The Three-Phase Optimal Design Test Meets Reality: Lessons Available, Part Two

John F. Fay
Becki Amendt
Gregory Hutto
Douglas Ray
Kevin Diggs
David Hartline
James Moore

Presenting Author

Dr. John F. Fay

850-883-2105

Odyssey Systems Consulting Group

john.fay.3.ctr@us.af.mil

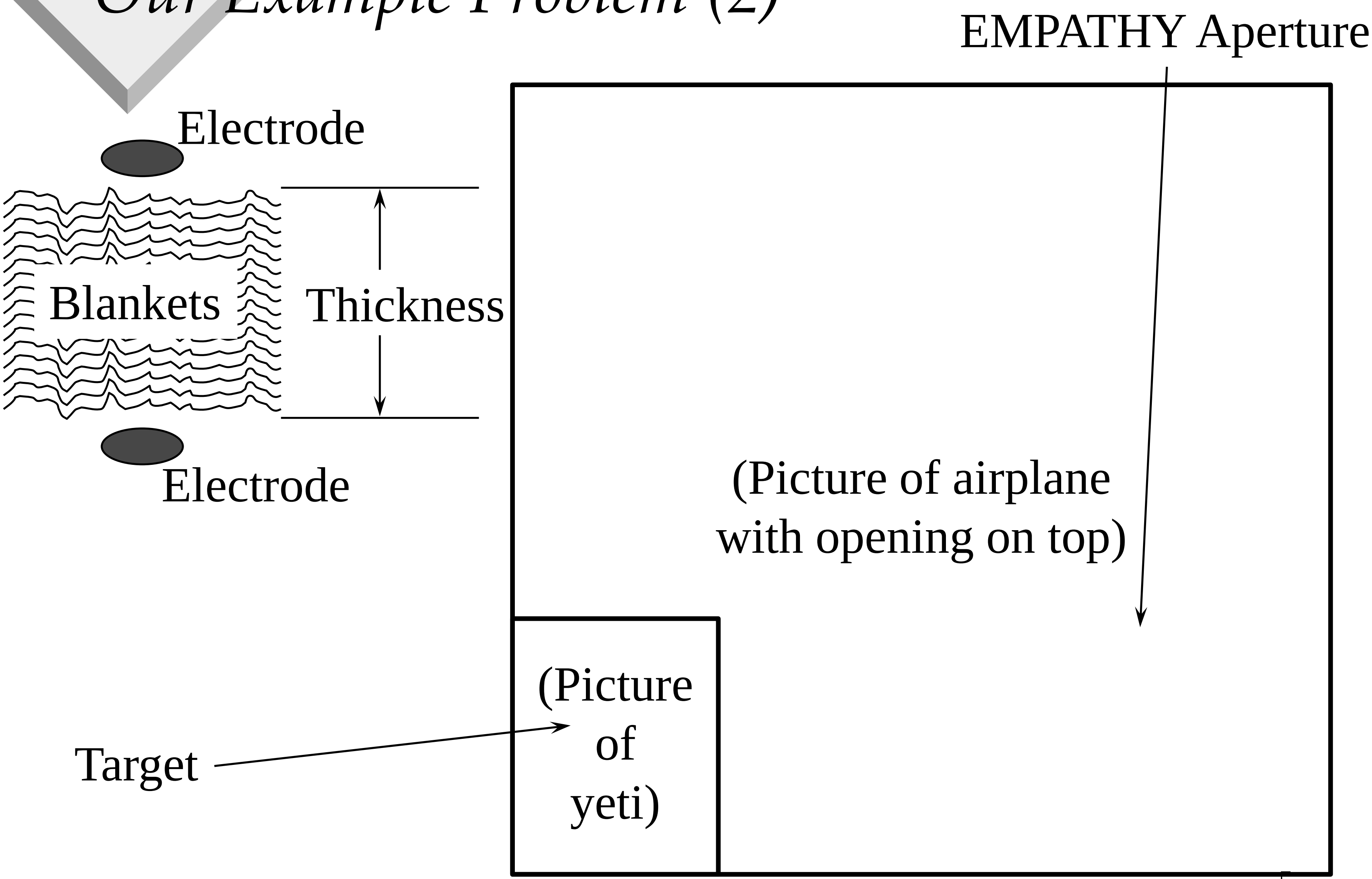
Outline

- Our Example Problem
- The Three-Phase Optimal Design Test
- Issues and How to Cope With Them
  - Data Range
  - Limited Precision
  - Specified vs. Actual Test Points
  - End of Test

