Investigation of the Hugh James Criteria Using Estimated Parameters

Justin Sweitzer, Practical Energetics Research, Inc.

Nicholas Peterson, US Army AMRDEC

Nausheen Al-Shehab, US Army ARDEC

Background – Hugh James Criteria

- The James criteria is a phenomenological model of shock initiation
  - "James Space" – energy fluence and specific kinetic energy
  - If critical values of energy fluence, E, and specific kinetic energy, Σ, are surpassed, then initiation is predicted.
- Concept extended by Hrousis, et al, to a generalized parameter, J
  - J < 1 → non-initiation with margin
  - J = 1 → marginal initiation
  - J > 1 → initiation with margin

$$J = \frac{E_c}{E} + \frac{\Sigma_c}{\Sigma}, \qquad E = \int P\, u\, dt\,, \qquad \Sigma = \frac{u^2}{2}$$

James, H. R. An extension to the critical energy criterion used to predict shock initiation thresholds. *Propellants Explos. Pyrotech.* **21,** 8–13 (1996).
Hrousis, C. a, Gresshoff, M. & Overturf, G. E. *Probabilistic Shock Initiation Thresholds and QMU Applications*. (2009).

Background – QMU Thresholds

- Hrousis' extensions result in a probability density function for initiation.
  - Mean value of 1.0 for initiation
  - Estimate of uncertainty in critical values leads to standard deviation
  - Assume normally distributed

Background – BLR Model

- Linear regression between categorical observations
- Probability density function yields likelihood that observation fits into a category

