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Automated Pointing of Indirect Weapon Systems – A Lesson in Kinematics

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3 May 2017

NDIA Armament Systems Forum, Fredericksburg, VA

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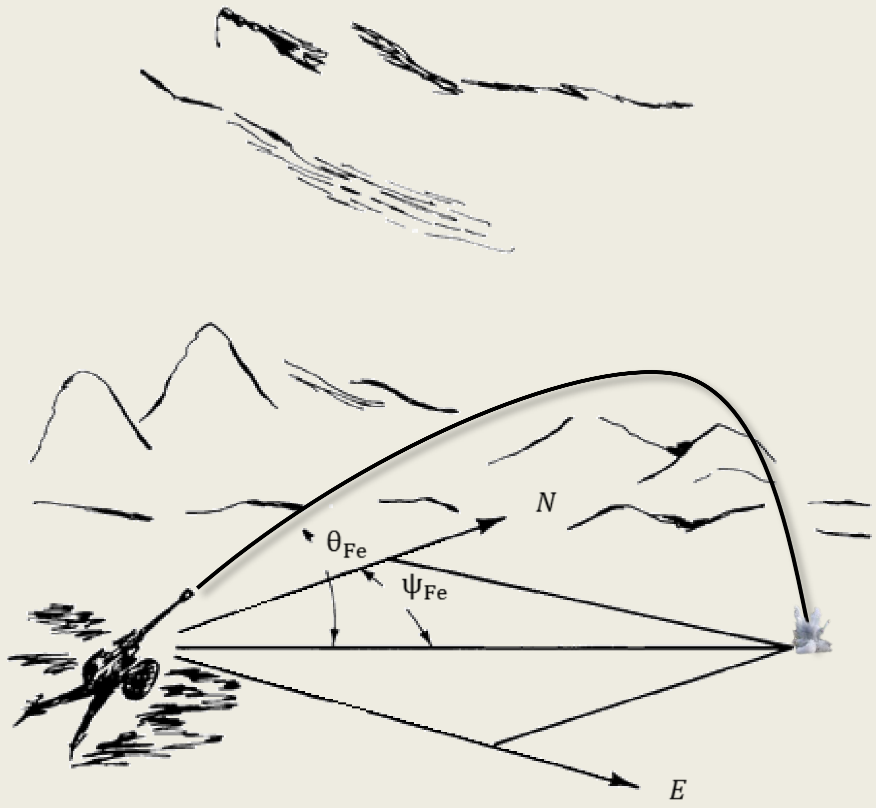
Agenda

- Laying of indirect fire weapons
- Issues faced by automated pointing systems
- Introduction to kinematics
- Finding a kinematic solution
- Implementation of a pointing algorithm
- Conclusion

Indirect Weapon Laying

- Gun laying is achieved by performing movements that align the axis of the gun barrel with the calculated lay angles.
- A gun is traversed in a horizontal plane and elevated in a vertical plane to range it to its target.
- The traverse and elevation values make up the aiming portion of the "firing solution".

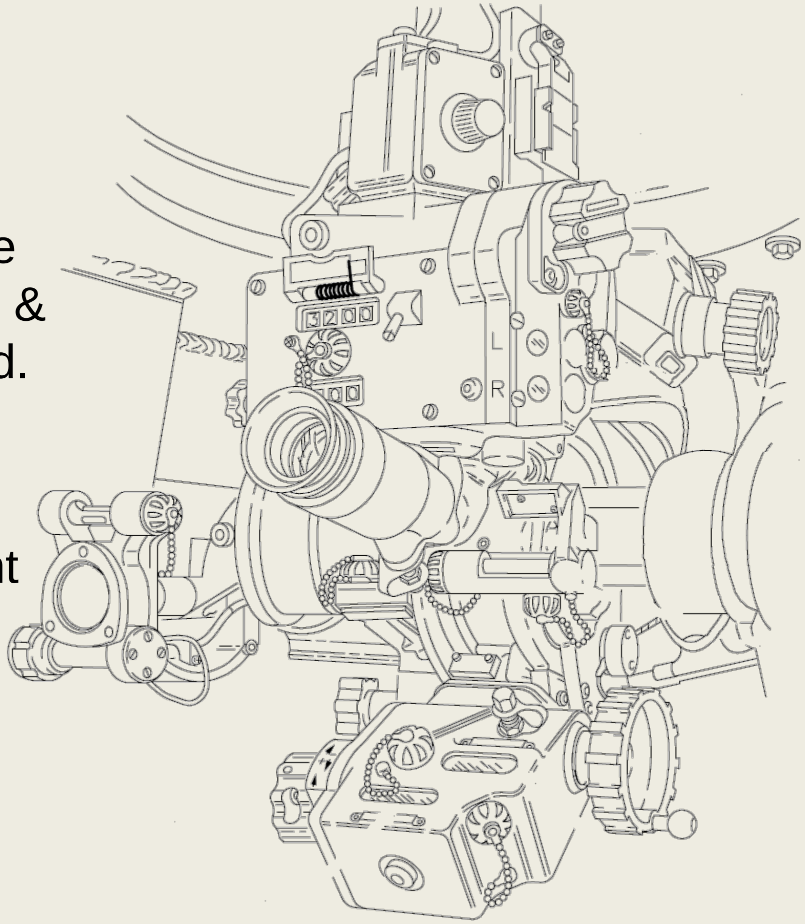


U.S. DoD MIL-HDBK-799 Fig 2-21 Pg. 2-30, Modified

Variable	Definition
ψ_{Fe}	Firing solution yaw rotation (Earth reference)
θ_{Fe}	Firing solution pitch rotation (Earth reference)