Our Example Problem

- Fictitious Weapon: Electro-Magnetic Pulse Against Thoroughly Hostile Yetis
  - Two high-voltage electrodes
  - Separated by stack of insulating blankets
  - Thicker stack → better chance of enough insulation between electrodes → better chance that charge does not bleed off slowly → better chance of electrical discharge when needed
  - Need thickness of stack required to give
    99.99% chance of discharge
    at 95% confidence level

Our Example Problem (2)



The Three-Phase Optimal Design Test

- We have an input
  - Varies continuously – thickness of stack
- We have an output
  - One or zero – success or failure – on or off – discharge or no discharge
  - Probabilistic function of input
    - The same input can give different outputs in different tests
    - Probability of a one increases as input increases

The Three-Phase Optimal Design Test (2)

- Invented by
  - Jeff Wu of Georgia Institute of Technology
  - Yubin Tian of Beijing Institute of Technology
- Published in the Journal of Statistical Planning and Inference, 2013
  - http://dx.doi.org/10.1016/j.jspi.2013.10.007

The Three-Phase Optimal Design Test (3)

- Phase I: Find the mean
  - Step I1: Obtain success and failure results
  - Step I2: Get an overlapping result
  - Step I3: Enhance the overlapping result
- Phase II: Optimize the mean and standard deviation
- Phase III (optional): Test at desired probability level to reduce uncertainty

The Three-Phase Optimal Design Test (4)