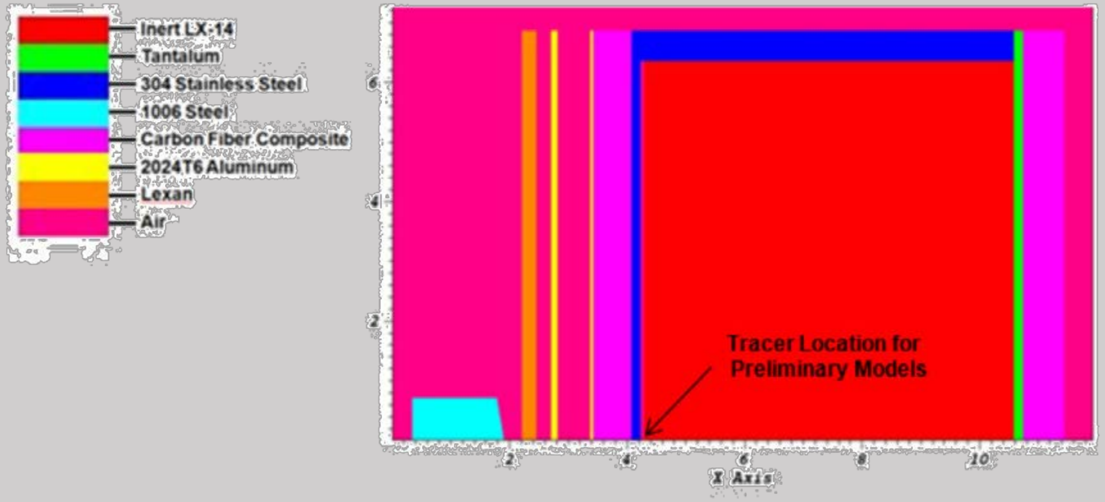
$$p(x) = \frac{1}{1 + e^{-ax-b}}$$

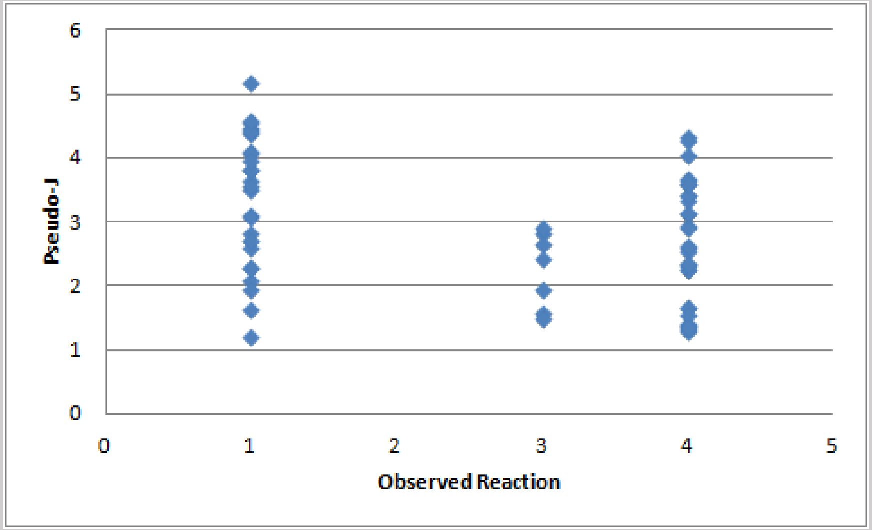
- Log-Likelihood function must be numerically optimized to maxima to fit slope and intercept of linear function $\mathrm{f}(\mathrm{x}) = ax + b$

$$LL = \sum_{i=1}^{n} y_i \, ln(p_i) + (1 - y_i) \, ln(1 - p_i)$$

Background – Prior Effort

- Previous effort substituted arbitrary values for critical values
  - Suitable critical values not located for main fill HE (LX-14)
  - Multiple Fragment Impact (FI) test data points were available
  - Hydrocode simulations of test data used to estimate J-parameter
  - Binary Logistic Regression (BLR) model tied to FI test observations





Sweitzer, J. C. & Peterson, N. R. Method for Prediction of Fragment Impact Response Using Physics Based Modeling and Statistical Analysis. *Procedia Eng.* **103,** 601–609 (2015).

Problem Statement

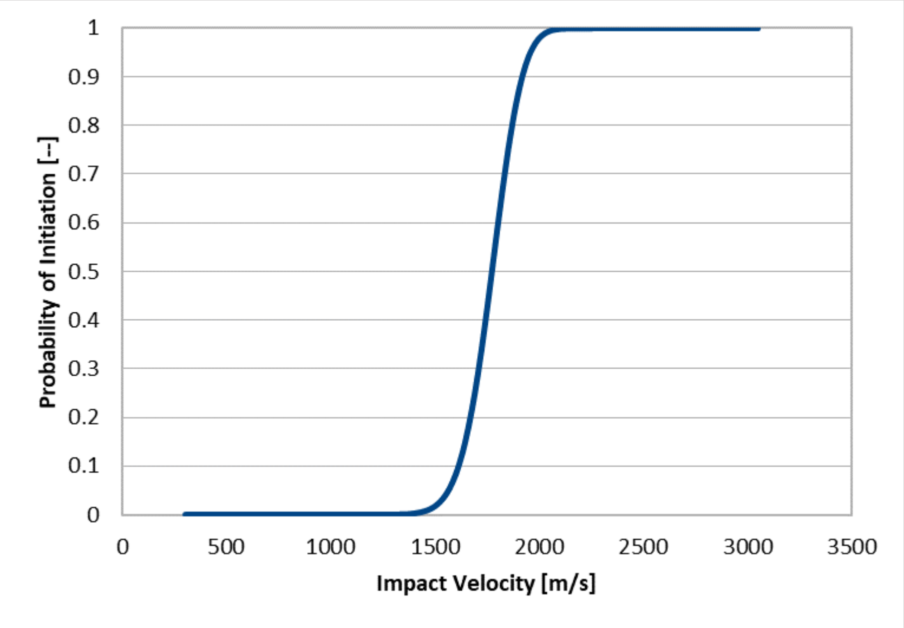
- The previous effort substituted arbitrary parameters for the critical values under the assumption that the BLR would correctly categorize results from FI tests.
- The validity of this assumption was not investigated in detail

Hypothesis:

A BLR model fit to experimental data is a good estimator of the true probability density.

