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UNCERTAINTY MODELING ENHANCEMENT CONCEPTS IN QRA METHODOLOGY

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Background

EXPLOSIVE RISK METHODOLOGY

- The DDESB has developed and continuously improved its approved methodology for estimating explosives risks over a 20-year period.
  - This methodology is described in detail in DDESB Technical Paper 14 (TP-14).
  - Current edition is Revision 4a; Revision 5 is currently being drafted
- In 2002-2003, it was recognized that a state-of-the-art tool must incorporate the uncertainties involved in estimating risk.
- Various reviews during the last 15 years have suggested potential changes to the risk model:
  - Using different statistical distributions to model individual sub-elements of risk (currently all submodels are represented by lognormal distributions).
  - Shifting sub-model point estimates to the mean of each element distribution (currently point estimates are treated as the distribution median).
- In 2017, the Risk Assessment Program Team completed a task to confirm that the Analytical methodology currently in use could successfully incorporate these two changes. This paper reports the findings of this study.

Risk Methodology Background

ANALYTICAL MODEL (REF. 1)

$E_{ep}(EF) = E_{ep}(\Delta t)*E_{ep}(S)*E_{ep}[E(\lambda*E)]*E_{ep}[E(p_{f/e})]$

$= \Delta t_o*\exp(0.5\sigma^2_{\Delta t})*S_o*\exp(0.5\sigma^2_S)*E_{ep}[E(\lambda*E)]*E_{ep}[E(p_{f/e})]$

Where:

$E_{ep}(\Delta t) = \Delta t_o*\exp(0.5\sigma^2_{\Delta t})$

$E_{ep}(S) = S_o*\exp(0.5\sigma^2_S)$

The Analytical Model computes risk as the product of elemental sub-models addressing the annual Probability of Event, Scaling Factors addressing handling conditions, the Probability of Fatality given an event, and the Exposure of personnel to the explosives hazard.

$E_{ep}[E(\lambda*E)] = \lambda_{oo}*E_{oo}*\exp[0.5(\sigma^2_{\lambda o} + \sigma^2_{Eo} + \sigma^2_{NEW1} + (r+1)^2*\sigma^2_e + 2(r+1)*\rho_{Ne}*\sigma_{NEW1}*\sigma_e)]$

$E_{ep}[E(p_{f/e})] = \Sigma_k p_{f/k,oo} *\exp[0.5(\sigma^2_{k1}+\sigma^2_{ko1})] - \Sigma_{i\neq k} p_{f/i,oo} *p_{f/k,oo} *\exp[0.5(\sigma^2_{ik}+\sigma^2_{iko})] + \Sigma_{i\neq j\neq k}\, p_{f/i,oo} *p_{f/j,oo} *p_{f/k,oo} *\exp[0.5(\sigma^2_{ijk} + \sigma^2_{ijko})] – (\Pi_k p_{f/k,oo})*\exp[0.5(\sigma^2_{1234} + \sigma^2_{1234o})]$

And variances are respectively:

$\sigma^2_{k1} = \sigma^2_k + \sigma^2_y + \sigma^2_{NEW2}$

$\sigma^2_{ko1} = \sigma^2_{ko} + \sigma^2_{yo}$

$\sigma^2_{ik} = \sigma^2_i + \sigma^2_k + 2(\sigma^2_y + \sigma^2_{NEW2})$

$\sigma^2_{iko} = \sigma^2_{io} + \sigma^2_{ko} + 2\sigma^2_{yo}$

$\sigma^2_{ijk} = \sigma^2_i + \sigma^2_j + \sigma^2_k + 3(\sigma^2_y + \sigma^2_{NEW2})$

$\sigma^2_{ijko} = \sigma^2_{io} + \sigma^2_{jo} + \sigma^2_{ko} + 3\sigma^2_{yo}$