INDIRECT WEAPON LAYING

- Prior to digitized fire control, gun laying was primarily performed using optical sights.
- The target deflection and quadrant values are verbally relayed and the weapon is traversed & elevated until the commanded lay is indicated.
- After each shot the optical sight must be checked and if necessary adjusted to account for any displacement due to firing.
- The process requires verification and manipulation of various dials, spirit levels, knobs, etc. and is susceptible to user error.
- Location and North finding is accomplished via survey,

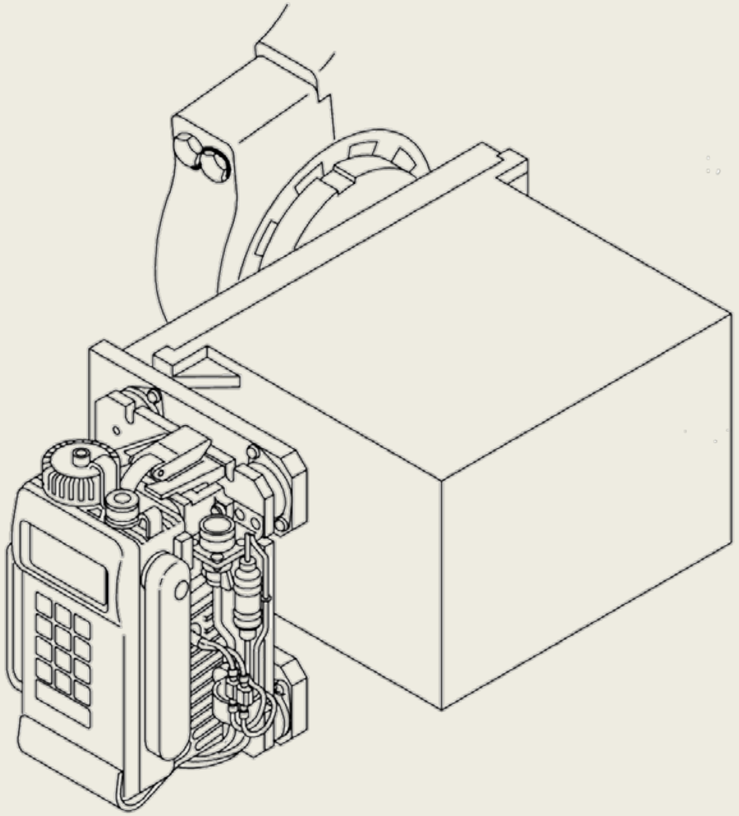


U.S. Dept. of the Army TM 9-2350-314-10 Pg. 2-135, Modified

Pictured is the optical fire control hardware used on the M109A6 Paladin Self Propelled Howitzer.

Indirect Weapon Laying

- Digitized fire control provides the same pointing capability but relies on sensors in place of optics.
- A GPS assisted inertial navigation unit provides the system with weapon position and Earth referenced attitude.
- The gunner uses information on a display to properly lay the weapon.
- Automated systems go a step further and actuate the azimuth and elevation axes to lay the weapon.



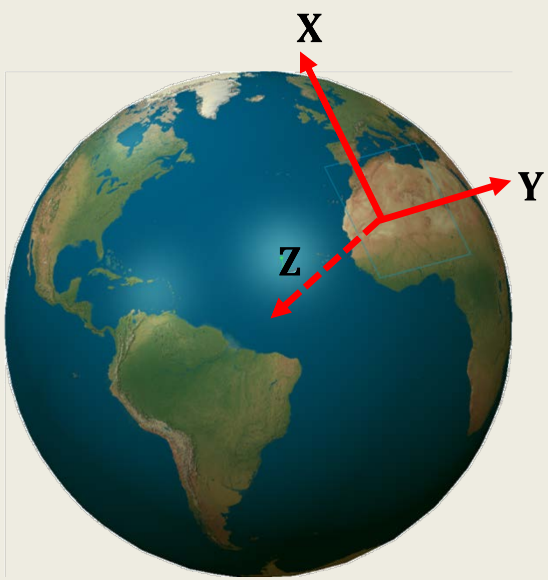
U.S. Dept. of the Army TM 9-2350-314-20-2-1 Pg. 3-66.3, Modified

Pictured is the PLGR & DRUH used on the M109A6 Paladin Self Propelled Howitzer.

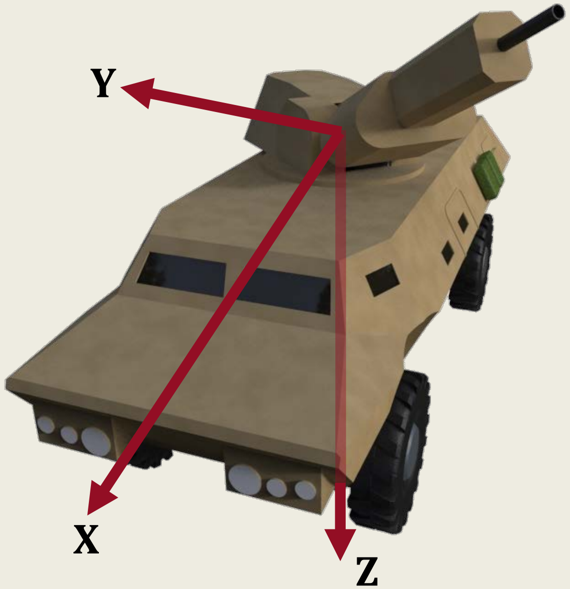
Automated Pointing Issues

- One of the challenges with automated systems is that the firing solution is represented by angles relative to an Earth reference frame while the actuators are controlled relative to a platform reference frame.

