The Three-Phase Optimal Design Test Meets Reality: Lessons Available, Part One

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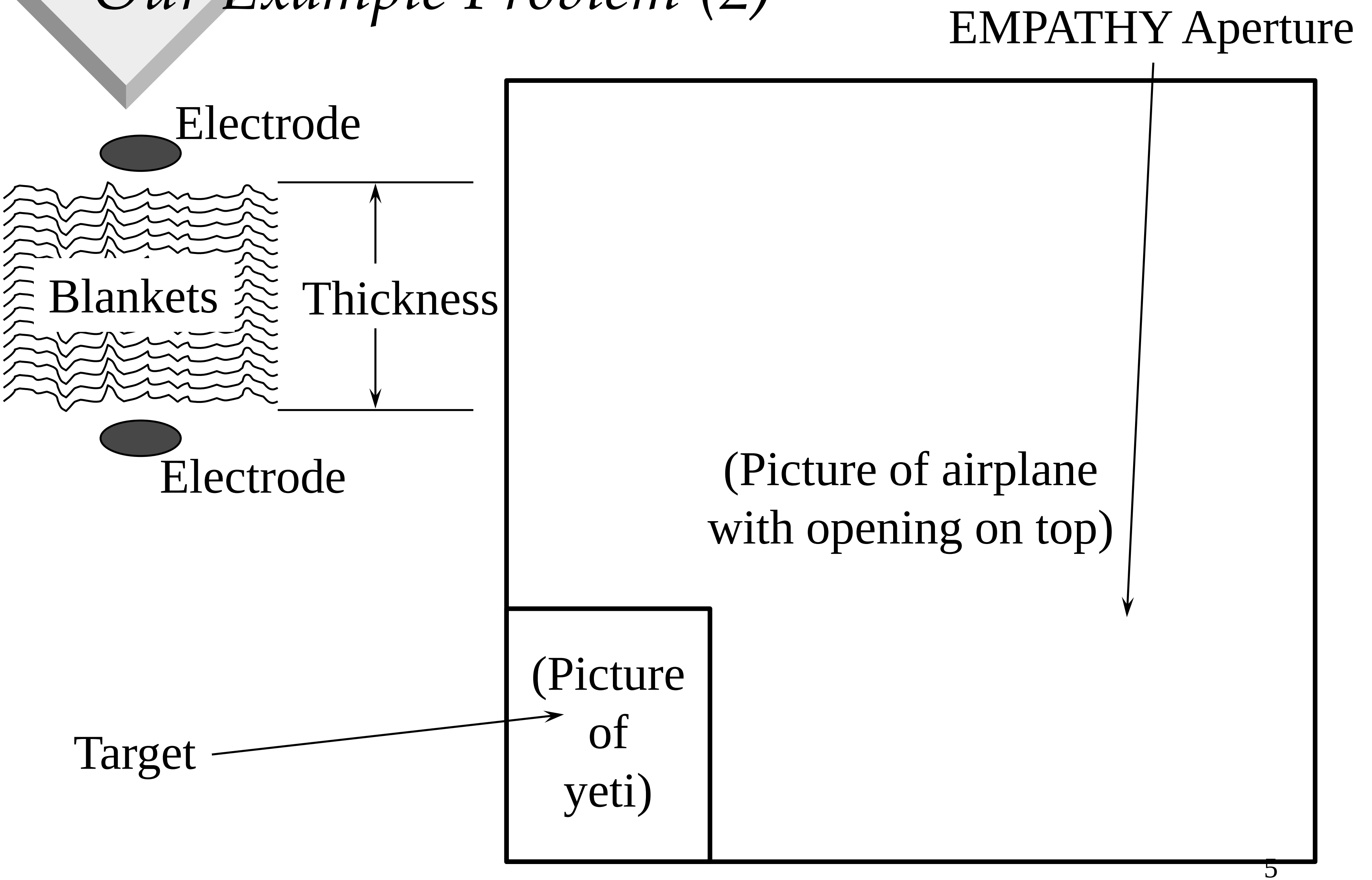
Outline

- Our Example Problem
- The Three-Phase Optimal Design Test
- Issues and How to Cope With Them
  - Phase I1 Test Points
  - Phase II and Phase III
  - Test Schedule and Range Availability

Our Example Problem

- Fictitious Weapon: Electro-Magnetic Pulse Against Thoroughly Hostile Yetis
    - Two high-voltage electrodes
    - Separated by stack of insulating blankets
    - Thicker stack → better chance of enough insulation between electrodes → better chance that charge does not bleed off slowly → better chance of electrical discharge when needed
    - Need thickness of stack required to give
      99.99% chance of discharge
      at 95% confidence level

Our Example Problem (2)



The Three-Phase Optimal Design Test

- We have an input
  - Varies continuously – thickness of stack
- We have an output
  - One or zero – success or failure – on or off – discharge or no discharge
  - Probabilistic function of input
    - The same input can give different outputs in different tests
    - Probability of a one increases as input increases

The Three-Phase Optimal Design Test (2)

- Invented by
  - Jeff Wu of Georgia Institute of Technology
  - Yubin Tian of Beijing Institute of Technology
- Published in the Journal of Statistical Planning and Inference, 2013
  - http://dx.doi.org/10.1016/j.jspi.2013.10.007

The Three-Phase Optimal Design Test (3)

- Phase I: Find the mean
  - Step I1: Obtain success and failure results
  - Step I2: Get an overlapping result
  - Step I3: Enhance the overlapping result
- Phase II: Optimize the mean and standard deviation
- Phase III (optional): Test at desired probability level to reduce uncertainty

The Three-Phase Optimal Design Test (4)

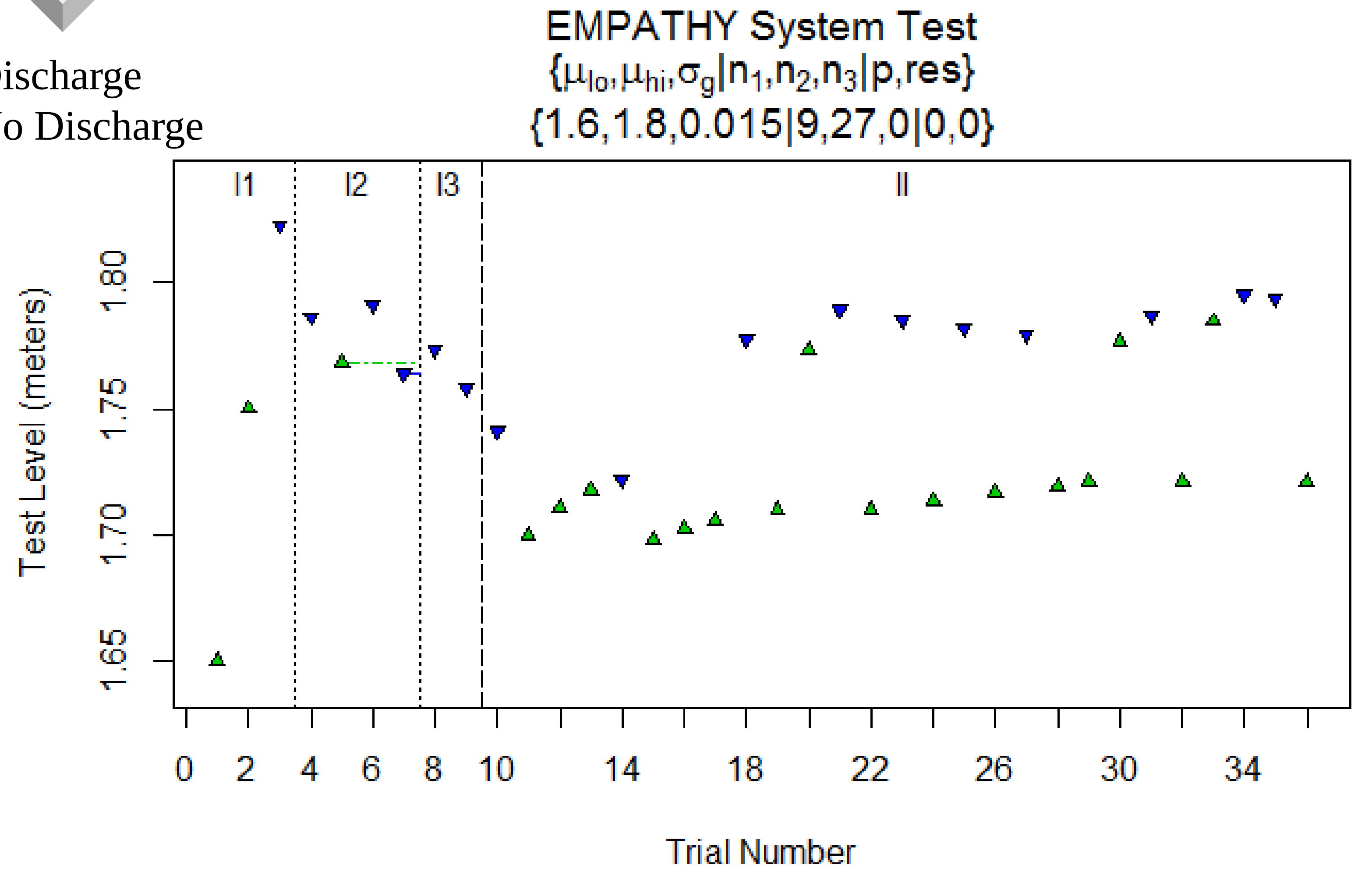
- Assumes probability curve follows normal distribution
- Requires starting values:
  - Approximate lower and upper bounds of range
  - Approximate standard deviation of probability curve