Methodology

- Validity of hypothesis tested by using the parameters of Hrousis, et al for uF-TATB to generate a matrix of simulated 'observations'
  - Hydrocode calculations performed with the known E_c, Σ_c
    - Yields 'True' probability density
  - Iterations with arbitrary E_c, Σ_c, random number compared to p-value from 'True' probability function.
    - Yields 'Observations' to fit BLR model



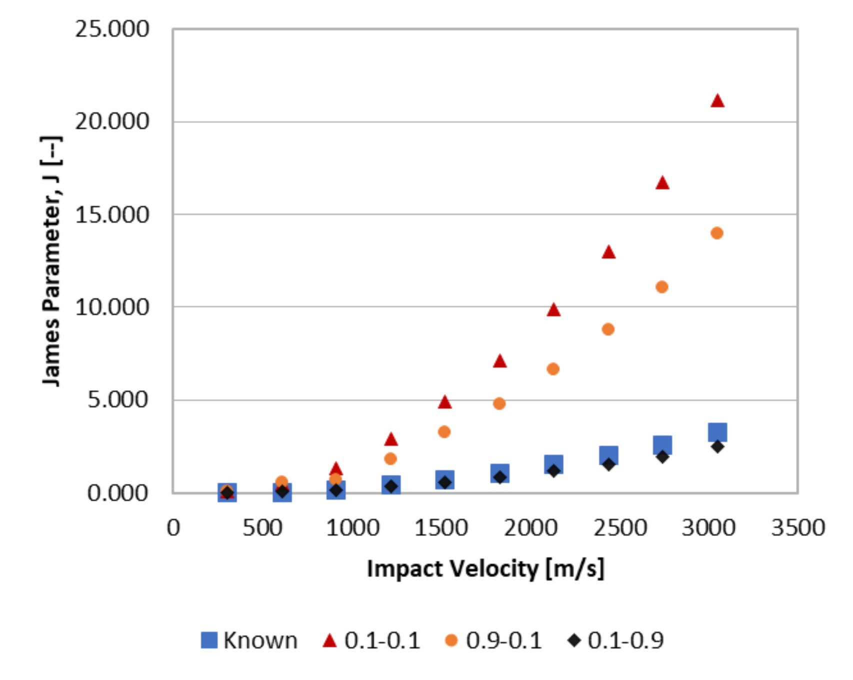
Probability of initiation at various impact velocities

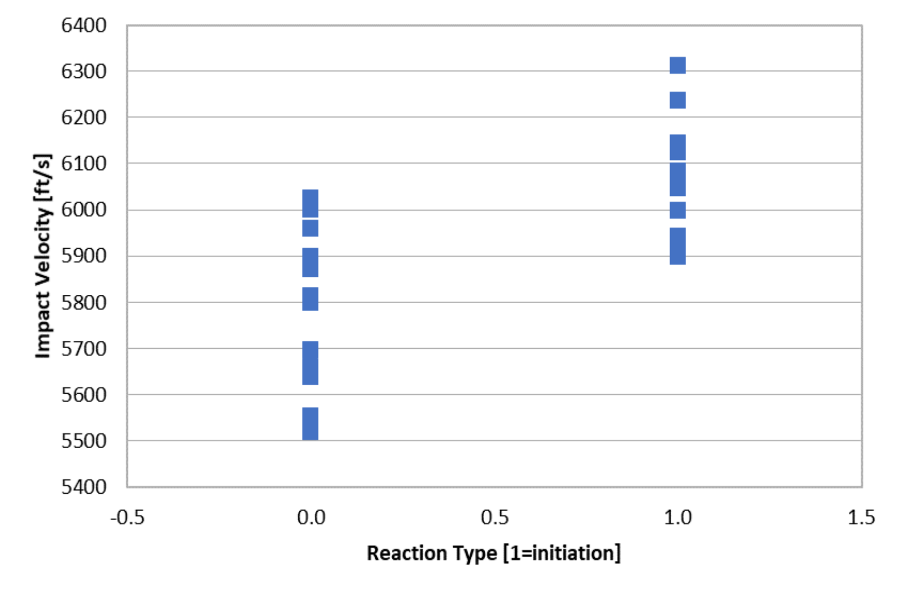
Hypothesis is tested by comparing pdf of BLR model to 'True' pdf.