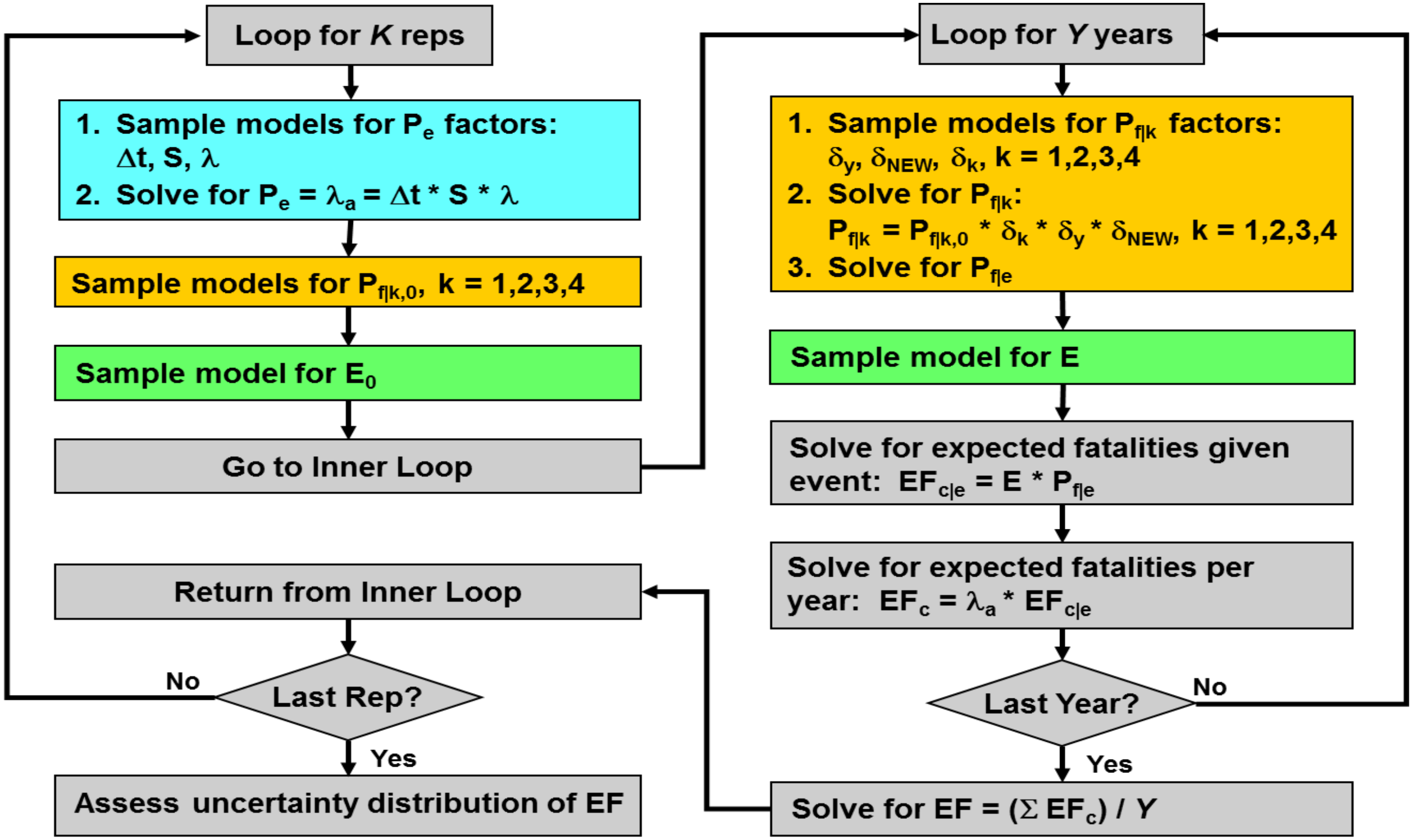
$\sigma^2_{1234} = \Sigma_k\sigma^2_k + 4(\sigma^2_y + \sigma^2_{NEW2})$

$\sigma^2_{1234o} = \Sigma_k\sigma^2_{ko} + 4\sigma^2_{yo}$

- *The DDESB-approved methodology directly calculates the Expected Value of Fatalities (EF) using an analytical approach.*
- *This analytical approach models the resulting risk using a lognormal distribution.*

Risk Methodology Background

MONTE CARLO EXPERIMENT (REF. 2)



Monte Carlo experiments provide another approach to combine the element distributions to estimate EF and the risk distribution.

