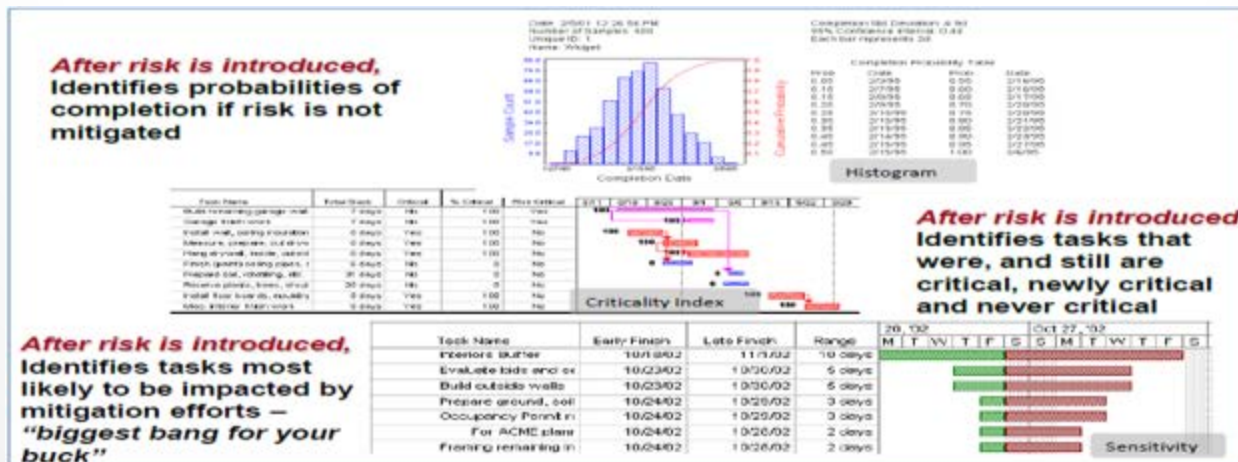
- Systems Engineering (SE)
 - Desired Features of Systems
 - Requirement Conflicts and Constraints



- Game Theory
 - Linear Programming with Multivariate Approach
 - Game Theory Based Stochastic Model (Players, Rules, Consequences & Pay Offs)



- Trade Studies
 - Determine & Focus on the Key Parameter Performance / Requirements
 - Optimize Decision Outcome



Methodologies

- Case Studies

- Linear Programming with Game Theory Approach
- Sensitivity Analysis using Random Variables

- Game Theory – Quantitative Approach

- For each trade-off alternative, calculate: $\sum_1^n f(C * CI + S * SI | C, S > 0)$
 - i. Consequences: the key factors are Cost (C) & Schedule (S) for each player, where CI is its cost impact and SI is its schedule impact (forecasted values)
 - ii. Players: Key Performance Parameters (KPPs)
 - iii. Pay-Offs: Potential cost and schedule savings for each player
 - iv. Rules: Goal to Minimize $(C * CI) + (S * SI)$ for each player

Subject to:

- $C_1 + C_2 + C_3 + \dots + C_n \leq TC$
- $S_1 + S_2 + S_3 + \dots + S_n \leq TD$
- $(C_1 * CI_1) + (C_2 * CI_2) + \dots + (C_n * CI_n) \leq TC$
- $(S_1 * SI_1) + (S_2 * SI_2) + \dots + (S_n * SI_n) \leq TS$

Where TC is total cost, TD is total duration, given that C, S, TC, TS > 0

- Select the optimal solution or the best alternative from the finite set of actions (available alternatives) based on the forecasted cost and schedule impact

Simulation

- Setup : input randomly selected data
- Process : for each program, a trade-off simulation is conducted where any key parameter shows potential requirement conflicts or impact to the overall performance; for example, a cost overrun or a schedule slip to achieve certain technical requirements is viewed as a trade-off opportunity to minimize overall impact to the system.
- Sensitivity Analysis : multiple scenarios based on randomly generated values are tested in order to conduct a sensitivity analysis to ensure that the proposed model is robust and generates a reliable stochastic outcome for any system trade-offs.
- Result : the preliminary result indicates that although a stochastic model can generate an objective outcome, there are other factors to consider when making decisions such as social impact, policy changes or political implications.