Our Example Problem (3)

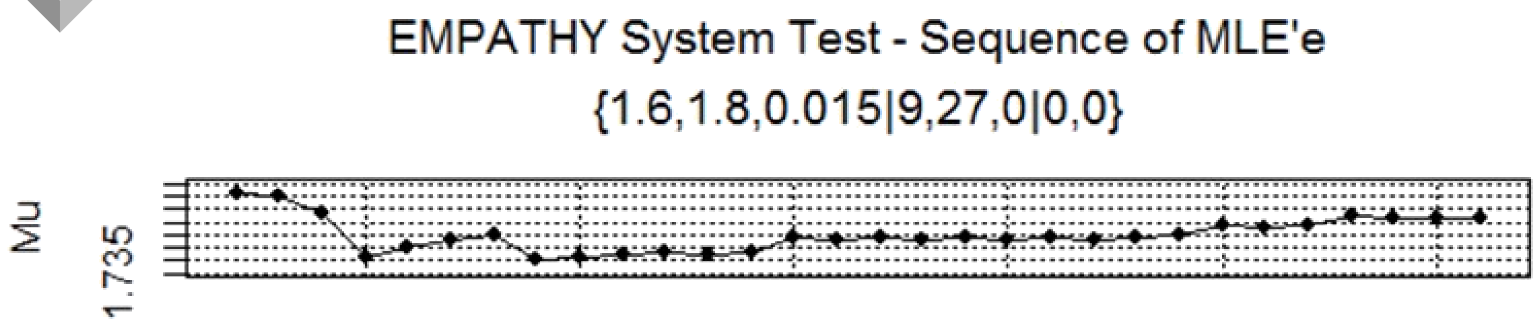
- Simulations show:
  - 1.6-meter stack of blankets is not enough insulation—no discharge
    - Lower end of “initial guess” interval
  - 1.8-meter stack of blankets is enough insulation—discharge
    - Upper end of “initial guess” interval
- Estimated Standard Deviation
  - Should be less than one sixth of range
  - We use 0.015 meters

Our Example Problem (4)

▼ - Discharge
▲ - No Discharge

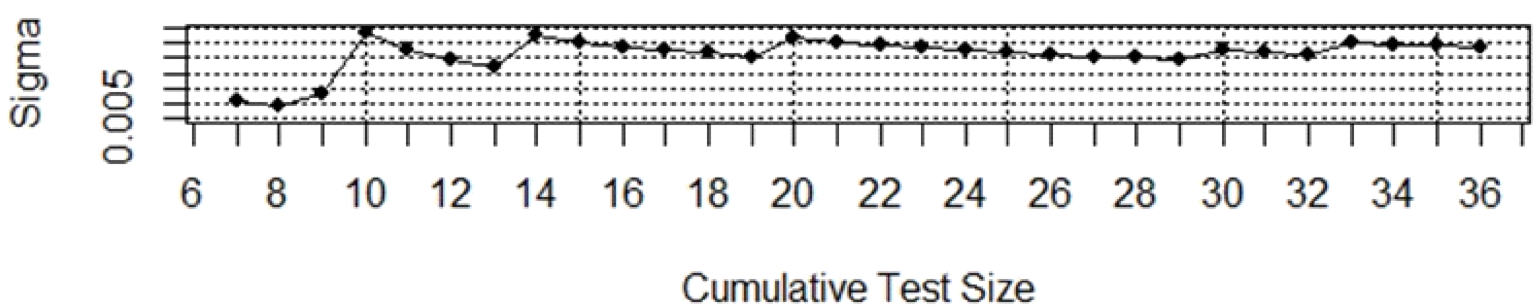


Our Example Problem (5)