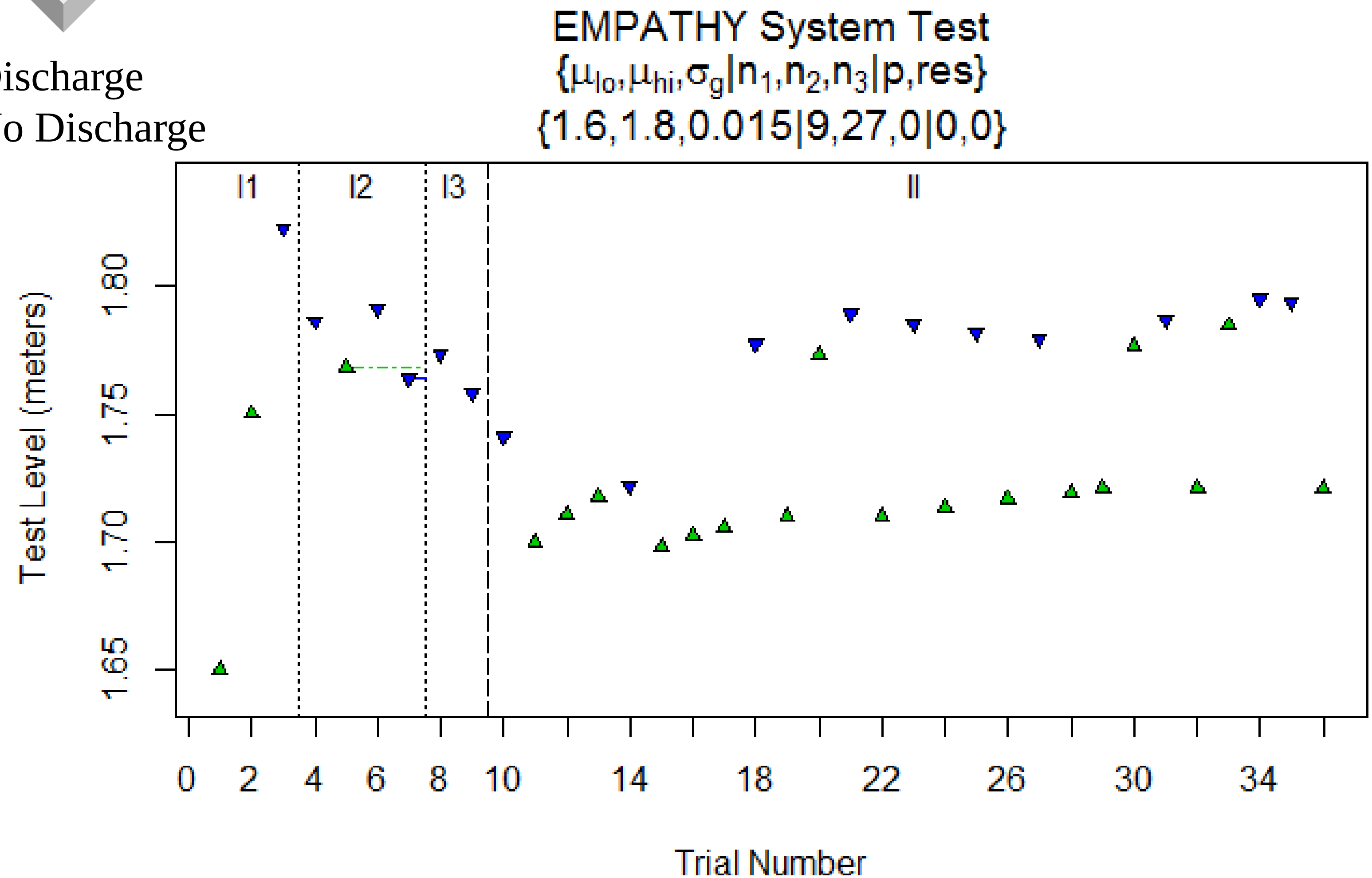
- Assumes probability curve follows normal distribution
- Requires starting values:
  - Approximate lower and upper bounds of range
  - Approximate standard deviation of probability curve

Our Example Problem (3)

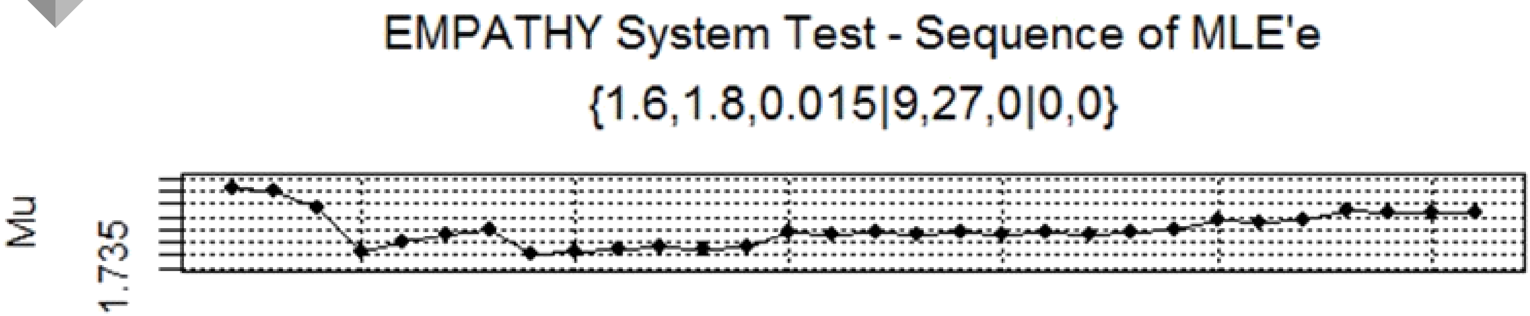
- Simulations show:
  - 1.6-meter stack of blankets is not enough insulation—no discharge
    - Lower end of “initial guess” interval
  - 1.8-meter stack of blankets is enough insulation—discharge
    - Upper end of “initial guess” interval
- Estimated Standard Deviation
  - Should be less than one sixth of range
  - We use 0.015 meters