- The Earth and platform reference frames are rarely coincident meaning there is a disconnect between the pointing control system and its target.



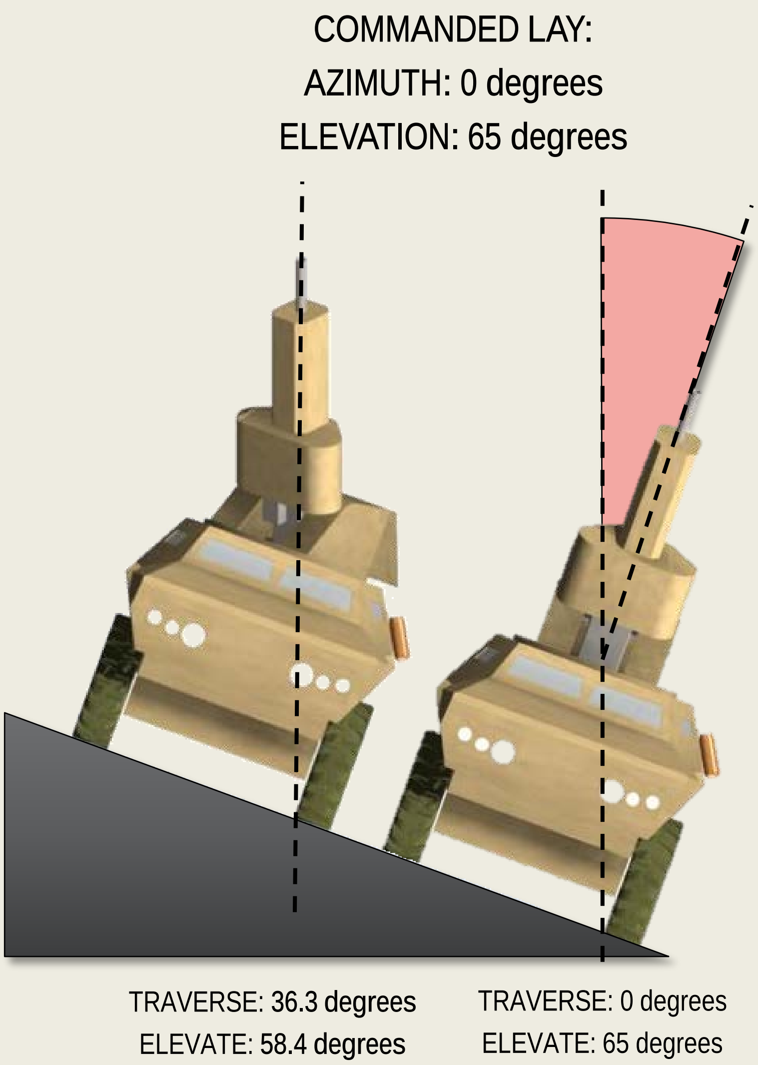
Earth Reference Frame



Platform Reference Frame

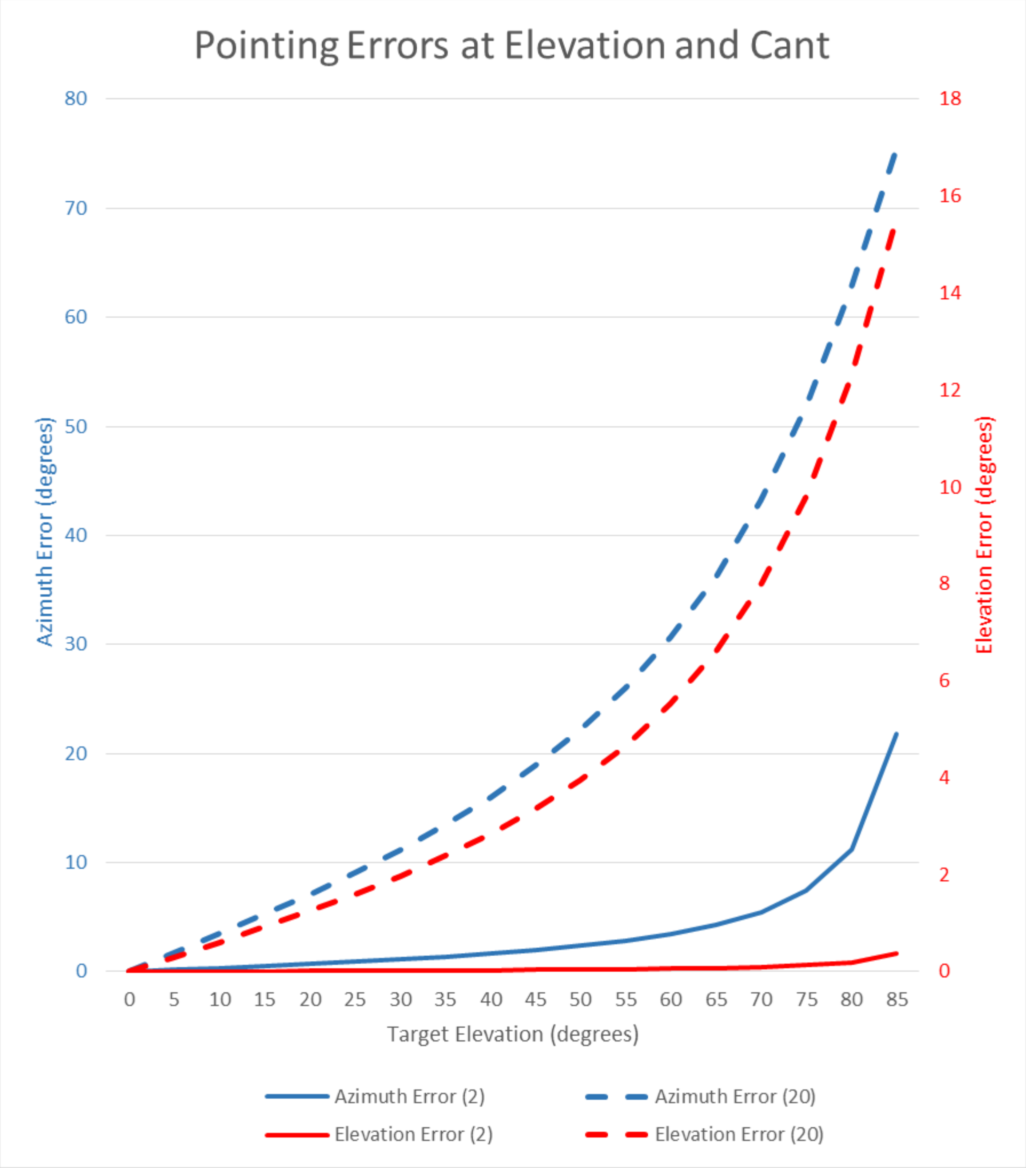
Automated Pointing Issues

- When the weapon system is level the difference between the Earth and platform references is limited to the offset between the Earth referenced azimuth and the platform's azimuth.
- However, when the system is on uneven ground, elevating by the target elevation results in an Earth referenced elevation that is below the target and an azimuth that has drifted off the commanded line of fire.
- At greater platform cants and higher target elevations this divergence becomes increasingly severe.
- A weapon system is rarely on perfectly level ground. This is an issue worth addressing.



Automated Pointing Issues

- The graph on the right shows how the error grows with greater platform cant and target elevation.
- The cant and weapon azimuth is oriented as shown on the previous slide.
- The error values depicted indicate the difference between the target value and the resulting value if the weapon is elevated relative to the platform without consideration for the platform cant.