Generate Random Variables for Testing

- If "Lognormal" Then

- `'// Generate Lognormal RVs`
- `'// the Probability Density Function of the lognormal distribution is given by`
- `'// $f(x) = [1/(x*sd*(2*pi)^{0.50})*exp(-0.5*[(\log(x) - u)/sd]^2)$`
- `'// where u and sd are the parameters of the distribution computed as follows`
- `'// Mean $E[X] = \exp(u+0.5sd^2)$ and $Var[X] = \exp[2u + sd^2] * \{(\exp sd^2)-1\}$`
- `'// In Excel, Lognormal is $e^N(m,s)$, which would be =EXP(NORMINV(RAND(),Mean,Stdev))`
- `'// However that will give me very large values. Next step is to scale the mean and standard deviation.`
- `'// In pseudo code,`
- `'// $adjMean = \ln(m^2 / \sqrt{m^2 + s^2})$`
- `'// $adjStd = \sqrt{\ln((m^2 + s^2) / m^2)}$`

- Else If "Triangular" Then

- `'// Generate Triangular RVs`
- `'// The function uses the (standard) inverse cumulative method for generating a random number.`
- `'// The triangular density function has a piecewise-quadratic cumulative distribution, and`
- `'// the If statement uses the inverse of the appropriate quadratic.`
- `'// In pseudo code,`
- `'// LowerRange = Mode - Minimum`
- `'// HigherRange = Maximum - Mode`
- `'// TotalRange = Maximum - Minimum`
- `'// CumulativeProb = Rnd()`
- `'// If $CumulativeProb < (LowerRange / TotalRange)$`
- `'// Then, $RandomTriangular = Minimum + \text{Sqr}(CumulativeProb * LowerRange * TotalRange)$`
- `'// Else, $RandomTriangular = Maximum - \text{Sqr}((1 - CumulativeProb) * HigherRange * TotalRange)$`

- Else "Normal" Then

- `'// Generate Normal RVs`
- `'// Normal distribution (see lognormal for detail)`
- `'// calculate a normal distribution using this formula: NORMINV(RAND(), Mean, Stdev)`

Setup

i Decision Version 1.0
using Monte Carlo Simulation

Problems or Suggestions

Instruction (Read Me)

Reset

<<< Indicates required user Inputs (please see the yellow-highlighted cells below)

Statistical Model Setup

Input Values (input)

1. Enter # of iterations to run (1,000 ~ 100,000):
2. Enter Alpha (0.01 ~ 0.99):
3. Enter # of WBS or Task ID: <<< Expandable

1: Enter Input

2: Run Simulation

3: View Output

Program Information

Total Program (* Required)

Org Budget (\$K)*:	\$	100,000
Revised Budget (\$K):	\$	125,000
Ceiling Price (Budget + Fee in \$K):		N/A
Org Duration (Hrs(K))*:		2,400
Revised Duration (Hrs(K)):		3,200
Contract Type:		CPIF
Select the Share Ratio of Overrun/Underrun (G/K):		60/40
Enter the fee amount (\$K):	\$	6,250
Enter the ceiling price if applicable (\$K):	\$	135,000
Enter the expected VAC (O/U) (\$K):	\$	(7,500)

Other Indirect Costs or Hours & Performance Factor

COST

Overhead (\$K):	\$	12,000
Cost of Money (\$K):	\$	8,000
Management Reserve (MR \$K):	\$	7,000
G&A (General & Administrative \$K):	\$	23,000
Other Costs such as Fringe, etc (\$K):	\$	-
Total Indirect Costs (\$K):	\$	50,000