Our Example Problem (4)

▼ - Discharge
▲ - No Discharge

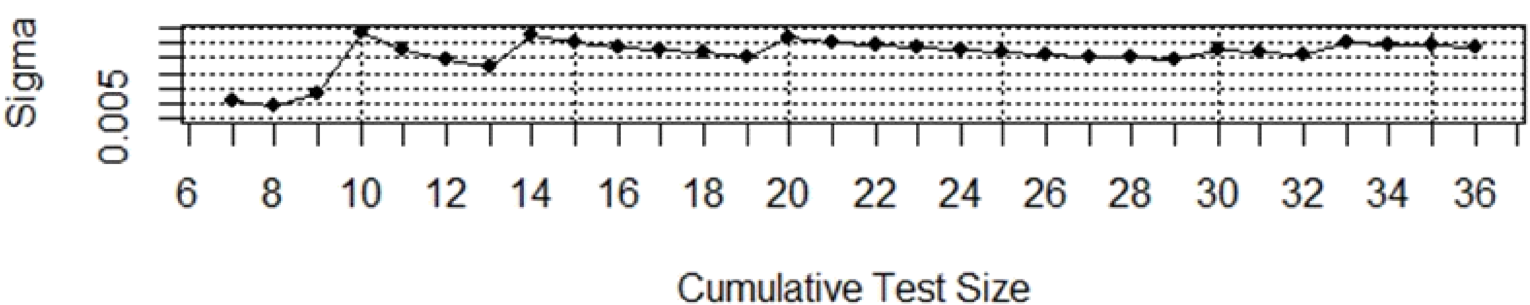


Our Example Problem (5)