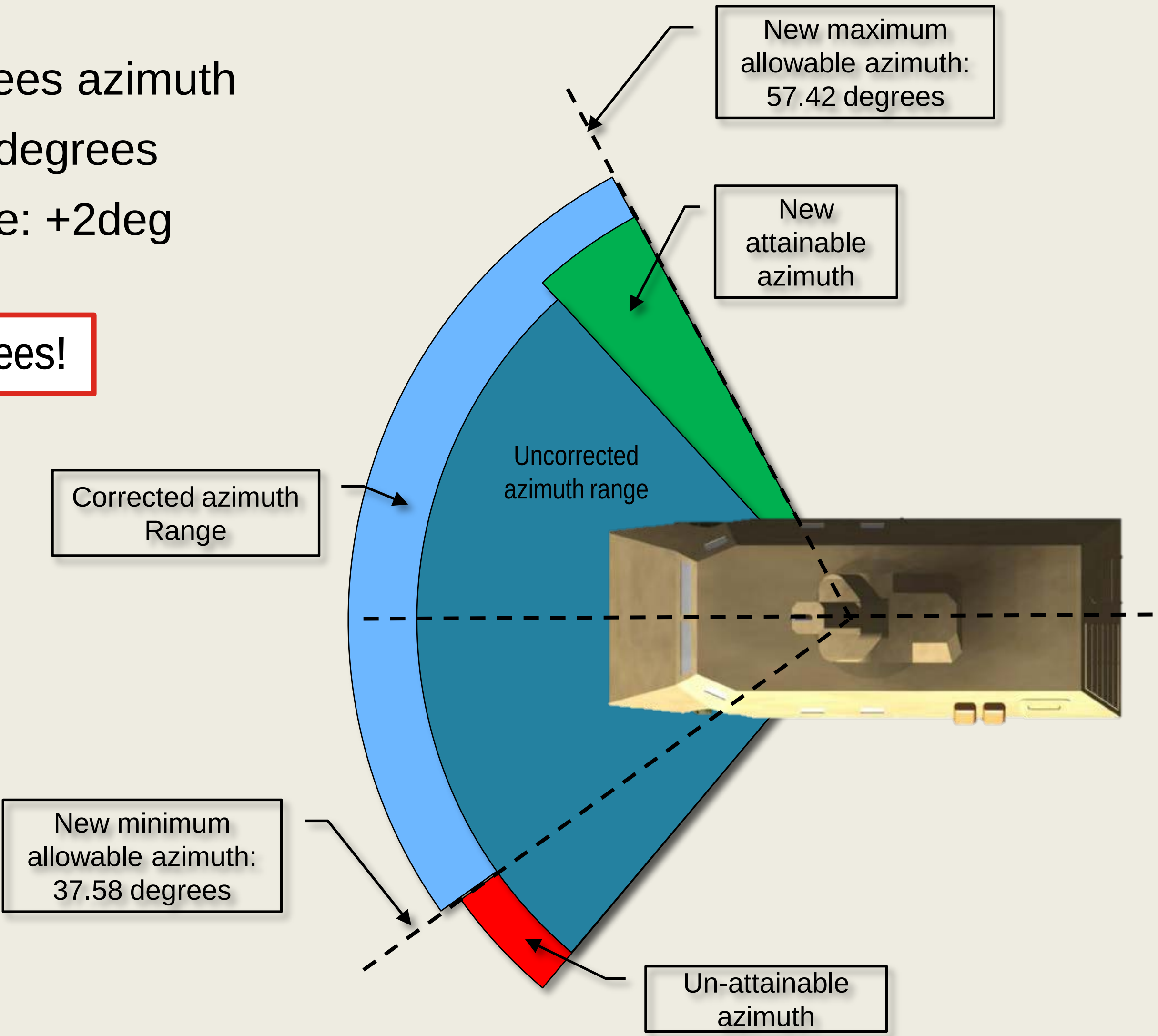


Automated Pointing Issues

Real world example:

- Limited firing fan +/-45 degrees azimuth
- Firing solution Elevation: 75degrees
- System cant along line of fire: +2deg

Azimuth error: +7.42 degrees!



Intro to Kinematics

- Kinematic equations can be used to solve a multitude of problems involving moving objects.

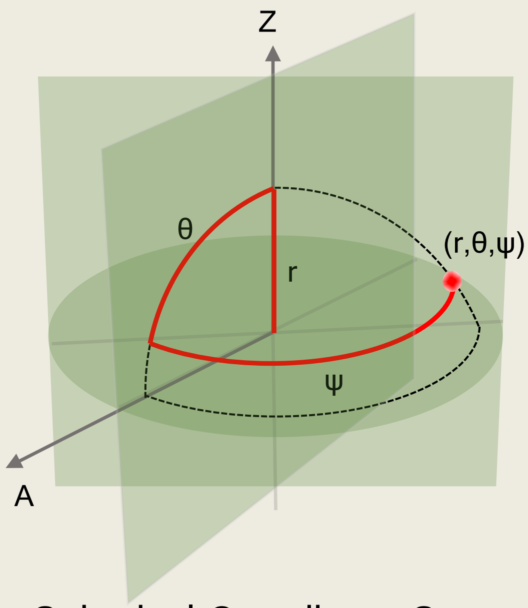
- While position of the system is important for calculating a ballistic solution, pointing of the weapon is concerned only with rotation.

- The goal is to determine a method that transforms the Earth referenced firing solution into values that represent the same solution but referenced to the platform.

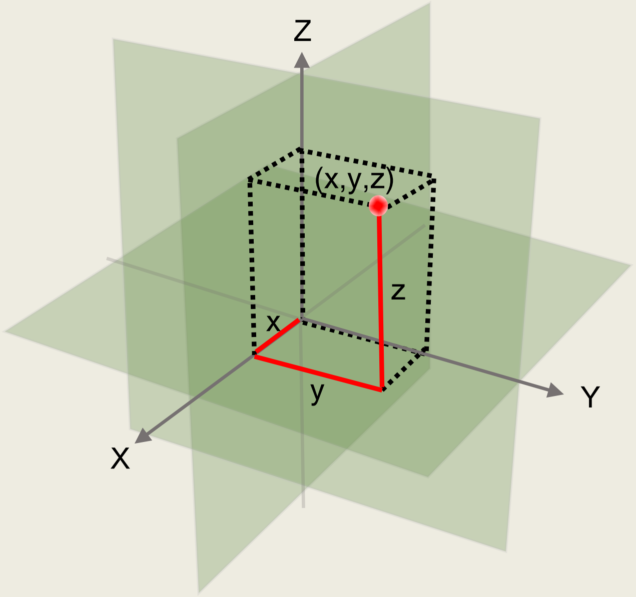
Kinematics: *the branch of mechanics that deals with pure motion, without reference to the masses or forces involved in it.*

Intro to Kinematics

- Two types of coordinate systems are used in the subsequent equations to define the orientation of a vector: Cartesian and Spherical.
- Firing solutions and servo commands use spherical coordinates with r set to 1.
- Subsequent kinematics equations operate on unit vectors represented by x,y,z Cartesian coordinates.