SCHEDULE

Total Indirect Hours (K):		800
---------------------------	--	-----

Total MODs

Cost MODs (\$K):	\$	25,000
Schedule MODs (Hrs(K)):		3,045

Input Process

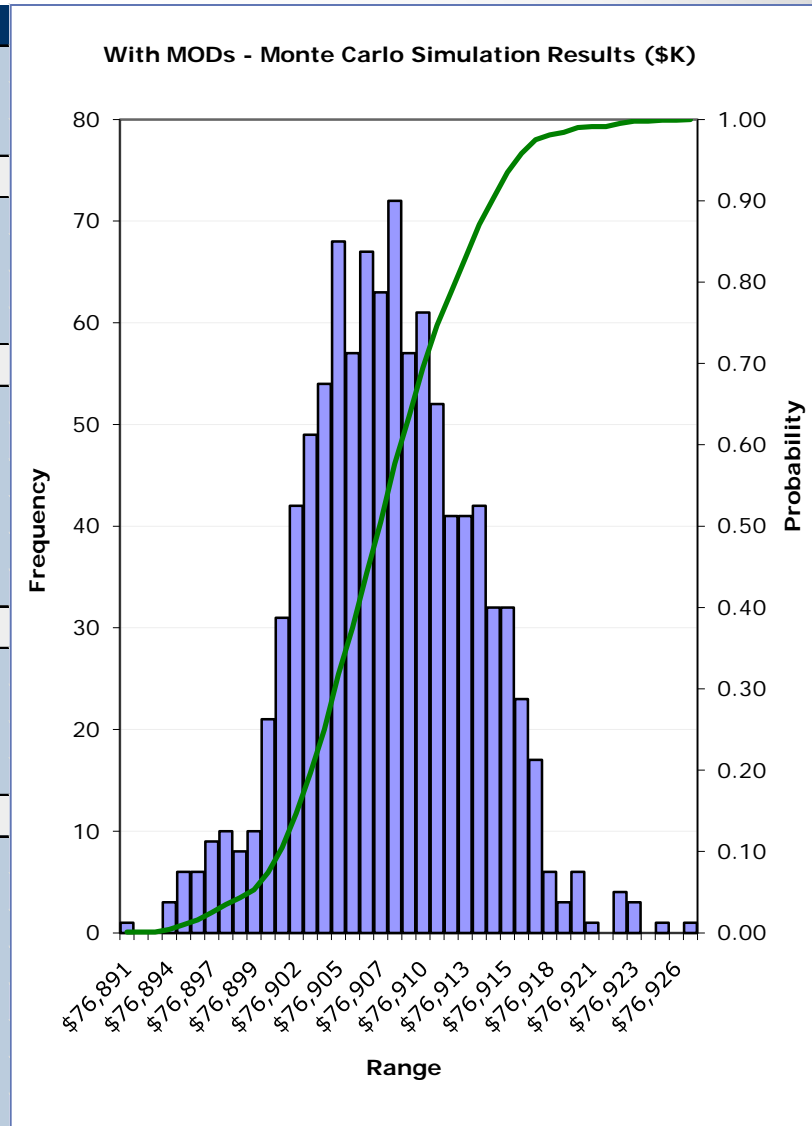
# of KPPs	KPP (Key Performance Parameter) Description	KPP Objective	KPP Threshold	KPP expected cost savings (\$K) w/ threshold	KPP expected schedule savings (Hrs(K)) w/ threshold
1	Weight	3000 lbs	4500 lbs	\$ 2,500.00	25.00
2	Temp	-25F	-15F	\$ 1,500.00	22.00
3	Speed	650 Knots	500 Knots	\$ 250.00	16.00
4	Fuel	J5, J8 & Diesel	J5 & J8	\$ 350.00	8.00