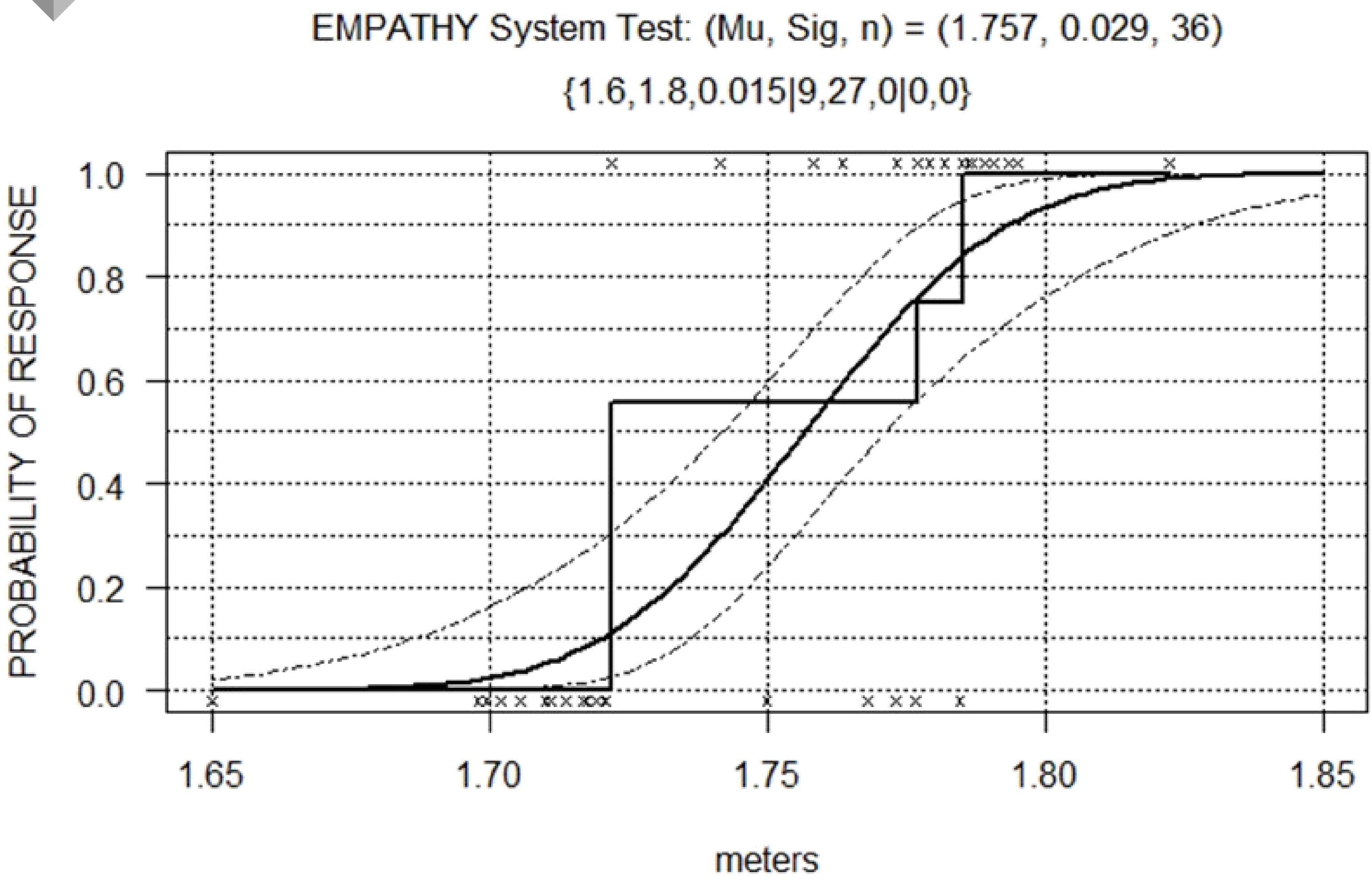


Nominal Values:
Mu = 1.750
Sigma = 0.050

Final Calculated Values:
Mu = 1.757
Sigma = 0.029



Our Example Problem (6)



Issues

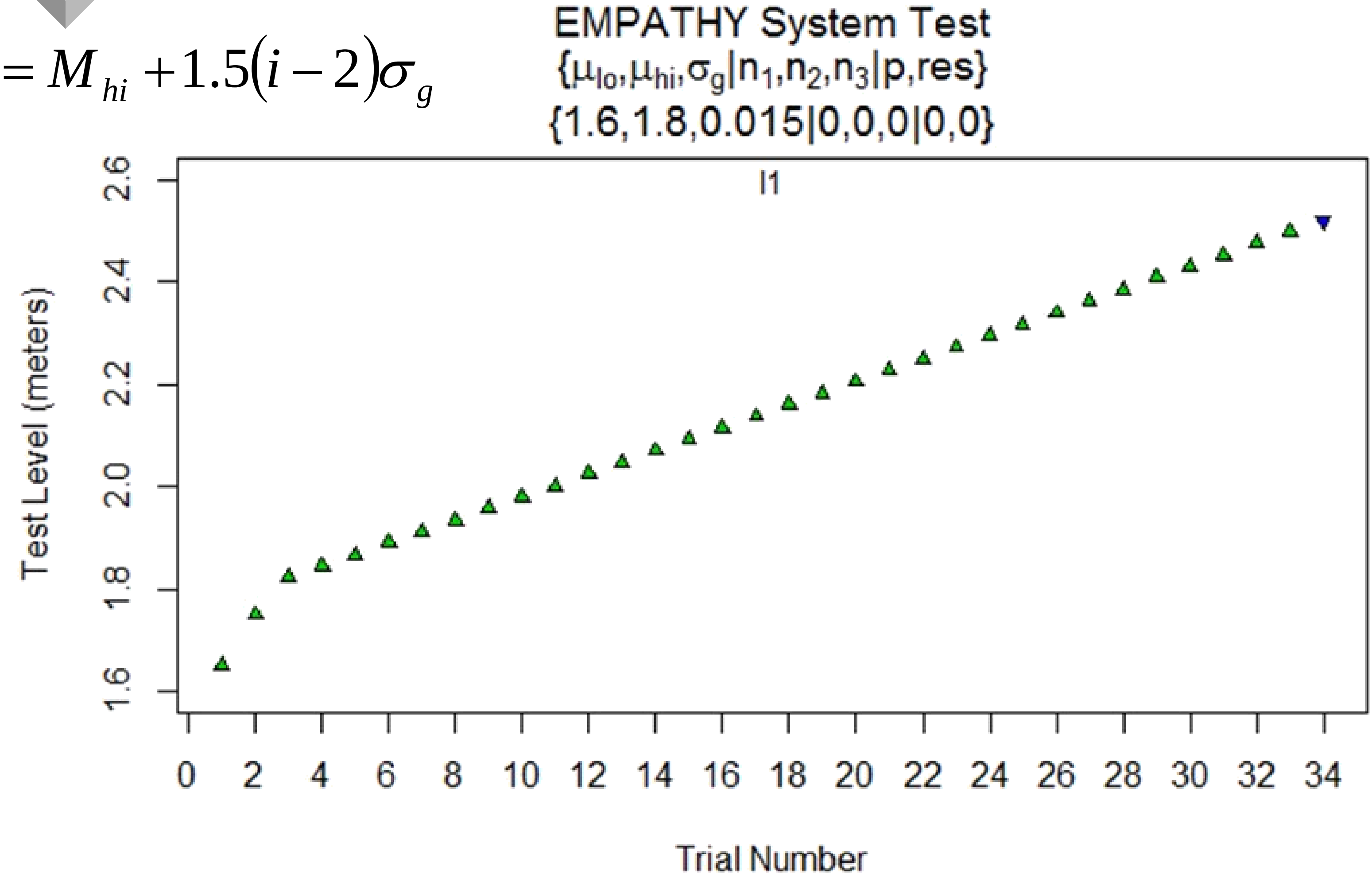
- Phase I1 Test Point Selection
- Phase II and Phase III Test Quantity
- Test Schedule and Range Availability

Phase I1 Test Point Selection

- If initial guess of test range is off
  - 3POD method moves away from initial guesses in steps of $1.5\sigma_g$
  - Authors' Opinion:
    - Step size should increase after fourth or fifth step
    - Very off-nominal case—will not happen unless initial guesses are very wrong