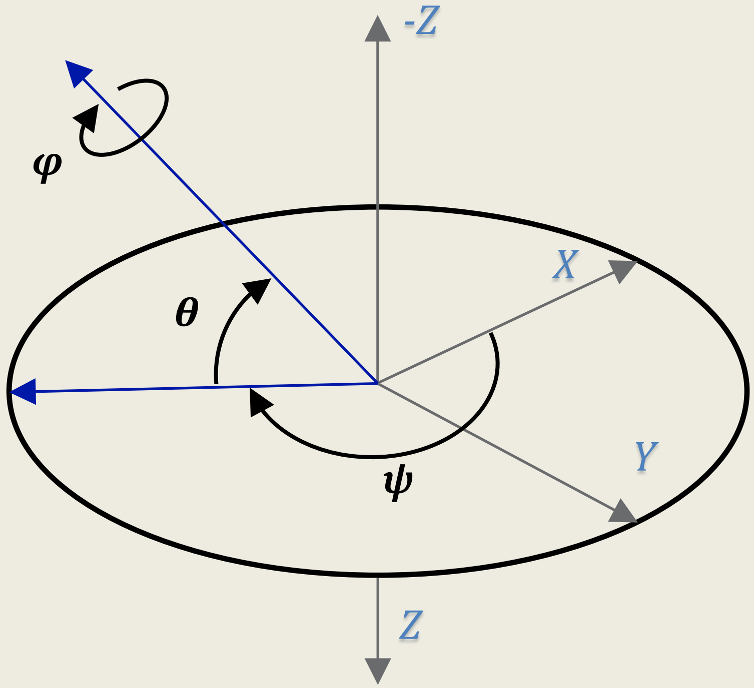
Spherical Coordinate System

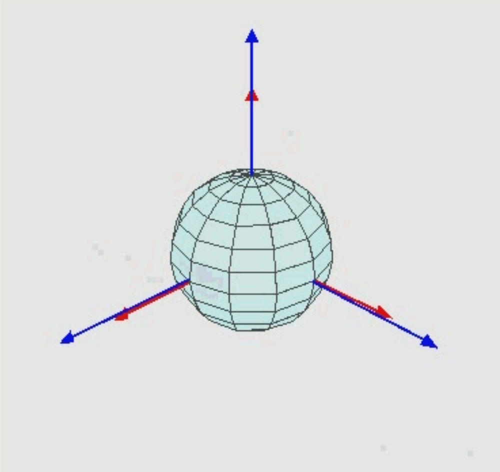


Cartesian Coordinate System

Intro to Kinematics

- Euler angles are used to describe the orientation of a rigid body with respect to a fixed reference frame.
- The image to the right depicts a zyx rotation with the rotation sequence:
  - Yaw about Z,
  - Pitch about Y
  - Roll about X





By Euler2.gif: Juansempere derivative work: Xavax - This file was derived from Euler2.gif:, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=24338647

Positive rotation defined as clockwise when looking outwards from the frame origin.

Intro to Kinematics

- A rotation matrix is used to perform a rotation in Euclidean space.
- When working in three dimensions there are three basic rotation matrices that are used to rotate vectors about the X, Y and Z axes.
- Using matrix multiplication a series of rotations can be strung together.
- If two vectors, p and p′, both describe the Cartesian coordinates of a point P using different reference frames, a rotation matrix can be used to represent the transformation of the vector coordinates from one reference frame to the other.

$$R_Z(\psi) = \begin{bmatrix} cos\psi & -sin\psi & 0 \\ sin\psi & cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_Y(\theta) = \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix}$$

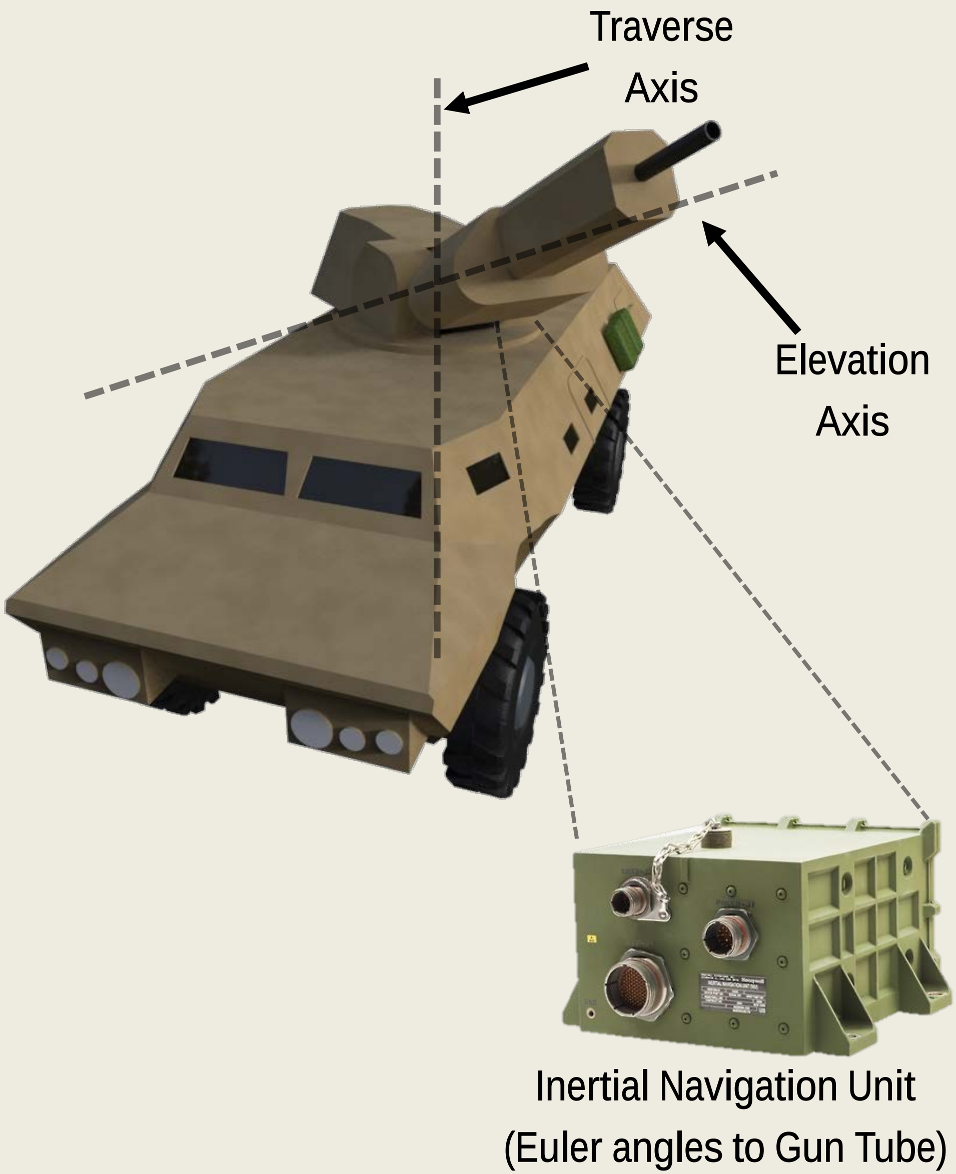
$$R_X(\varphi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\varphi & -sin\varphi \\ 0 & sin\varphi & cos\varphi \end{bmatrix}$$