Work ID*	Work Description	Org Budget (\$K)*	Org Duration (Hrs(K))*	MOD (\$K) - both addtl scope & descope	MOD (Hrs(K)) - both addtl scope & descope	Cost Risk (\$K)	Schedule Risk (Hrs(K))	Cost Opportunities (\$K)	Schedule Opportunities (Hrs(K))
C001	CLIN 1	\$ 1,500	48		-	\$ 15	0.061		
C002	CLIN 2	\$ 1,500	48		-			\$ 20	0.08
C003	CLIN 3	\$ 2,000	64	\$ 1,000	150	\$ 35	0.141		
C004	CLIN 4	\$ 5,000	160		-			\$ 10	0.04
C005	CLIN 5	\$ 12,500	400	\$ 12,500	550	\$ 850	3.412		
C006	CLIN 6	\$ 7,500	240	\$ 1,500	145	\$ 85	0.342		
C007	CLIN 7	\$ 2,500	80	\$ 6,500	1,500	\$ 120	0.482		
C008	CLIN 8	\$ 2,500	80	\$ 3,500	700	\$ 200	0.802		
C009	CLIN 9	\$ 5,000	160		-			\$ 150	0.61
C010	CLIN 10	\$ 10,000	320		-			\$ 200	0.08

- A list of players or key performance parameters have its objective and threshold values and their associated cost and schedule savings (pay offs)
- The model is expandable to accept up to 1M inputs
- The model also takes risks and opportunities as user inputs

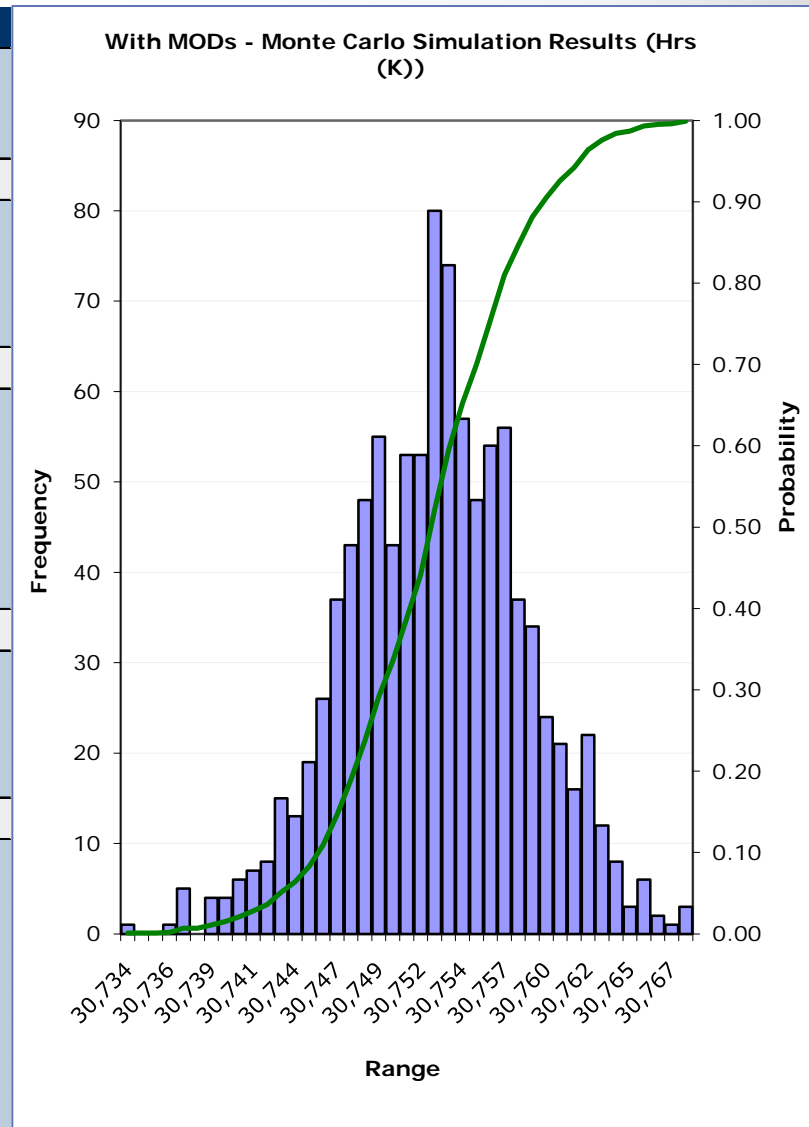
Sensitivity using RVs (\$K)