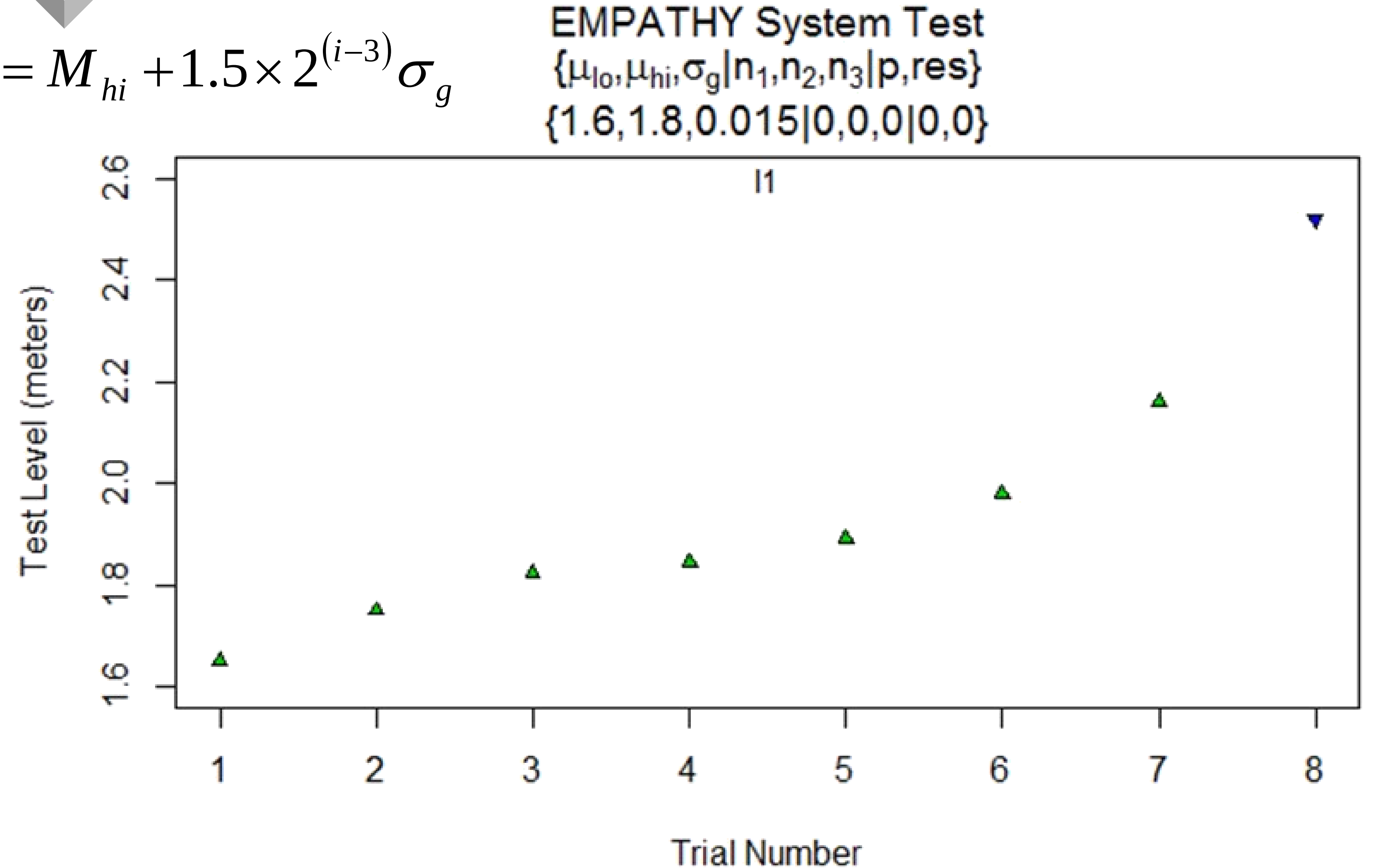
Example – Nominal Wu and Tian method

$$x_i = M_{hi} + 1.5(i-2)\sigma_g$$



Example – Proposed method

$$x_i = M_{hi} + 1.5 \times 2^{(i-3)} \sigma_g$$



Phase II and Phase III

- Phase II:
  - Enhance estimate of mean value
  - Estimate standard deviation
  - Method: Choose test points that maximize Fisher Information Matrix determinant
- Phase III:
  - Reduce uncertainty at specified probability value
  - Robbins-Monro-Joseph (RMJ) Procedure: Choose test points at estimate of specified probability value

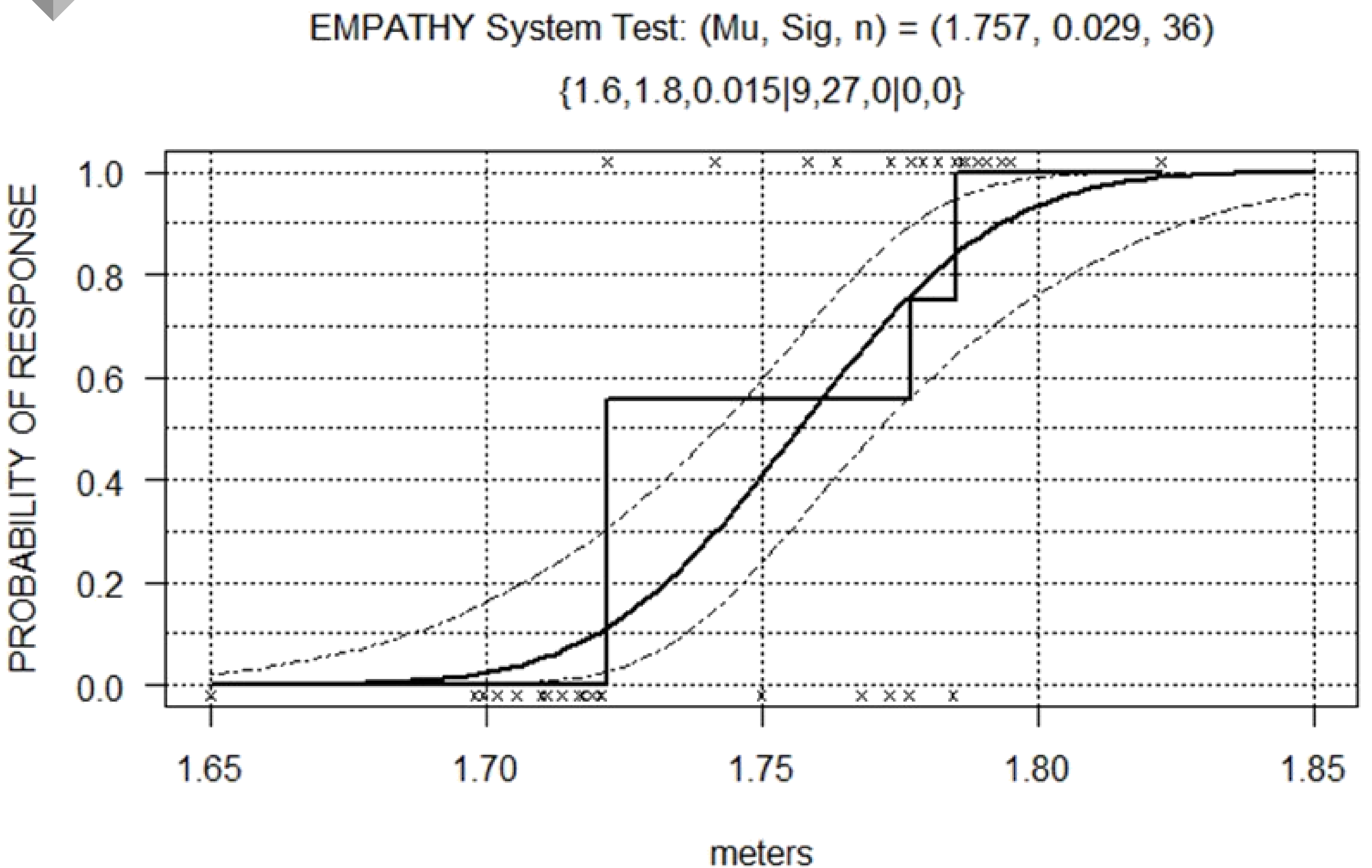
Phase II and Phase III (2)

- Issue:
  - Limited number of tests for Phase II and Phase III
  - Extreme probability level desired
    - All tests are expected to give a discharge or all tests are expected not to give a discharge
    - Ambiguity: Are we aiming at
      - 99% level?
      - 99.9% level?
      - 99.99% level?