$$p = Rp' \rightarrow p' = R^T p$$

KINEMATICS SOLUTION

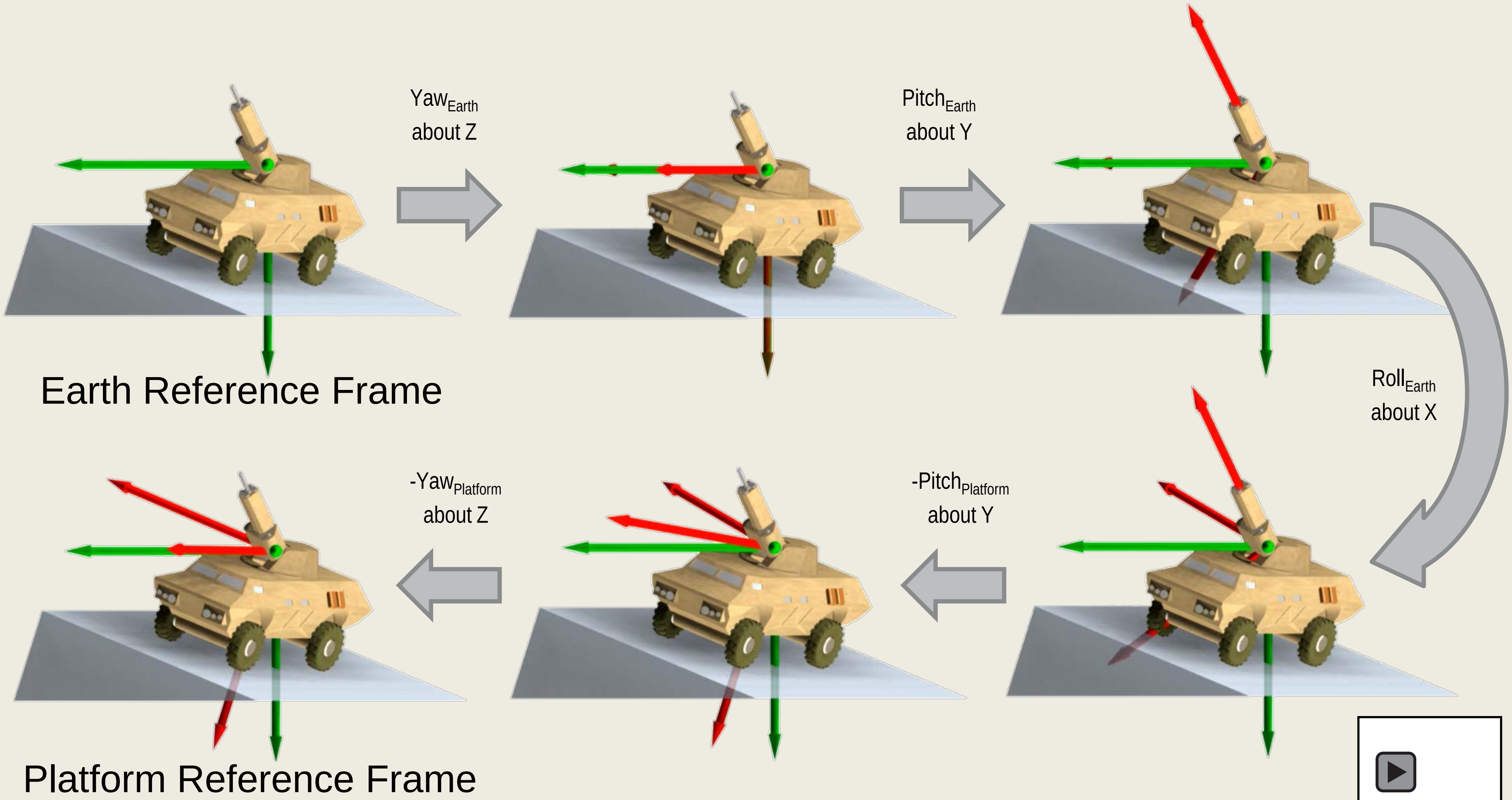
- An automated pointing system typically requires a device to measure the angle of the gun tube relative to Earth and sensors on the rotating axes to measure the angle of the gun tube relative to the platform.

- Using the available sensor data a series of calculations can be performed that transform the Earth referenced firing solution into a platform referenced firing solution.



Kinematics Solution

The first step in finding a solution is to define the series of rotations necessary to move from the Earth reference frame to the platform reference frame.



Kinematics Solution

The five rotations are represented in the equation below. Note that cosine and sine functions are represented as c and s for brevity.