	E_c MJ/m2	Σ_c MJ/kg
Known	0.26	0.67
Variation 1	0.1	0.1
Variation 2	0.1	0.9
Variation 3	0.9	0.9
Variation 4	0.9	0.1

Calculation Results

- Quadratic relationship between impact velocity and calculated Jmax in all cases.
- P-value vs random number results in overlap of observations
  - Some impacts with J < 1 initiate
  - Some impacts with J > 1 do not
- Consistent with observations in FI & Gap Tests





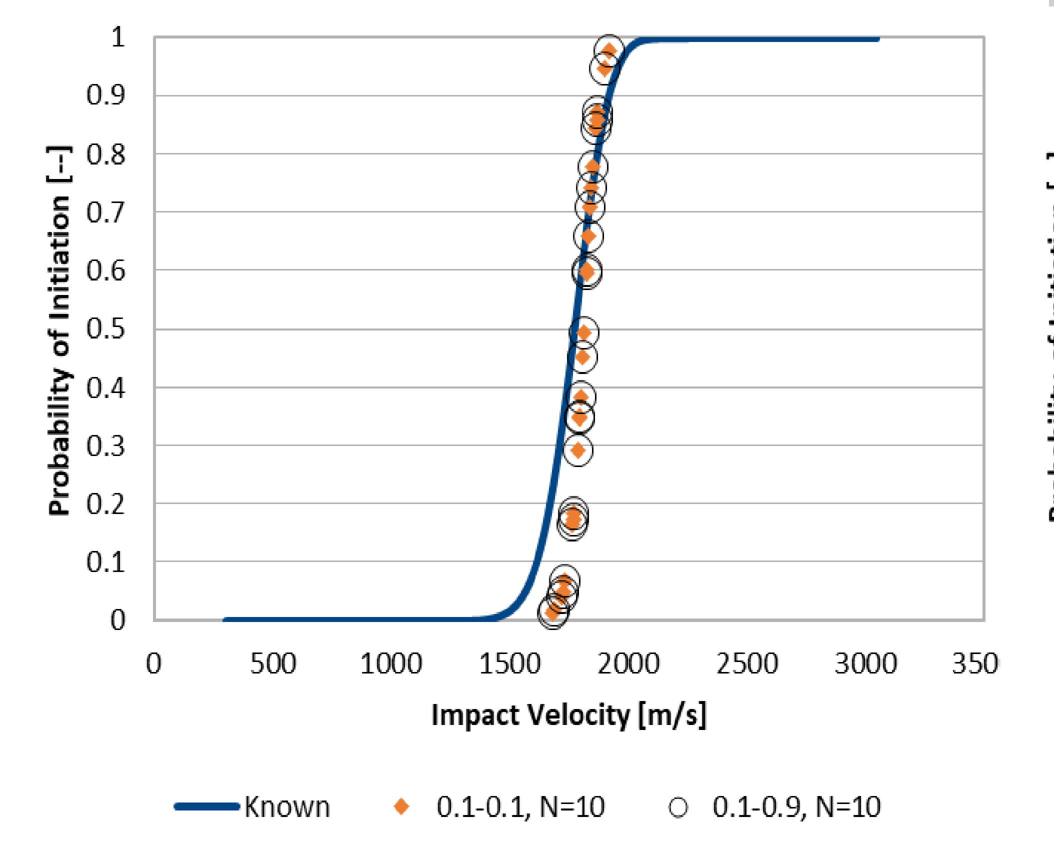
Generated Observation Matrix

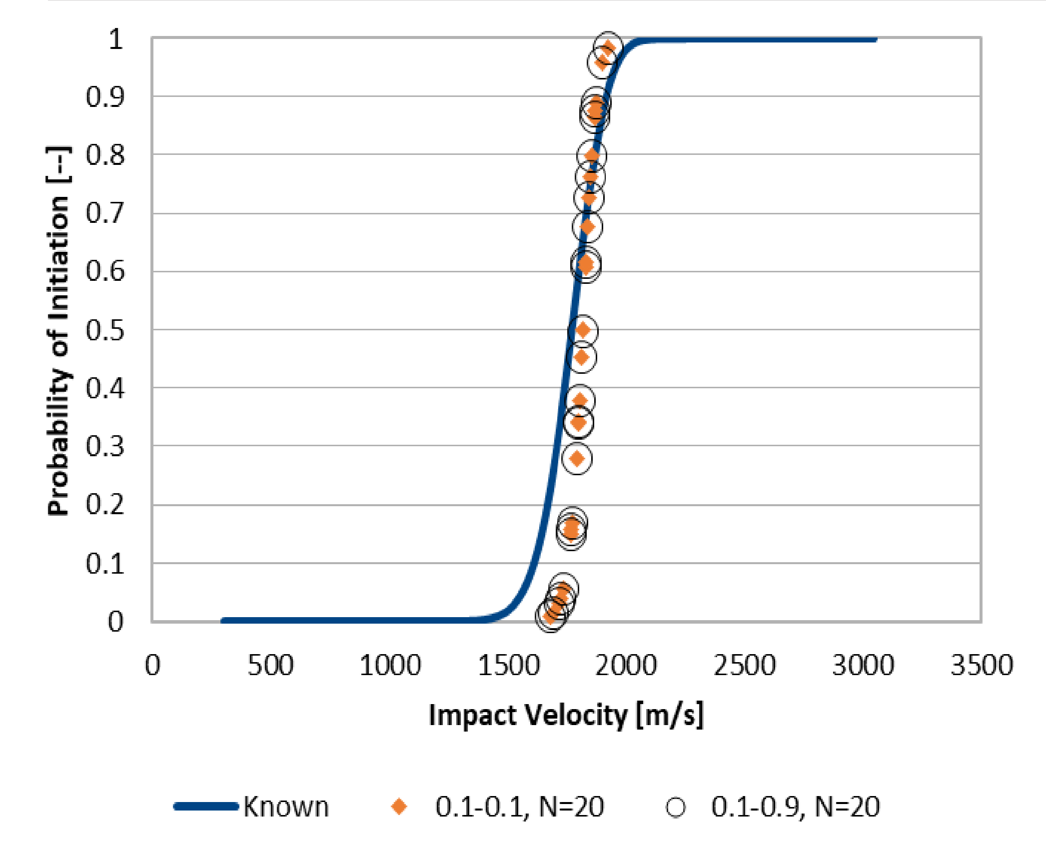
- Set of 25 'observations' generated
  - Randomized impact velocity
  - J-values from known critical parameters used for probability density
  - BLR models fit to increments of 5 data points for each set of arbitrary parameters
    - N = 5, 10, 15, 20, 25