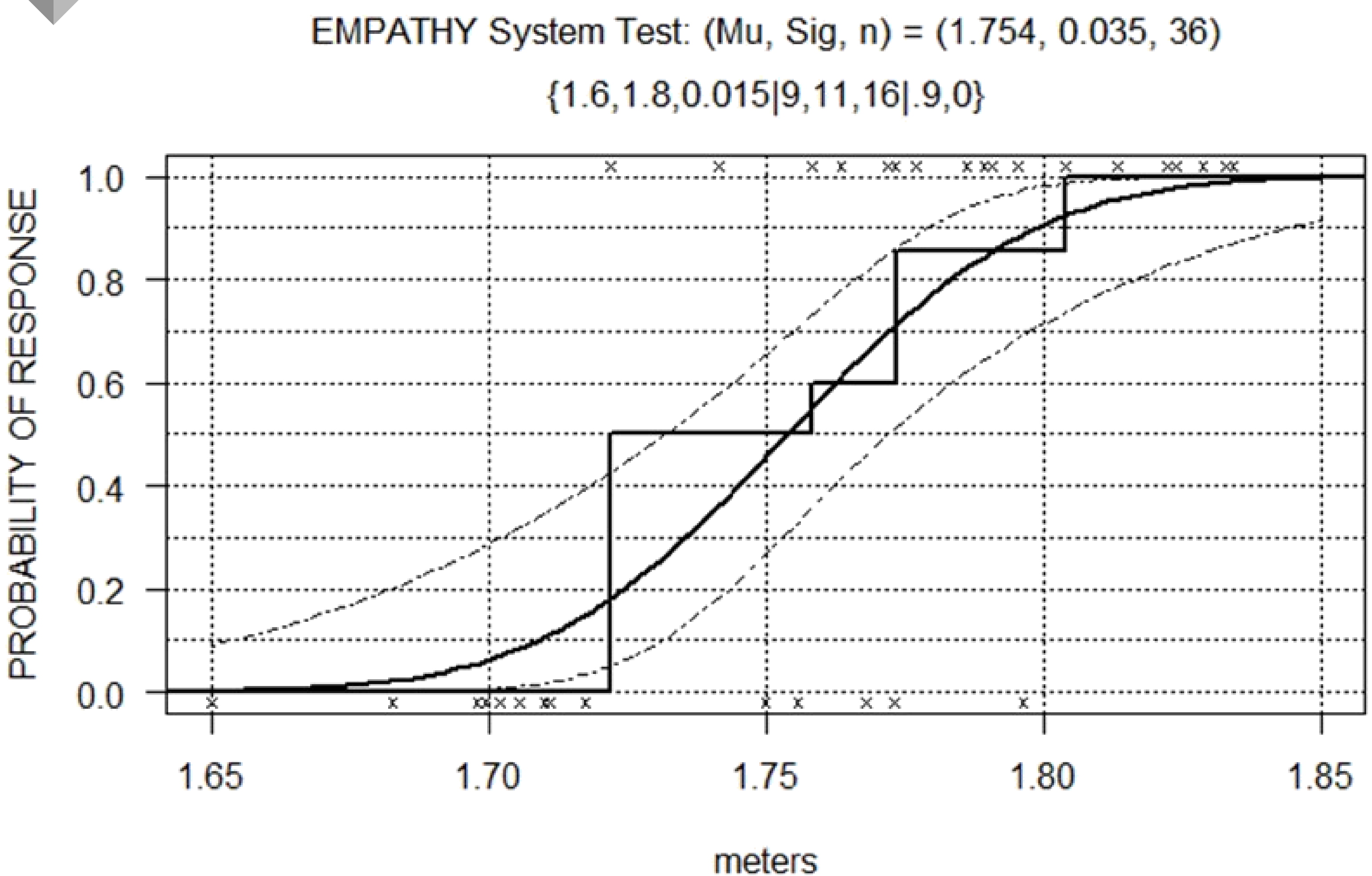
Phase II and Phase III (3)

- Resolution
  - Rule of thumb:
    - If possible, select enough tests for Phase III that at least one “anomalous” result is expected
    - If not possible, skip Phase III and use all tests for Phase II
      - Better definition of mean and standard deviation

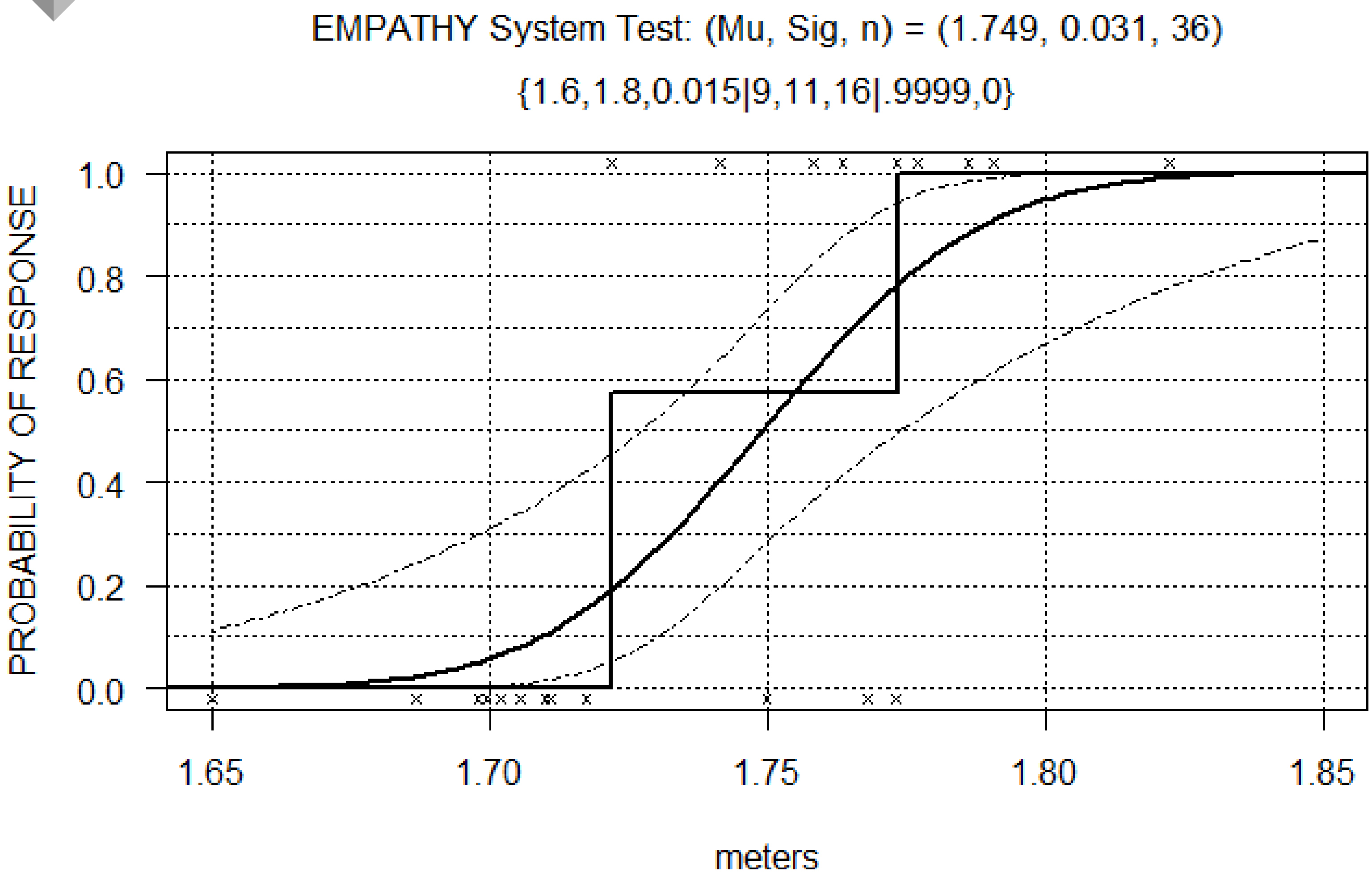
Example – Phase I/II – 36, Phase III – 0



Example – Phase I/II – 20, Phase III – 16 / 0.9



Example – Phase I/II – 20, Phase III – 16 / 0.9999



Testing Schedule and Test Range Availability

- Issues:
  - Test range time is expensive
    - Much more expensive than test items
  - Program schedule is paramount
    - Making a single test item takes significant time
    - Can create multiple test items in parallel

Testing Schedule and Range Availability (2)