Summary Statistics				
Sample Size (N):		1000		
Central Tendency				
Mean:	\$76,907.32	Median:	\$76,907.22	
StErr:	\$0.17			
Spread				
StDev:	\$5.28	Q(.75):	\$76,910.85	
Max:	\$76,926.74	Q(.25):	\$76,903.71	
Min:	\$76,891.31	IQ Range:	\$7.13	
Range:	\$35.43			
Shape				
Skewness:	0.104846746			
Kurtosis:	0.092029194			
Quantiles, Percentiles, Intervals				
	90% Interval		95% Interval	
Q: 0.05	\$76,898.81	Q: 0.025	\$76,896.74	
Q: 0.95	\$76,915.78	Q: 0.975	\$76,916.98	
Alpha (a):	0.05	Q(a/2):	\$76,896.74	
% Interval:	95%	Q(1-a/2):	\$76,916.98	



Sensitivity using RVs (Hours in K)

Summary Statistics				
Sample Size (N):		1000		
Central Tendency				
Mean:	30,752.31	Median:	30,752.43	
StErr:	0.18			
Spread				
StDev:	5.54			
Max:	30,768.23	Q(.75):	30,755.96	
Min:	30,733.60	Q(.25):	30,748.56	
Range:	34.63	IQ Range:	7.40	
Shape				
Skewness:	-0.070701505			
Kurtosis:	0.103211459			
Quantiles, Percentiles, Intervals				
	90% Interval		95% Interval	
Q: 0.05	30,743.09	Q: 0.025	30,741.01	
Q: 0.95	30,761.65	Q: 0.975	30,762.83	
Alpha (a):	0.05	Q(a/2):	30,741.01	
% Interval:	95%	Q(1-a/2):	30,762.83	



Result 1 – List of Alternatives

i Decision to Make the Optimal Solution															
Sort by # (Back to Original)				Rank by Forecast Budget						Rank by Forecast Duration					
# of alternatives	Scenarios	Org Budget (\$K)	Org Duration (Hrs(K))	Mod Budget (\$K)	Mod Duration (Hrs(K))	KPP Forecast Cost Savings (\$K)	KPP Forecast Schedule Savings (Hrs (K))	Forecast Budget (\$K)	Forecast Duration (Hrs (K))	VAC (\$K)	Share Ratio Burden (Govt, %)	Share Ratio Burden (Govt, \$K)	Fees (\$K)	Total Price (\$K)	
0	Contract MODs without KPP Savings	\$ 100,000	2,400	\$ 25,000	3,045	0	0	\$ 132,847	5,753	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 143,597	
1	Weight Objective & Temp Objective & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	0		\$ 107,834	2,707	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 118,584	
2	Weight Threshold & Temp Objective & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	2500	25	\$ 105,334	2,682	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,084	
3	Weight Objective & Temp Threshold & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	1500	22	\$ 106,334	2,685	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 117,084	
4	Weight Threshold & Temp Threshold & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	4000	47	\$ 103,834	2,660	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 114,584	
5	Weight Objective & Temp Objective & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	250	16	\$ 107,584	2,691	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 118,334	
6	Weight Threshold & Temp Objective & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	2750	41	\$ 105,084	2,666	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 115,834	
7	Weight Objective & Temp Threshold & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	1750	38	\$ 106,084	2,669	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,834	
8	Weight Threshold & Temp Threshold & Speed Threshold & Fuel Objectiv	\$ 100,000	2,400	\$ -	-	4250	63	\$ 103,584	2,644	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 114,334	
9	Weight Objective & Temp Objective & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	350	8	\$ 107,484	2,699	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 118,234	
10	Weight Threshold & Temp Objective & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	2850	33	\$ 104,984	2,674	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 115,734	
11	Weight Objective & Temp Threshold & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	1850	30	\$ 105,984	2,677	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,734	
12	Weight Threshold & Temp Threshold & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	4350	55	\$ 103,484	2,652	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 114,234	
13	Weight Objective & Temp Objective & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	600	24	\$ 107,234	2,683	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 117,984	
14	Weight Threshold & Temp Objective & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	3100	49	\$ 104,734	2,658	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 115,484	
15	Weight Objective & Temp Threshold & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	2100	46	\$ 105,734	2,661	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,484	
16	Weight Threshold & Temp Threshold & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	4600	71	\$ 103,234	2,636	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 113,984	