STANDARD TEST CASES

INPUT VARIABLES		Part 1: Original Values - Point Estimate = Median			
		Case 1	Case 2	Case 3	Case 4
median value of delta t	Δt_o	2.28E-02	2.28E-02	2.28E-01	2.28E-01
std dev of delta t	$\sigma_{\Delta t}$	6.93E-01	2.31E-01	2.23E-01	6.93E-01
median value of scale factor	S_o	3.00E+00	3.00E+00	1.00E+01	3.00E+00
std dev of SF	σ_S	1.70E-01	1.70E-01	7.68E-01	1.70E-01
median value of lambda	λ_o	1.00E-05	3.00E-06	1.00E-04	3.00E-05
std dev of lambda	σ_λ	5.36E-01	3.66E-01	5.36E-01	6.49E-01
Ep Median Daily Exposure	E_{oo}	2.28E-02	2.28E-02	2.28E+01	2.28E+02
Rand Var std dev Exposure	σ_e	9.99E-01	2.31E-01	7.68E-01	6.08E-02
Ep std dev of Exposure	σ_{Eo}	4.05E-01	0.00E+00	6.93E-01	0.00E+00
Ep Median Pf|e blast	$P_{f\|1oo}$	2.96E-09	2.96E-09	2.96E-09	2.96E-09
Ep Median Pf|e bldg collapse	$P_{f\|2oo}$	2.35E-10	1.00E-15	3.66E-10	4.87E-02
Ep Median Pf|e debris	$P_{f\|3oo}$	1.75E-08	2.00E-10	3.35E-03	1.06E-06
Ep Median Pf|e glass	$P_{f\|4oo}$	1.00E-10	1.00E-15	1.00E-10	1.00E-15
Std Dev Pf|e due to Yield	σ_y	1.21E+00	0.00E+00	3.15E-02	1.60E-01
Ep std dev Pf|e due to Yield	σ_{yo}	3.66E-01	3.66E-01	3.66E-01	3.66E-01
Std Dev Pf|e due to NEW	σ_{NEW}	9.69E-01	0.00E+00	0.00E+00	1.36E-01
std dev for variation in o/p	σ_1	7.15E-01	7.15E-01	7.15E-01	0.00E+00
std dev for variation in b/c	σ_2	7.06E-01	7.06E-01	7.06E-01	2.55E-01
std dev for variation in debris	σ_3	8.66E-01	8.66E-01	8.66E-01	1.10E-01
std dev for variation in glass	σ_4	8.83E-01	8.83E-01	8.83E-01	0.00E+00
Ep std dev for overpressure	σ_{1o}	3.49E-01	3.49E-01	3.49E-01	3.49E-01
Ep std dev for bldg damage	σ_{2o}	5.71E-01	5.71E-01	5.71E-01	5.71E-01
Ep std dev for debris	σ_{3o}	6.35E-01	6.35E-01	6.35E-01	6.35E-01
Ep std dev for glass	σ_{4o}	8.83E-01	8.83E-01	8.83E-01	8.83E-01

Experimental Parameters	
Outer Loop : K Reps	50,000
Inner Loop: Y Years	50,000
Random Number Seed	35611

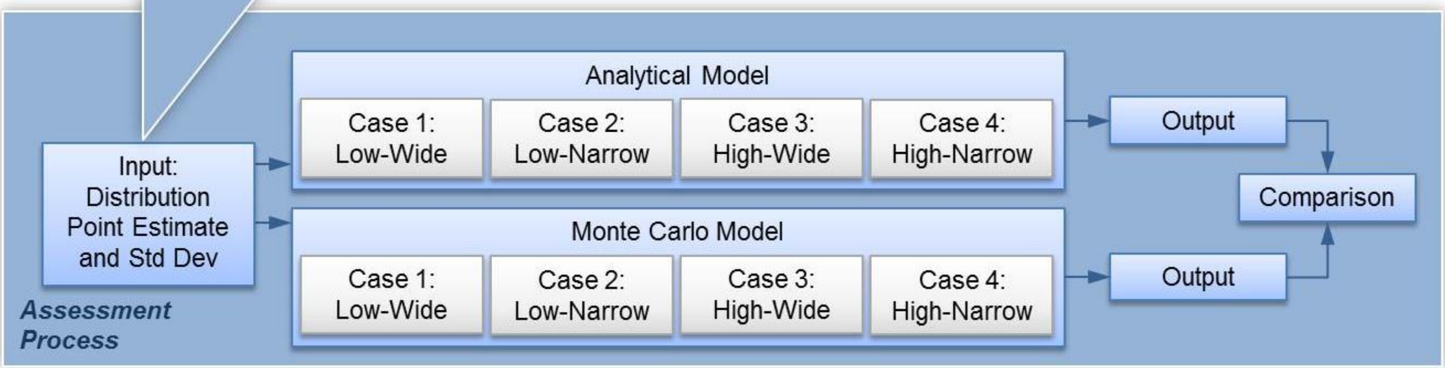
In 2003, four standard test cases were devised to represent the four corners of the envelope:

- Low Risk, Low Uncertainty
- Low Risk, High Uncertainty
- High Risk, Low Uncertainty
- High Risk, High Uncertainty

ANALYSIS PROCESS OVERVIEW

Part	Distribution	Point Estimate	Completion References
1	Lognormal	Median	CE1-17-00300
2	Lognormal	Mean	CE1-17-00300
3a	Triangular	Mode	CE1-17-00301
3b	Normal	Median	CE1-17-00301
3c	Mixed*	Median / Mode	CE1-17-00302
4	Mixed*	Mean / Mode	CE1-17-00302

* Mixed Distribution Selections detailed in Figure 3



The study was divided into four parts to examine the impacts on the model if various combinations of distributions and point estimate usage were implemented.

PART 1: ALL LOGNORMAL DISTRIBUTIONS, POINT ESTIMATES USED AS MEDIAN

Part 1 demonstrated that the Analytical and Monte Carlo tools used to validate original methodology produce the same results as in 2003.

Solution Method	Part 1, Case 1 (Low-Wide)			Part 1, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method (2003)	5.26E-15	9.29E-15	1.83E-14	2.54E-17	1.93E-17	6.14E-17
Analytical Method (2017)	5.26E-15	9.29E-15	1.83E-14	2.54E-17	1.93E-17	6.14E-17
Monte Carlo (2003)	5.28E-15	9.39E-15	1.81E-14	2.55E-17	1.91E-17	6.08E-17
Monte Carlo (2017)	5.28E-15	9.39E-15	1.81E-14	2.55E-17	1.91E-17	6.08E-17

Solution Method	Part 1, Case 3 (High-Wide)			Part 1, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method (2003)	9.00E-05	2.20E-04	3.37E-04	4.85E-04	8.42E-04	1.68E-03
Analytical Method (2017)	9.00E-05	2.20E-04	3.37E-04	4.85E-04	8.42E-04	1.68E-03
Monte Carlo (2003)	9.01E-05	2.18E-04	3.38E-04	4.87E-04	8.45E-04	1.68E-03
Monte Carlo (2017)	9.01E-05	2.18E-04	3.38E-04	4.87E-04	8.45E-04	1.68E-03

PART 2: ALL LOGNORMAL DISTRIBUTIONS, POINT ESTIMATES USED AS MEAN

Part 2 repeated a previous study that demonstrated the Analytical uncertainty model is valid when sub-model point estimates are treated as the means of element distributions.

Solution Method	Part 2, Case 1 (Low-Wide)			Part 2, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	3.25E-16	5.67E-16	1.12E-15	1.48E-17	1.13E-17	3.60E-17
Experimental (Monte Carlo)	3.26E-16	5.71E-16	1.13E-15	1.49E-17	1.13E-17	3.58E-17
Δ%	0.31%	0.71%	0.89%	0.68%	0.00%	0.56%

Solution Method	Part 2, Case 3 (High-Wide)			Part 2, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	1.74E-05	4.27E-05	6.54E-05	2.29E-04	3.97E-04	7.93E-04
Experimental (Monte Carlo)	1.74E-05	4.23E-05	6.54E-05	2.30E-04	3.98E-04	7.90E-04
Δ%	0.00%	0.94%	0.00%	0.44%	0.25%	0.38%

COMPARISON – PARTS 1 AND 2

Part 2 also confirmed previous results demonstrating that the risk estimate is reduced when Point Estimates are applied as the Mean of elemental distributions rather than as the Median.