Impact Velocity m/s	J (KNOWN) --	Z --	P --	Random Number --	Result
1798	1.167	1.111	0.867	0.629	1
1719	1.075	0.501	0.692	0.836	0
1924	1.137	0.912	0.819	0.023	1
1873	1.019	0.128	0.551	0.313	1
1870	1.108	0.717	0.763	0.939	0
1768	1.095	0.632	0.736	0.584	1
1867	1.001	0.007	0.503	0.637	0
1798	0.947	-0.356	0.361	0.679	0
1693	0.996	-0.027	0.489	0.022	1
1829	1.035	0.233	0.592	0.184	1
1725	1.025	0.164	0.565	0.840	0
1837	1.200	1.332	0.909	0.119	1
1737	1.096	0.641	0.739	0.475	1
1816	0.966	-0.226	0.410	0.753	0
1683	1.057	0.381	0.648	0.251	1
1772	1.031	0.204	0.581	0.889	0
1901	1.144	0.961	0.832	0.442	1
1829	1.166	1.107	0.866	0.110	1
1848	1.124	0.828	0.796	0.122	1
1854	1.094	0.625	0.734	0.455	1
1803	1.030	0.201	0.580	0.964	0
1790	1.025	0.168	0.567	0.502	1
1843	1.153	1.018	0.846	0.923	0
1811	0.978	-0.144	0.443	0.296	1
1770	1.096	0.643	0.740	0.267	1

BLR Model Fit

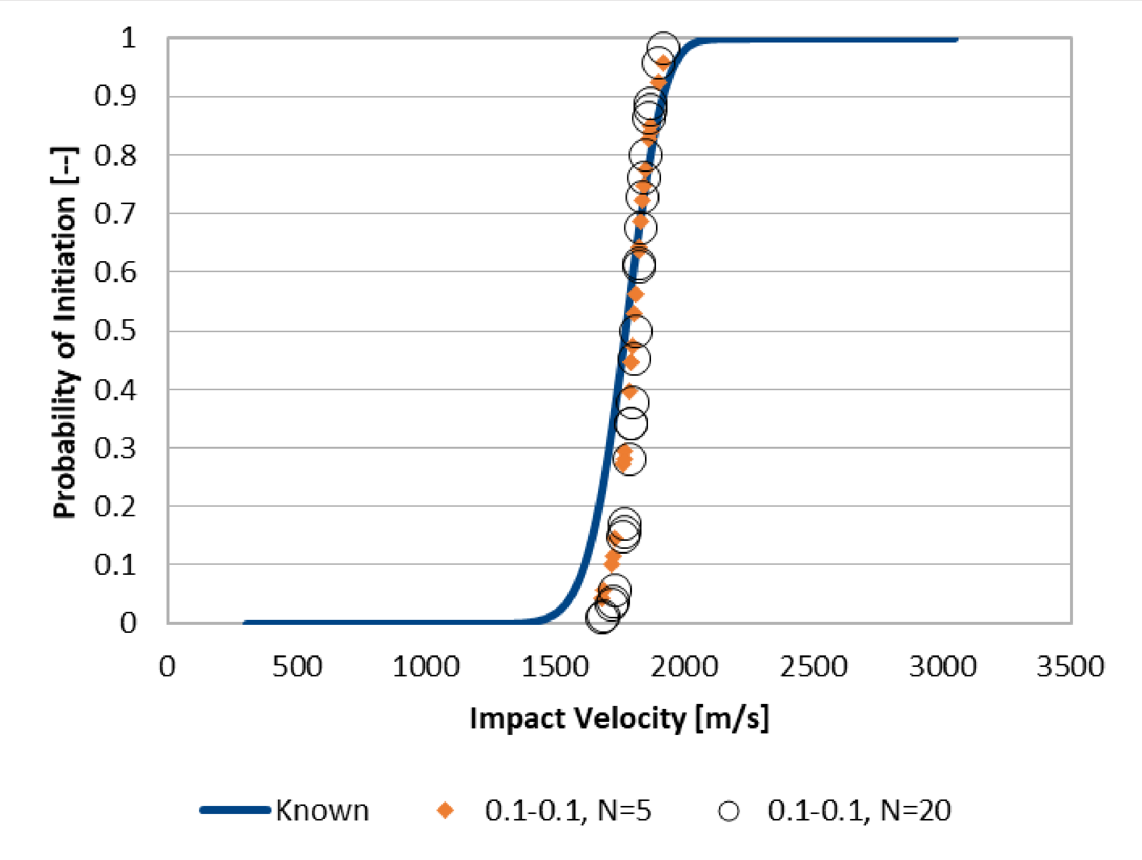
- The BLR model predictions remain consistent regardless of critical parameter values used.