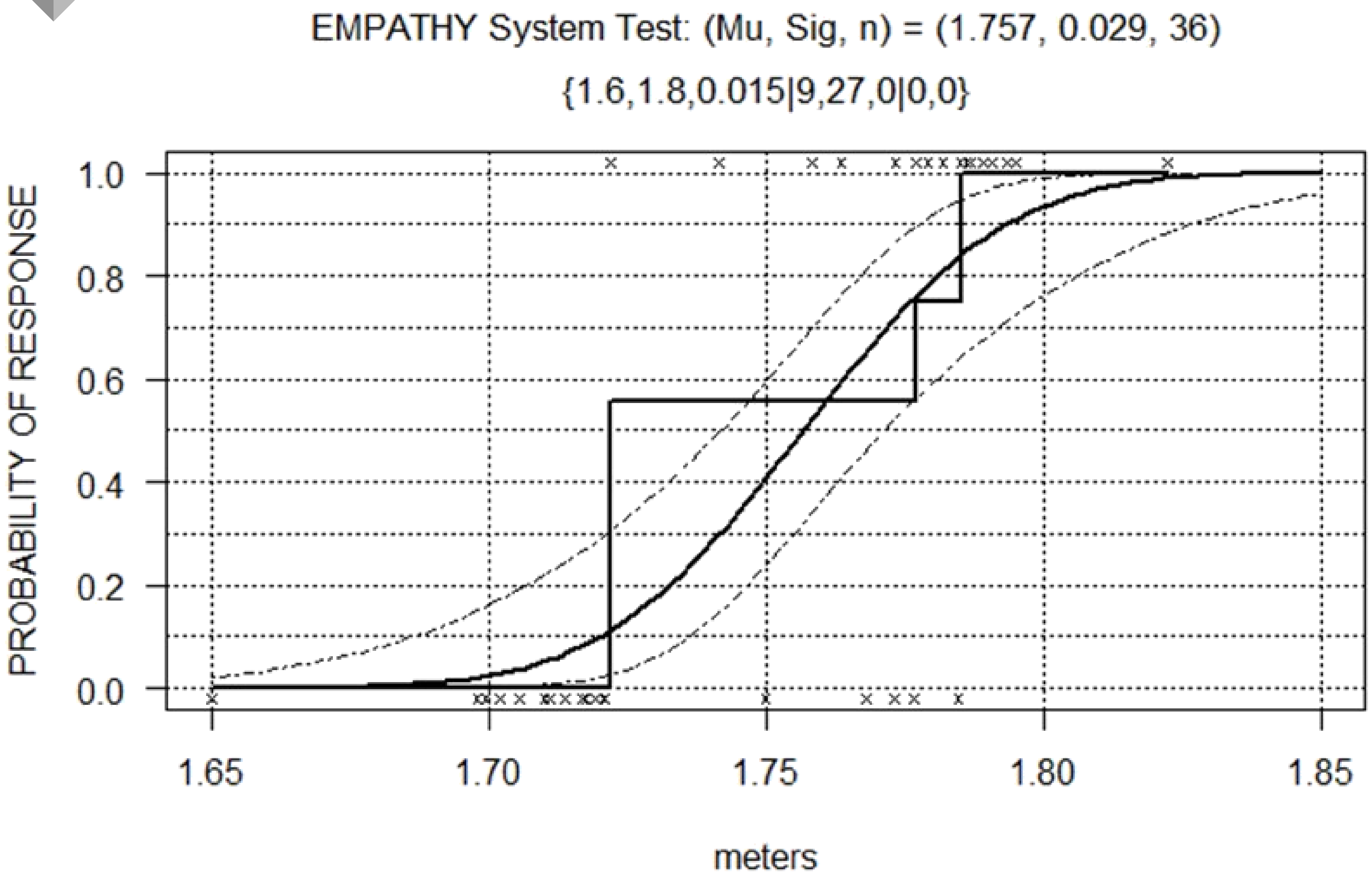


Nominal Values:
Mu = 1.750
Sigma = 0.050

Final Calculated Values:
Mu = 1.757
Sigma = 0.029



Our Example Problem (6)



Issues

- Data Range
- Limited Precision
- Specified vs. Actual Test Point Values
- End of Test

Data Range

- Issue:
  - Method is mathematical
    - No knowledge of physical limitations on system
    - Can specify unreasonable test points
      - Negative thickness of stack of blankets
      - Stack thickness beyond system capability
- Resolution:
  - Use common sense

Example (5)

- EMPATHY system testing:
  - If first several tests give "discharge" result:
    - Thickness of blanket stack decreases
    - Next test point requires negative thickness
      - Not physically real
  - If first several tests give "no discharge" result:
    - Diameter of EMPATHY case is 2.14 meters
    - Hard upper limit on blanket stack thickness
      - May result in system not meeting requirement

Limited Precision

- Issue:
  - 3POD method can specify test points to unlimited precision
  - Test articles cannot be built to unlimited precision
- Resolution:
  - Points close to optimal point are still good
  - Do your best

Limited Precision (2)

- Three different things:
  - Test point specification – result of 3POD method
  - Test item fabrication – built to specified point at limited precision
  - Test item measurement – may be more precise than test item fabrication ability

Limited Precision (3)