- The total of 16 alternatives (4² scenario combinations) are found based on the 4 key players (performance parameters) of Weight, Temperature, Speed and Fuel Consumption
- The result from the requirement modification decisions (Alt 0) and the simulated alternatives (1 to 16) are compared against each other to select the best optimal solution / alternative

Result 2 - Sort by Forecasted Budget

# of alternatives	Scenarios	Org Budget (\$K)	Org Duration (Hrs(K))	Mod Budget (\$K)	Mod Duration (Hrs(K))	KPP Forecast Cost Savings (\$K)	KPP Forecast Schedule Savings (Hrs (K))	Forecast Budget (\$K)	Forecast Duration (Hrs (K))	VAC (\$K)	Share Ratio Burden (Govt, %)	Share Ratio Burden (Govt, \$K)	Fees (\$K)	Total Price (\$K)
16	Weight Threshold & Temp Threshold & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	4600	71	\$ 103,234	2,636	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 113,984
12	Weight Threshold & Temp Threshold & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	4350	55	\$ 103,484	2,652	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 114,234
8	Weight Threshold & Temp Threshold & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	4250	63	\$ 103,584	2,644	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 114,334
4	Weight Threshold & Temp Threshold & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	4000	47	\$ 103,834	2,660	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 114,584
14	Weight Threshold & Temp Objective & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	3100	49	\$ 104,734	2,658	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 115,484
10	Weight Threshold & Temp Objective & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	2850	33	\$ 104,984	2,674	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 115,734
6	Weight Threshold & Temp Objective & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	2750	41	\$ 105,084	2,666	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 115,834
2	Weight Threshold & Temp Objective & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	2500	25	\$ 105,334	2,682	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,084
15	Weight Objective & Temp Threshold & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	2100	46	\$ 105,734	2,661	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,484
11	Weight Objective & Temp Threshold & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	1850	30	\$ 105,984	2,677	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,734
7	Weight Objective & Temp Threshold & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	1750	38	\$ 106,084	2,669	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,834
3	Weight Objective & Temp Threshold & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	1500	22	\$ 106,334	2,685	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 117,084
13	Weight Objective & Temp Objective & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	600	24	\$ 107,234	2,683	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 117,984
9	Weight Objective & Temp Objective & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	350	8	\$ 107,484	2,699	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 118,234
5	Weight Objective & Temp Objective & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	250	16	\$ 107,584	2,691	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 118,334
1	Weight Objective & Temp Objective & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	0		\$ 107,834	2,707	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 118,584
0	Contract MODs without KPP Savings	\$ 100,000	2,400	\$ 25,000	3,045	0	0	\$ 132,847	5,753	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 143,597

- To view the best alternative by the forecasted budget after the simulation, the results are shown above
- As expected, the best alternative to minimize the cost and schedule impact is alternative #16 which takes the potential cost and schedule savings in account for all key performance parameters.
- By implementing the threshold values for all key performance parameters (Alt #16), the model forecasted a \$4,600K savings to complete the project