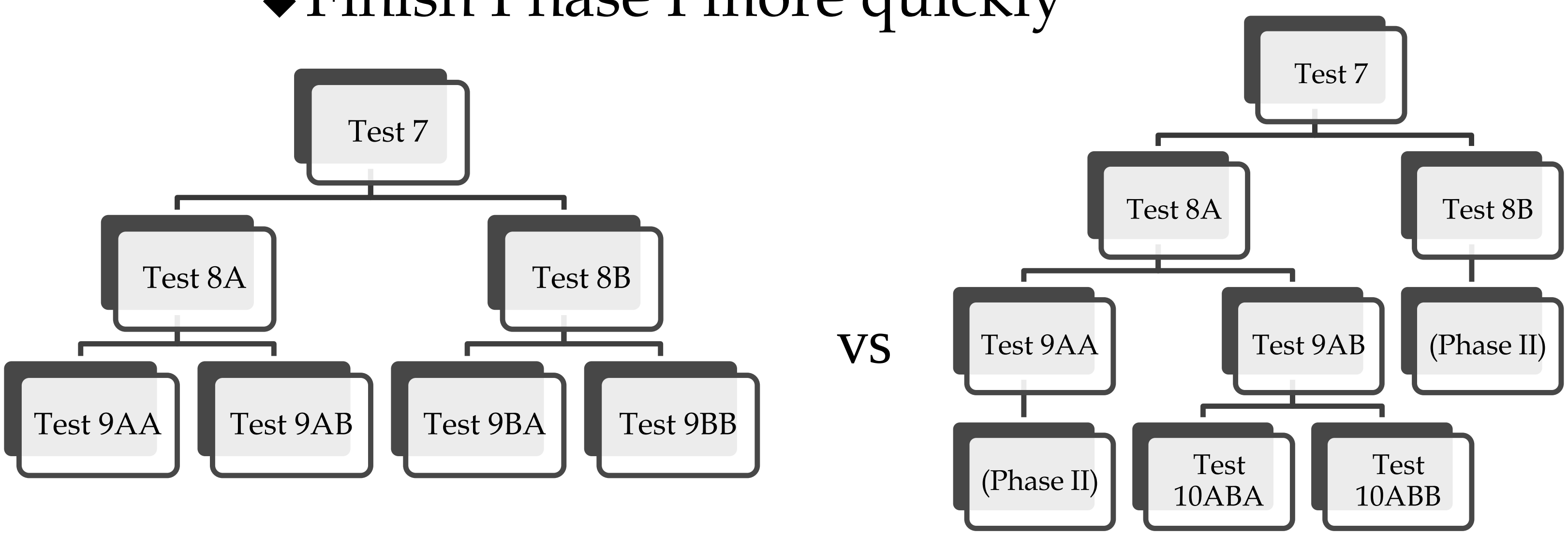
- Resolution: Phase I
  - Moderate speedup needed
    - Two tests per day
    - Case 1: 3POD specifies two tests at once
      - Create two test items
    - Case 2: 3POD specifies one test at a time
      - Create three test items:
        - Item for next test
        - Item for test after next if next test gives One
        - Item for test after next if next test gives Zero
      - Double testing speed, waste one test item in three

Testing Schedule and Range Availability (3)

- Resolution: Phase I (2)
  - Larger speedup needed
    - Three tests per day
    - Create seven test items:
      - Item for next test
      - Item for test after next if next test gives One
      - Item for test after next if next test gives Zero
        - One/Zero results may give same test point
      - Items for third test given One/One, One/Zero, Zero/One, Zero/Zero results
    - Triple testing speed, waste half the test items

Testing Schedule and Range Availability (4)

- Resolution: Phase I (3)
  - Larger speedup case
    - Can predict test points tree to uneven depth
    - Finish Phase I more quickly



Testing Schedule and Range Availability (5)

- Resolution: Phase II
  - Predict up to six tests in advance using 3POD method
    - Assume likelier outcome happens each time
  - Create test items at each test point and test simultaneously
  - Why it works:
    - Phase II places test points near "$\mu \pm 1.2\ \sigma$"
    - "μ" and "σ" do not change quickly in Phase II

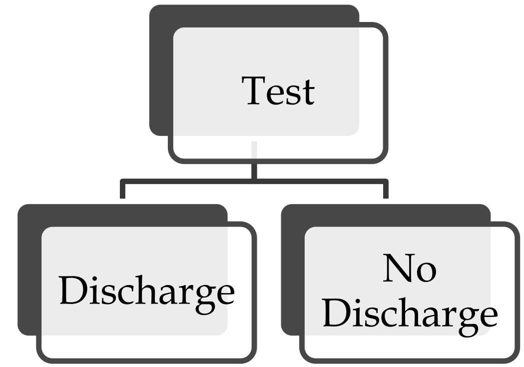
Testing Schedule and Range Availability (6)

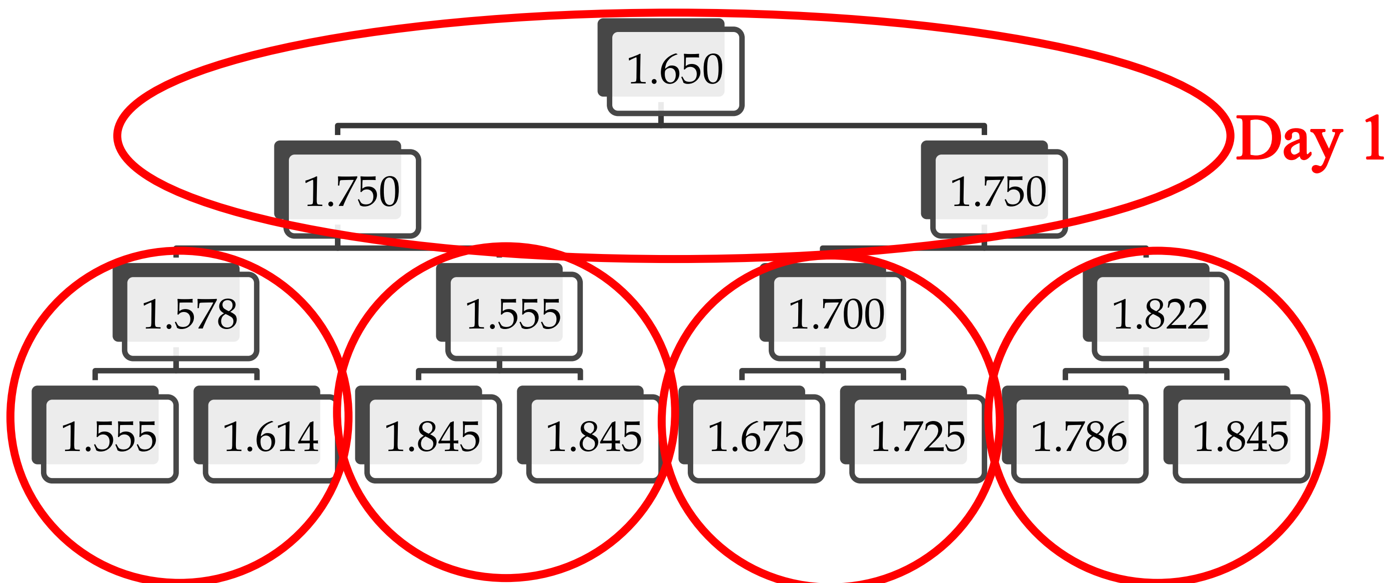
- Resolution: Phase III
  - Predict up to six tests in advance using 3POD method
    - Assume likelier outcome happens each time
  - Create test items at each test point and test simultaneously
  - Why it works:
    - Phase III test points determined by "$\mu + k\,\sigma$"
    - "$\mu + k\,\sigma$" does not change quickly in Phase III

Our Example Problem (7)

- EMPATHY system:
  - Blankets settle overnight to final thickness
  - Required 16-hour interval between making test article and performing test
  - One can remove blankets from unused test articles and create new test articles from the electrodes