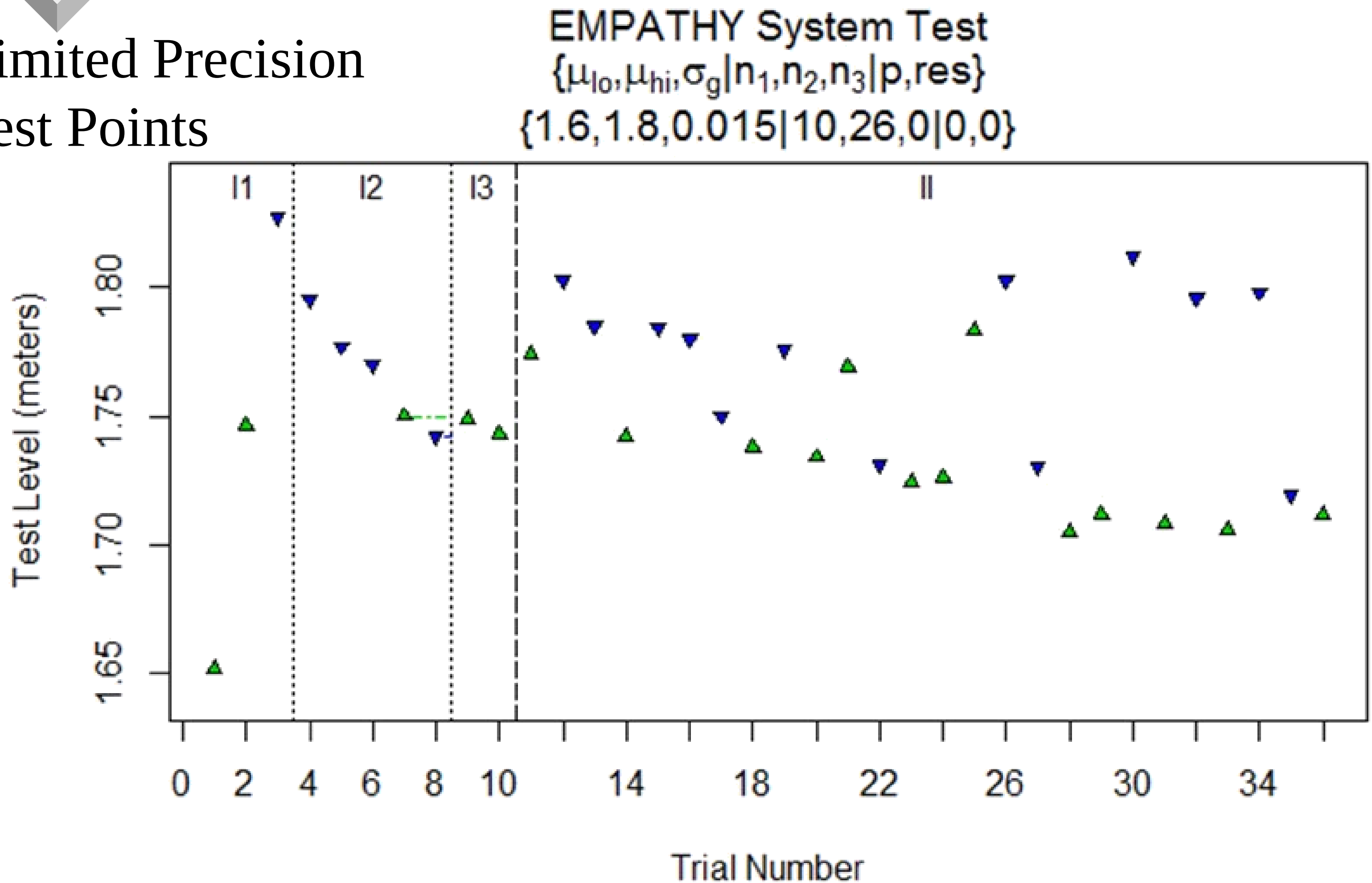
- Resolution (2)
  - Specify test points to 3POD recommended precision—do not round to specifiable precision
    - Scatter will center around recommended point
  - Use measured values in 3POD method calculations
    - Not specified values
    - Not rounded measured values
    - Not 3POD method's recommended values

Example (6)

- EMPATHY blankets settle irregularly
  - Final thickness controlled only to ±0.01 m
    - One centimeter scatter on either side
  - Can be measured to ±0.0005 m
    - One-millimeter uncertainty overall