Result 3 - Sort by Forecasted Duration

# of alternatives	Scenarios	Org Budget (\$K)	Org Duration (Hrs(K))	Mod Budget (\$K)	Mod Duration (Hrs(K))	KPP Forecast Cost Savings (\$K)	KPP Forecast Schedule Savings (Hrs (K))	Forecast Budget (\$K)	Forecast Duration (Hrs (K))	VAC (\$K)	Share Ratio Burden (Govt, %)	Share Ratio Burden (Govt, \$K)	Fees (\$K)	Total Price (\$K)
16	Weight Threshold & Temp Threshold & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	4600	71	\$ 103,234	2,636	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 113,984
8	Weight Threshold & Temp Threshold & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	4250	63	\$ 103,584	2,644	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 114,334
12	Weight Threshold & Temp Threshold & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	4350	55	\$ 103,484	2,652	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 114,234
14	Weight Threshold & Temp Objective & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	3100	49	\$ 104,734	2,658	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 115,484
4	Weight Threshold & Temp Threshold & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	4000	47	\$ 103,834	2,660	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 114,584
15	Weight Objective & Temp Threshold & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	2100	46	\$ 105,734	2,661	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,484
6	Weight Threshold & Temp Objective & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	2750	41	\$ 105,084	2,666	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 115,834
7	Weight Objective & Temp Threshold & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	1750	38	\$ 106,084	2,669	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,834
10	Weight Threshold & Temp Objective & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	2850	33	\$ 104,984	2,674	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 115,734
11	Weight Objective & Temp Threshold & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	1850	30	\$ 105,984	2,677	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,734
2	Weight Threshold & Temp Objective & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	2500	25	\$ 105,334	2,682	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 116,084
13	Weight Objective & Temp Objective & Speed Threshold & Fuel Threshold	\$ 100,000	2,400	\$ -	-	600	24	\$ 107,234	2,683	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 117,984
3	Weight Objective & Temp Threshold & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	1500	22	\$ 106,334	2,685	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 117,084
5	Weight Objective & Temp Objective & Speed Threshold & Fuel Objective	\$ 100,000	2,400	\$ -	-	250	16	\$ 107,584	2,691	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 118,334
9	Weight Objective & Temp Objective & Speed Objective & Fuel Threshold	\$ 100,000	2,400	\$ -	-	350	8	\$ 107,484	2,699	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 118,234
1	Weight Objective & Temp Objective & Speed Objective & Fuel Objective	\$ 100,000	2,400	\$ -	-	0		\$ 107,834	2,707	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 118,584
0	Contract MODs without KPP Savings	\$ 100,000	2,400	\$ 25,000	3,045	0	0	\$ 132,847	5,753	\$ (7,500)	60%	\$ (4,500)	\$ 6,250	\$ 143,597

- To view the best alternative by the forecasted duration after the simulation, the results are shown above
- As expected, the best alternative to minimize the cost and schedule impact is alternative #16 which takes the potential cost and schedule savings in account for all key performance parameters
- By implementing the threshold values for all key performance parameters (Alt #16), the model forecasted 71K hours savings to complete the project.

Data Source

- For the initial simulation
 - Simulated RV data was used to verify the model's behaviors
 - This was to ensure that the model would generate the mathematical alternatives in order to assist the users to make an optimal solution
- For the future studies
 - Real-Life data will be used in future studies
 - A case study will be performed to analyze the data from the randomly selected programs in order to validate that the model shows the same results by producing a set of mathematical alternatives that contains an optimal solution

Simulation Generated Expected Results

- The overall simulation used a set of random variables (RVs) in order to test the model's behaviors
- Since the simulation was conducted using the pre-set or pre-determined data, the model was expected to produce the most efficient way, which was selecting the threshold for all possible players (key performance parameters) in order to save money and time
- As expected, a result of running a toy-problem simulation using RVs, the model generated the most cost saving and schedule saving option which was the alternative 16

Conclusions

- Making a decision based on a mathematical model can provide objective ways to convince stakeholders.
- When there is a conflict, system trade-offs are necessary to ensure that all possible alternatives are evaluated and an optimal solution has been considered to maximize the benefit
- When each trade-off depends on the current state of the system and the decision maker's actions to be optimal, a way to make this optimal decision is by utilizing a stochastic model based on the Game Theory and Linear Programming approach
- The preliminary result of simulations shows that although a stochastic model provides a mathematically optimal solution, there are other factors to consider when making decisions, such as social impact, policy changes or political implications
- The social factors are often not negotiable and impact overall system performance in the end

Authors - Contact Information

- Jason Dever, George Washington University,
jdever@gwu.edu
- Jennifer Mills, George Washington University,
jenmills@gwmail.gwu.edu
- Steven Stuban, George Washington University,
stuban@gwu.edu
- Bereket Tanju, George Washington University,
btanju@email.gwu.edu