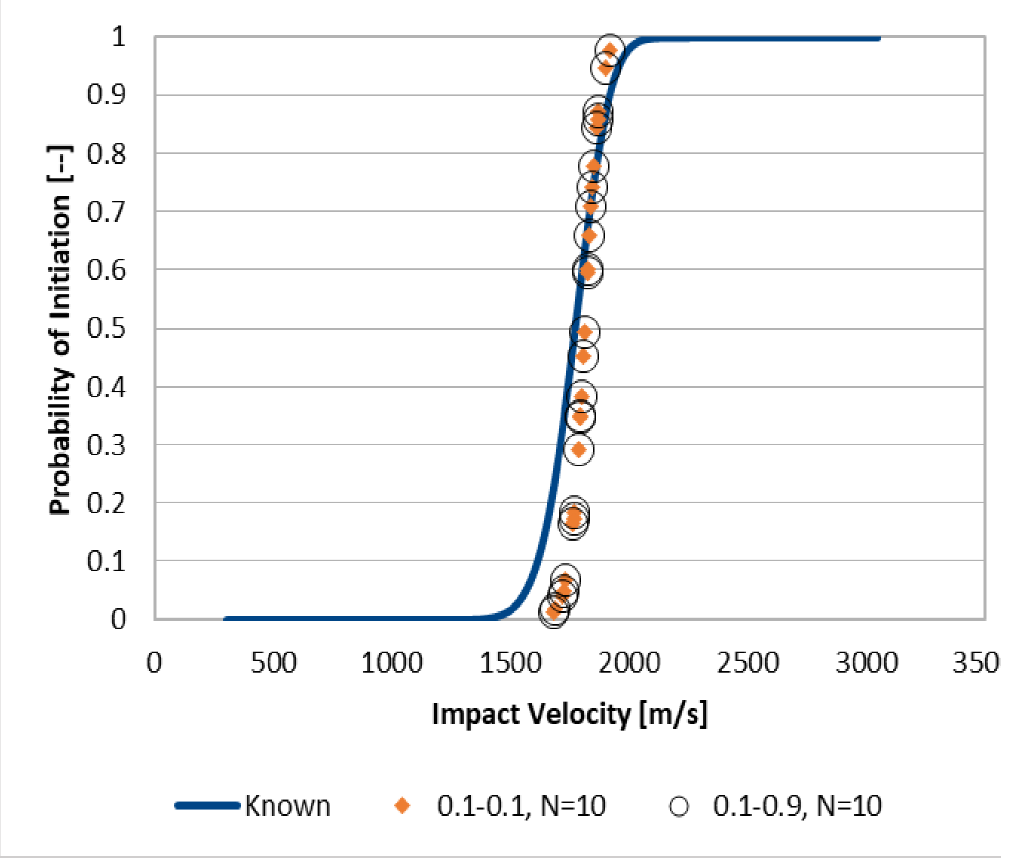
Sample Size Dependence

- Some sensitivity to sample size, but generally consistent with N=5 through N=20
  - Provided that all observations are in the vicinity of initiation threshold
- Caveat: The BLR model fit requires overlap in the observations, ie sub-threshold initiation and supra-threshold non-initiation.