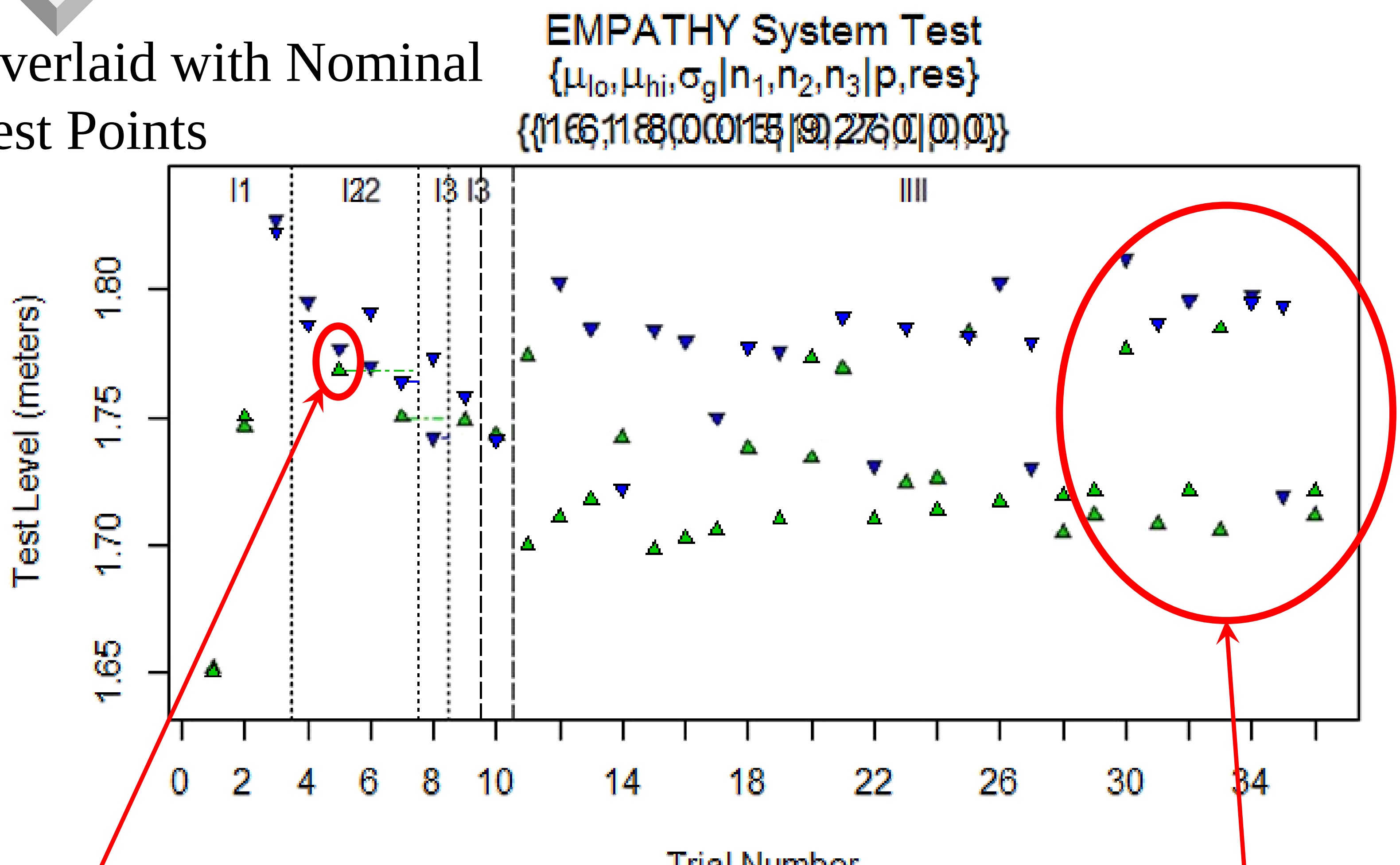
Example with Limited Precision

Limited Precision
Test Points



Example with Limited Precision

Overlaid with Nominal Test Points



Divergence

Convergence

Effect of Limited Precision

Test	Nominal		Limited Precision		
	Test Value	Outcome	Spec Value	Actual Value	Outcome
1	1.65	No Discharge	1.65	1.652	No Discharge
2	1.75	No Discharge	1.75	1.746	No Discharge
3	1.8225	Discharge	1.82250	1.827	Discharge
4	1.78625	Discharge	1.78650	1.795	Discharge
5	1.76812	No Discharge	1.77050	1.777	Discharge
6	1.79075	Discharge	1.76150	1.770	Discharge
7	1.76362	Discharge	1.75800	1.750	No Discharge
Final Values	Mu	Sigma		Mu	Sigma
	1.757	0.029		1.754	0.034