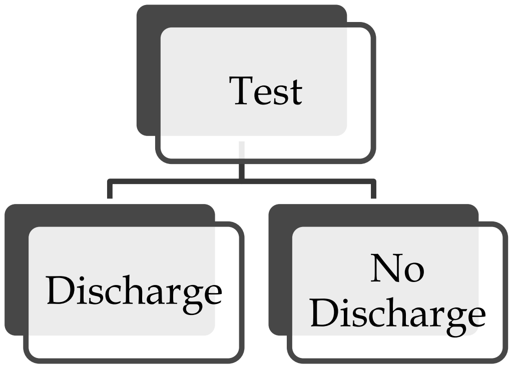
Example: Start of Day 1

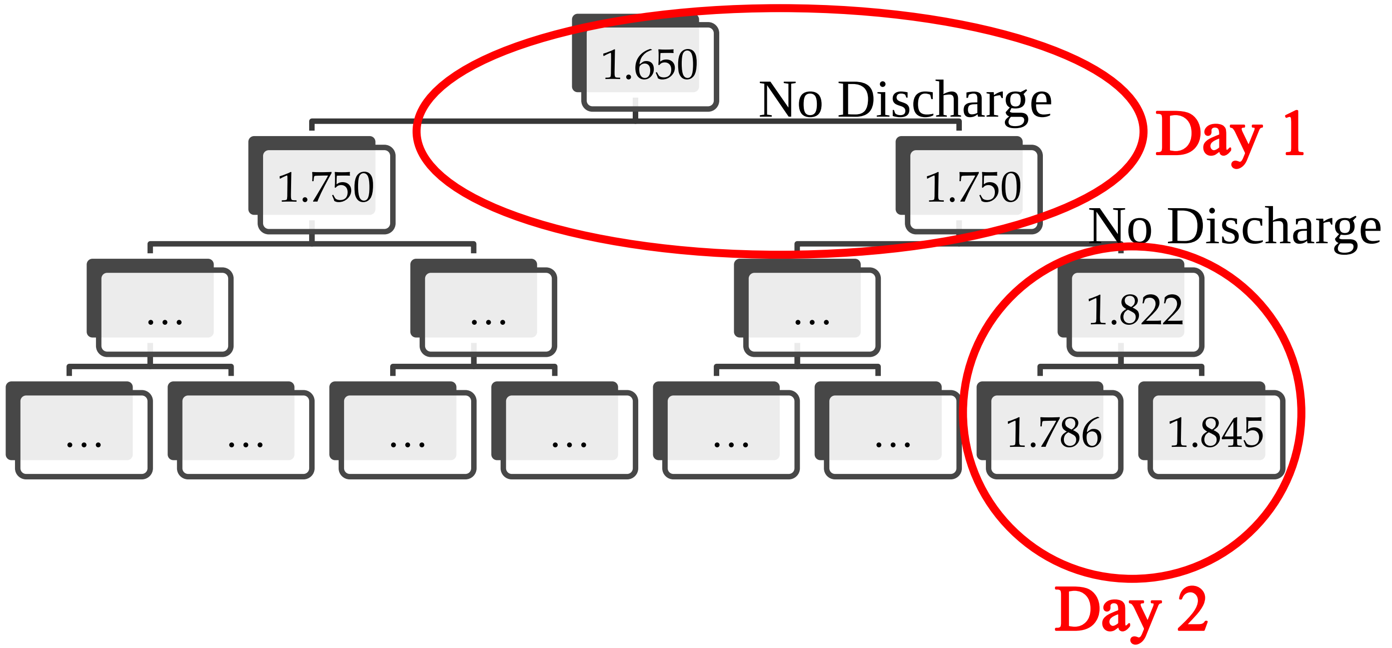




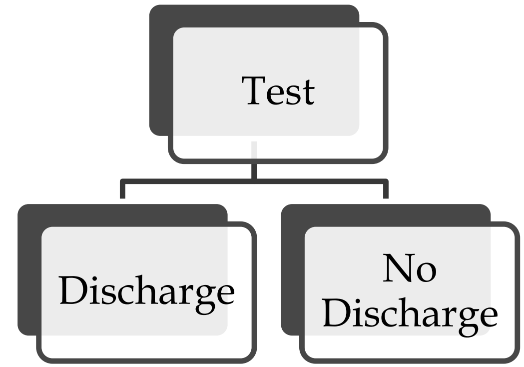
Day 2 is one of these—figure out which one at the end of Day 1 testing