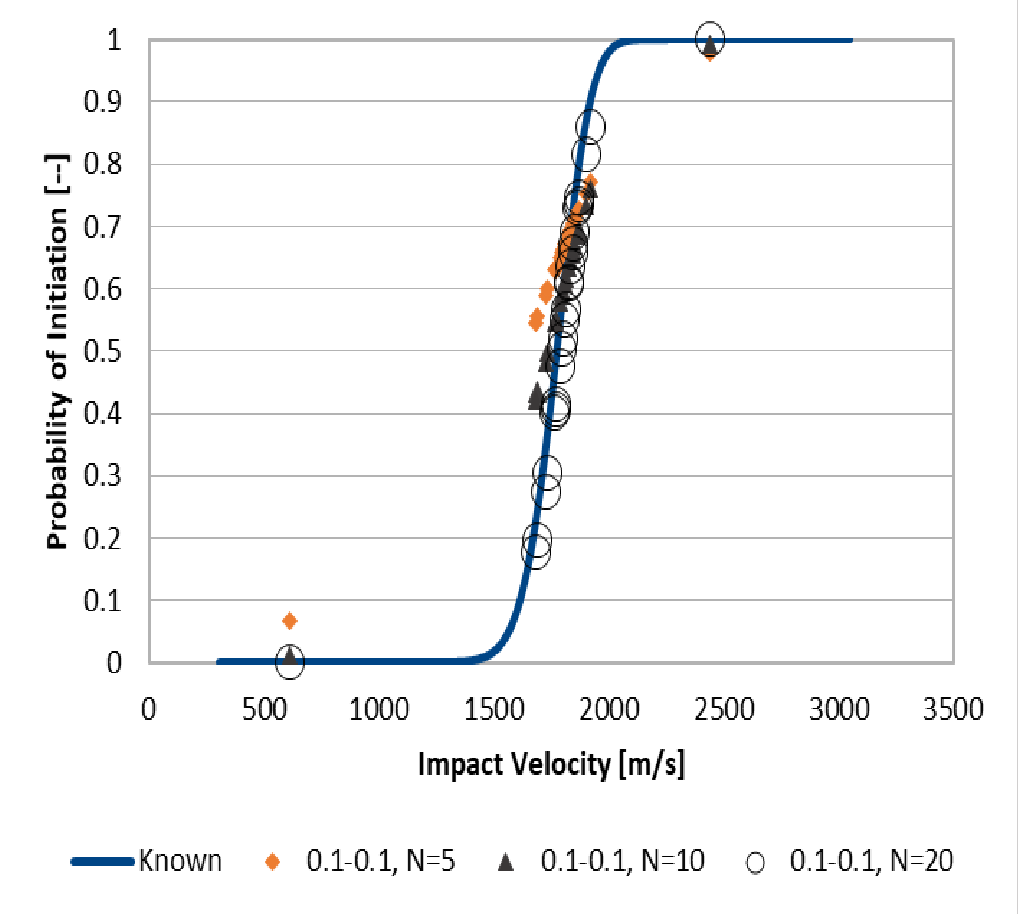
Improved BLR Model Fit – Anchor Points

- The BLR model correctly categorized ~76% of observations.
  - Deviation mostly limited to the high and low impact velocities.
- Observations were purposely generated to be near the initiation threshold.
- Predictive capability could be improved by adding ‘Anchor Points’ at velocity extremes.



Improved BLR Model Fit – Anchor Points

- First two observations replaced with anchor points
  - 500 m/s – Non-initation
  - 2500 m/s – Initiation
- Ordered observations skew the sample size for smaller sets (N=5,10 diverge more)
  - N=5,   72% categorized correctly
  - N=10, 88% categorized correctly
  - N=20, 96% categorized correctly
- Approaches the 'True' probability curve



Conclusions

- This technique is effective as an estimator of initiation threshold in the absence of well-characterized Hugh James parameters, given that some test data is available.
  - Generally small sample sizes produce very reasonable estimates of initiation threshold, provided that they are all near-margin (76%, N=5)
  - Much improved accuracy is possible by providing anchor points, but larger sample sizes are necessary (72%, N=5 -> 96%,N=20)
- Critical parameters and associated standard deviation in J could potentially be backed-out of this analysis.
  - Complicated by numeric optimization of LL function in regression analysis.

Questions?