Divergence

Final values still pretty close

Limited Precision (4)

- Issue:
  - Standard deviation of distribution may be near limit of precision of creating test items
  - Specified test points in Phase I2 may all round to the same value, preventing overlap

Limited Precision (5)

- Resolution:
  - Alternative 1: Use "engineering judgment" to modify test points for Phase I2
    - If tests never achieve overlap, standard deviation is less than measurement precision
  - Alternative 2: Add fictitious "test points" at changeover point to create artificial overlap

Example: Alternative 1

❖ Can build, measure only to 0.01:

Test	Specified Test Point	Rounded Test Point	Actual Test Point	Test Result
1	1.65	1.65	1.65	No Discharge
2	1.75	1.75	1.75	Discharge
3	1.7	1.70	1.70	No Discharge
4	1.725	1.72	1.72	No Discharge
5	1.735	1.74	1.74	No Discharge
6	1.7545	1.75	1.75	Discharge
7	1.7355	1.74	1.74	No Discharge
8	1.753	1.75	1.76	Discharge
9	1.737	1.74	1.73	…

Settling around two points

Example: Alternative 2

- Can build, measure only to 0.01:

Test	Specified Test Point	Rounded Test Point	Test Result
1	1.65	1.65	No Discharge
2	1.75	1.75	Discharge
3	1.7	1.70	No Discharge
4	1.725	1.72	No Discharge
5	1.735	1.74	No Discharge
6	1.7545	1.75	Discharge
7	1.7355	1.74	No Discharge
8	1.753	1.75	No Discharge
9	1.737	1.74	Discharge

Fictitious test points and results

Specified vs. Actual Test Points

- Issue:
  - 3POD method
    - Assumes actual test points same as specified test points
    - Declares overlap based on test result without checking actual test point
  - 3POD algorithm may specify leaving Phase I2 without achieving overlap
- Resolution: Use common sense

End of Test

- Issue:
  - 3POD method does not specify number of tests or ending criterion
  - Number of tests often governed by economics and other factors
  - "99.99% value" vs.
    "acceptable 99.99% value"

End of Test (2)

- Resolution
  - Specify ending criterion in advance:
    "99.99 percent probability, with 95 percent confidence, that a thickness of 2.14 meters will not allow a spark between electrodes"
  - Continue testing until
    - Criterion is met
    - Criterion will still be met if next three* tests give less probable result

*arbitrary number

End of Test (3)

- Issue:
  - Calculation of 99.99%-at-95% point
    - Issue: Different methods give different results
      - Logit link vs. Probit link vs. other links
        - (define exact shape of probability curve)
      - Which do you believe?

End of Test (4)

- If criterion is never met:
  - More testing will* tighten 95% confidence bounds
  - Possibility that criterion cannot be met
    - It may take 2.15 meters of blankets to prevent spark between the electrodes

*usually

End of Test (5)

- “Point of No Return”
  - Situation: Phase II
    - Predicted 99.99%-at-95% “threshold” point exceeds maximum value
    - Hard limit on number of tests possible
  - Suggestion: Predict test into the future
    - Assume no further anomalies
    - Determine whether remainder of test shots can bring threshold point down to an acceptable level
    - If not, consider declaring failure early and saving test resources

Conclusions

- 3POD method can be successfully applied to a “real-world” situation
- “Lessons Learned?”
  - Lessons are available
  - Learning them is everybody’s job

Any questions?