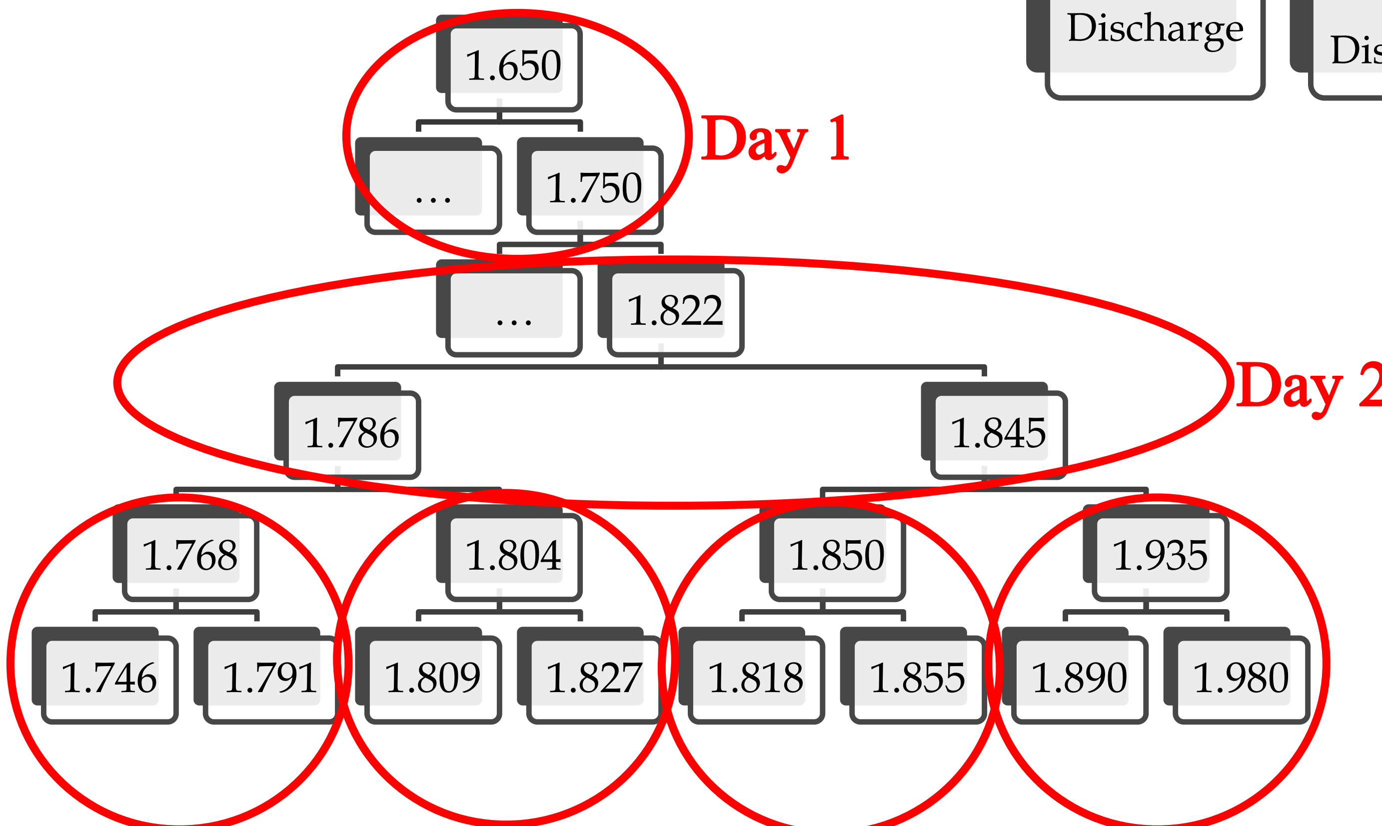
Example: End of Day 1





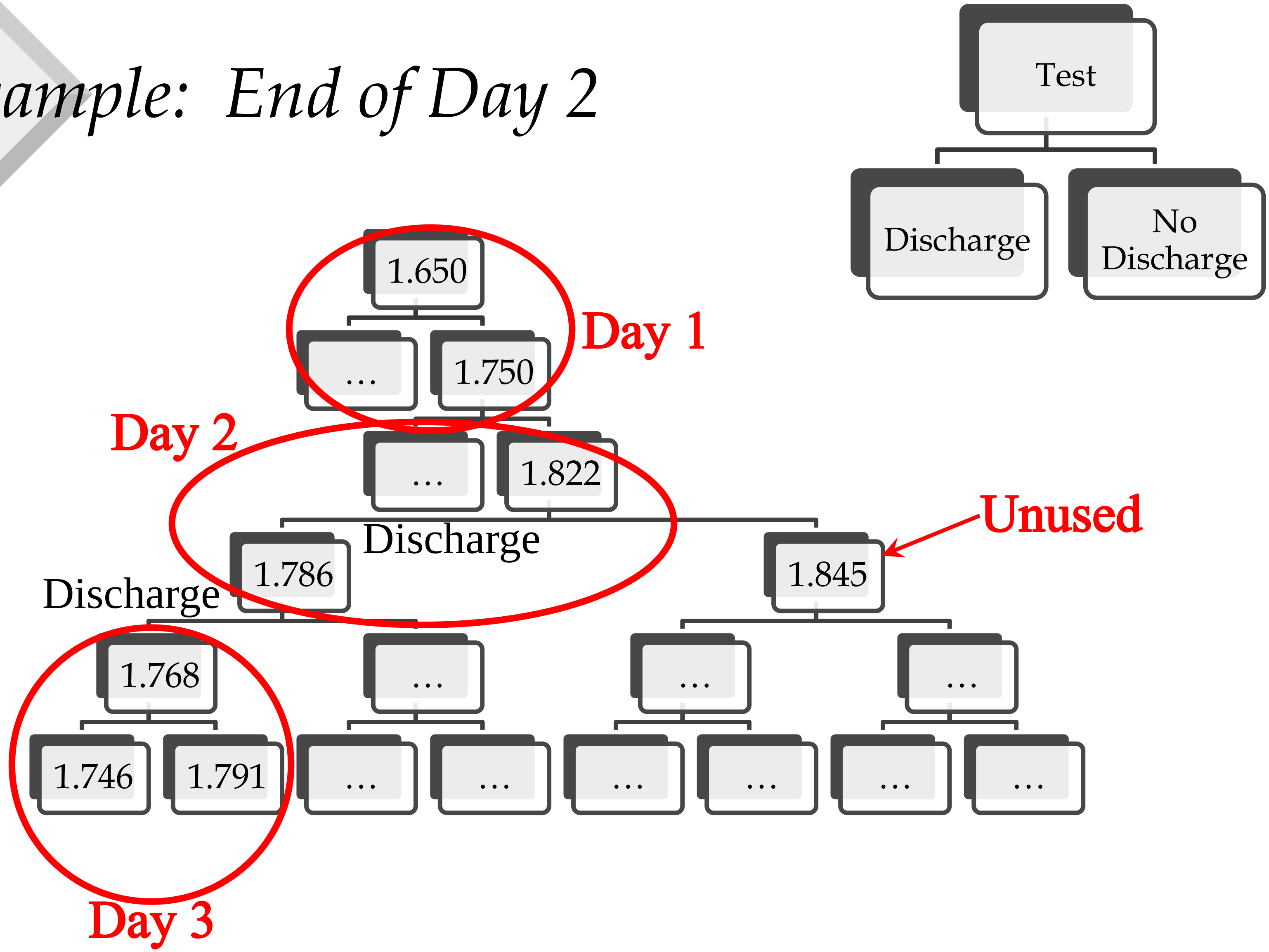
Example: Start of Day 2





Day 3 is one of these—figure out which one at the end of Day 2 testing

Example: End of Day 2

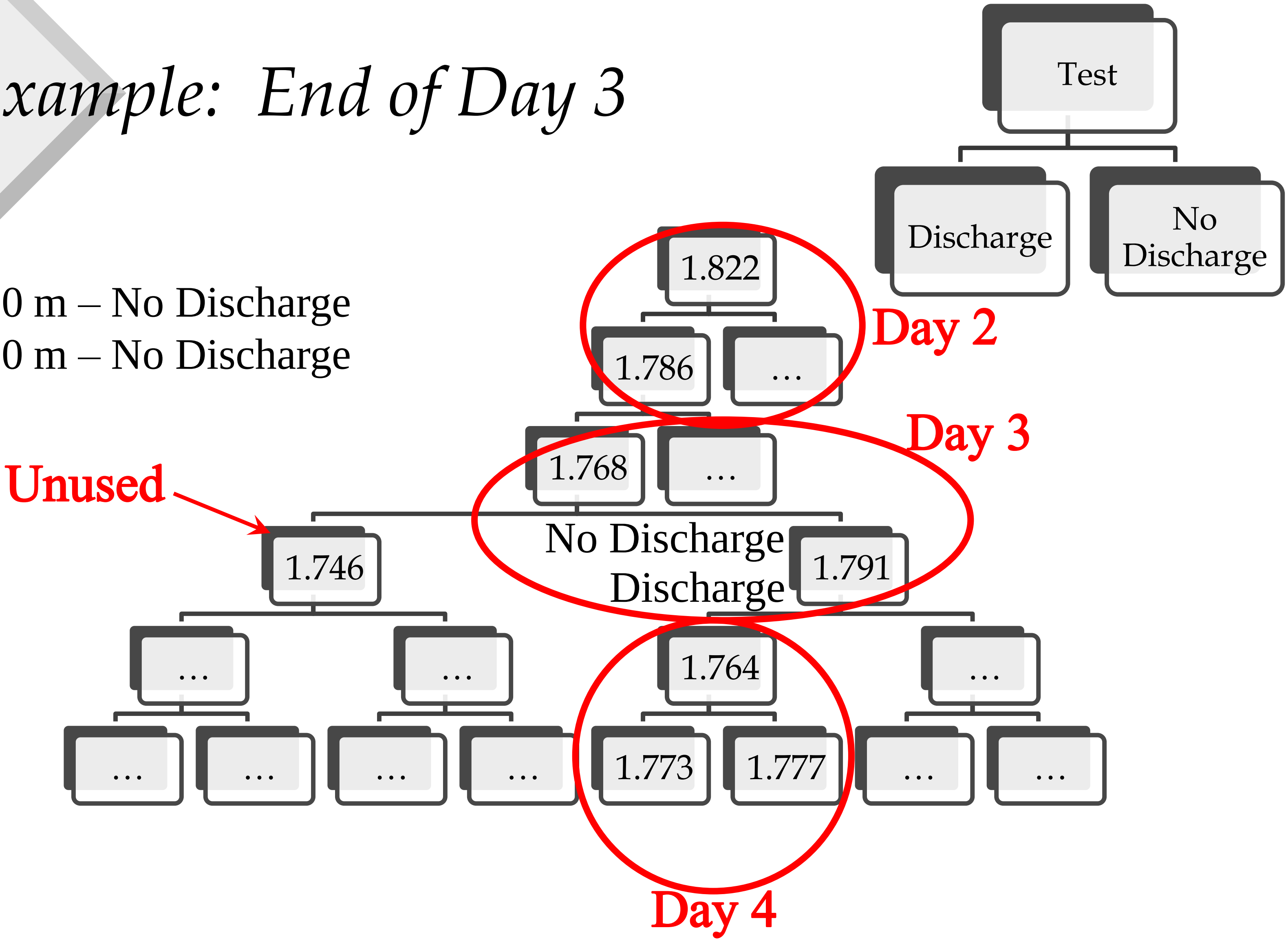


Example: End of Day 3

Day 1:

1.650 m – No Discharge

1.750 m – No Discharge



Example: End of Day 4

- Test Day 4:
  - Test 7: T = 1.76362 m → Discharge
  - Test 8: T = 1.77337 m → Discharge
- Test Points for Day 5:
  - Test 9: T = 1.75837 m
  - Test 10:
    - If Test 9 is Discharge: T = 1.74150 m
    - If Test 9 is No Discharge: T = 1.77515 m

(Test 8 through Test 9: Phase I3)

(Test 10 alternatives: Phase II)

Example: End of Day 5

- Test Day 5:
  - Test 9: T = 1.75837 m → Discharge } Phase I3
  - Test 10: T = 1.77337 m → Discharge } Phase II
- Test Points for Day 6:
  - T = 1.74150 m, 1.74510 m, 1.77368 m, 1.74853 m, 1.77119 m, 1.75080 m
  - If test schedule is not pressing, make only first four test items
    - Synchronizes test days with multiples of six tests

Conclusions

- 3POD method can be successfully applied to a “real-world” situation
- “Lessons Learned?”
  - Lessons are available
  - Learning them is everybody’s job