Yaw$_{Earth}$ about Z	Pitch$_{Earth}$ about Y	Roll$_{Earth}$ about X	-Pitch$_{Platform}$ about Y	-Yaw$_{Platform}$ about Z

$$R = \begin{bmatrix} c\psi_{Ge} & -s\psi_{Ge} & 0 \\ s\psi_{Ge} & c\psi_{Ge} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\theta_{Ge} & 0 & s\theta_{Ge} \\ 0 & 1 & 0 \\ -s\theta_{Ge} & 0 & c\theta_{Ge} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\varphi_{Ge} & -s\varphi_{Ge} \\ 0 & s\varphi_{Ge} & c\varphi_{Ge} \end{bmatrix} \begin{bmatrix} -c\theta_{Gp} & 0 & -s\theta_{Gp} \\ 0 & 1 & 0 \\ s\theta_{Gp} & 0 & -c\theta_{Gp} \end{bmatrix} \begin{bmatrix} -c\psi_{Gp} & s\psi_{Gp} & 0 \\ -s\psi_{Gp} & -c\psi_{Gp} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Variable	Definition
ψ_{Ge}	Gun attitude yaw rotation (Earth reference)
θ_{Ge}	Gun attitude pitch rotation (Earth reference)
φ_{Ge}	Gun attitude roll rotation (Earth reference)
ψ_{Gp}	Gun attitude yaw rotation (Platform reference)
θ_{Gp}	Gun attitude pitch rotation (Platform reference)

Kinematics Solution

- The yaw (azimuth) and pitch (elevation) values that form a calculated firing solution are treated as a unit vector and represented using spherical coordinates.

- The following equations are then used to convert those values to Cartesian coordinates for use with the calculated rotation matrix.

$$x_{\mathrm{Fe}} = \cos(\psi_{\mathrm{Fe}})\, cos(\theta_{\mathrm{Fe}})$$

$$y_{\mathrm{Fe}} = \sin(\psi_{\mathrm{Fe}}) \cos(\theta_{\mathrm{Fe}})$$

$$z_{\mathrm{Fe}} = -sin(\theta_{\mathrm{Fe}})$$

Variable	Definition
ψ_{Fe}	Firing solution yaw rotation (Earth reference)
θ_{Fe}	Firing solution pitch rotation (Earth reference)
x_{Fe}, y_{Fe}, z_{Fe}	Firing solution Cartesian coordinates (Earth reference)

Kinematics Solution

- Multiplication of the transpose of the rotation matrix by the firing solution vector results in the firing solution vector in terms of the platform reference frame.

$$\begin{bmatrix} x_{\mathrm{Fp}} \\ y_{\mathrm{Fp}} \\ z_{\mathrm{Fp}} \end{bmatrix} = R^T \begin{bmatrix} x_{\mathrm{Fe}} \\ y_{\mathrm{Fe}} \\ z_{\mathrm{Fe}} \end{bmatrix}$$

Variable	Definition
x_{Fe}, y_{Fe}, z_{Fe}	Firing solution Cartesian coordinates (Earth reference)
x_{Fp}, y_{Fp}, z_{Fp}	Firing solution Cartesian coordinates (Platform reference)

Kinematics Solution

- The Cartesian coordinates representing the firing solution referenced to the platform are then converted back to spherical coordinates.

- These values are then sent directly to the platform referenced servos to point at the firing solution.

$$\psi_{\mathrm{Fp}} = \mathrm{atan}\,2\left(\frac{y_{\mathrm{Fp}}}{x_{\mathrm{Fp}}}\right)$$

$$\theta_{\mathrm{Fp}} = \mathrm{atan2}\left(\frac{-z_{\mathrm{Fp}}}{\sqrt{x_{\mathrm{Fp}}{}^2 + y_{\mathrm{Fp}}{}^2}}\right)$$

Variable	Definition
ψ_{Fp}	Firing solution yaw rotation (Platform reference)
θ_{Fp}	Firing solution pitch rotation (Platform reference)
x_{Fp}, y_{Fp}, z_{Fp}	Firing solution Cartesian coordinates (Platform reference)

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IMPLEMENTING A POINTING ALGORITHM

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Points to consider when implementing equations:

- As the weapon moves, the weight transfer can cause the platform to reorient relative to Earth, necessitating repeat application of the algorithm.
- There are many trigonometric function calls which could take significant computation time on less powerful processors.
- As long as the pointing device indicates that the target has not been reached the aiming process will continue to converge on the solution.

Implementing a Pointing Algorithm

Other applications of kinematic equations:

- Calculation of platform's attitude relative to Earth.
- Determination of maximum platform cant angle. (Safe to fire?)
- Independent axis reference. (i.e. Move elevation relative to platform and traverse relative to Earth)

CONCLUSION

- The equations presented can be used directly on systems with an identical sensor configuration or the same methods can be applied to other configurations.

- Several benefits are offered by calculating the kinematic solution
  - The system can determine if the lay is achievable prior to moving.
  - The most direct path to the target is achieved, possibly resulting in a faster lay time (quicker round out).

- While this solution is not new or necessarily unique, it is not always known by developers. Many systems could potentially benefit by updating their pointing algorithms.

Questions???

Information Sources

1. MIL-HDBK-799 (AR), *Fire Control Systems – General*, 5 April 1996, Dept. of Defense
2. Stapp, Joshua: *Kinematics of Laying an Automated Weapon System*. Picatinny Arsenal Technical Report ARWSE-TR-XXXXX 2016. Print.
3. Stapp, Joshua: *Calculation of Weapon Platform Attitude and Cant Using Available Sensor Feedback*. Picatinny Arsenal Technical Report ARWSE-TR-XXXXX 2016. Print.