Analytical Solution Method	Case 1 (Low-Wide)			Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Part 1: Point Estimate = Median	5.26E-15	9.29E-15	1.83E-14	2.54E-17	1.93E-17	6.14E-17
Part 2: Point Estimate = Mean	3.25E-16	5.67E-16	1.12E-15	1.48E-17	1.13E-17	3.60E-17
% Reduction	93.82%	93.90%	93.88%	41.73%	41.45%	41.37%

Analytical Solution Method	Case 3 (High-Wide)			Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Part 1: Point Estimate = Median	9.00E-05	2.20E-04	3.37E-04	4.85E-04	8.42E-04	1.68E-03
Part 2: Point Estimate = Mean	1.74E-05	4.27E-05	6.54E-05	2.29E-04	3.97E-04	7.93E-04
% Reduction	80.67%	80.59%	80.59%	52.78%	52.85%	52.80%

PART 3A: ALL TRIANGULAR DISTRIBUTIONS, POINT ESTIMATES USED AS MEDIAN

- Part 3a explored the validity of using non-lognormal element distributions. In 3a, each sub-model distribution was represented by a triangular distribution.
- Results show excellent agreement in modeled Expected Value and comparable values for Standard Deviation and 95th percentiles.

Solution Method	Part 3a, Case 1 (Low-Wide)			Part 3a, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	6.00E-11	6.46E-11	1.73E-10	2.29E-15	2.16E-15	6.19E-15
Experimental (Monte Carlo)	5.96E-11	7.08E-11	1.99E-10	2.28E-15	2.44E-15	7.15E-15
Δ%	0.72%	9.58%	14.85%	0.43%	13.20%	15.47%

Solution Method	Part 3a, Case 3 (High-Wide)			Part 3a, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	1.01E-02	1.13E-02	2.97E-02	7.42E-04	7.48E-04	2.07E-03
Experimental (Monte Carlo)	1.01E-02	1.29E-02	3.41E-02	7.37E-04	9.31E-04	2.58E-03
Δ%	0.74%	13.72%	14.93%	0.72%	24.41%	24.46%

PART 3B: ALL NORMAL DISTRIBUTIONS, POINT ESTIMATES USED AS MEDIAN

- Part 3b explored the validity of using another non-lognormal distribution. In 3b, each sub-model distribution was represented by a normal distribution.
- Results again show excellent agreement in modeled Expected Value and comparable values for Standard Deviation and 95th percentiles.

Solution Method	Part 3b, Case 1 (Low-Wide)			Part 3b, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	3.25E-16	1.21E-16	5.51E-16	1.48E-17	5.61E-18	2.53E-17
Experimental (Monte Carlo)	3.25E-16	1.22E-16	5.47E-16	1.48E-17	5.00E-18	2.40E-17
Δ%	0.10%	0.58%	0.64%	0.31%	10.81%	5.23%

Solution Method	Part 3b, Case 3 (High-Wide)			Part 3b, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	1.75E-05	6.69E-06	3.00E-05	2.28E-04	8.76E-05	3.92E-04
Experimental (Monte Carlo)	1.75E-05	6.02E-06	2.85E-05	2.29E-04	7.84E-05	3.72E-04
Δ%	0.18%	10.00%	5.07%	0.45%	10.46%	5.09%

MIXED DISTRIBUTIONS – PARTS 3C AND 4

Input Distribution	Variable	Normal	Lognormal	Triangular
Delta t	Median of delta t	X		
	Std dev of delta t			
Scale Factor	Median of Scale Factor			X
	Std dev of Scale Factor			
Lambda	Median of lambda		X	
	Std dev of lambda			
Daily Exposure	Ep Median Daily Exposure	X		
	Ep std dev of Exposure			
	Rand Var std dev Exposure			X
Blast	Ep Median Pf|e blast		X	
	Ep std dev for blast			
	Std dev for variation in blast	X		
Building Collapse	Ep Median Pf|e bldg collapse		X	
	Ep std dev for bldg collapse			
	Std dev for variation in bldg collapse	X		
Debris	Ep Median Pf|e debris		X	
	Ep std dev for debris			
	Std dev for variation in debris	X		
Glass	Ep Median Pf|e glass		X	
	Ep std dev for glass			
	Std dev for variation in glass	X		
Yield	Ep std dev Pf|e due to Yield		X	
	Std dev Pf|e due to Yield	X		
NEW	St dev Pfe due to NEW		X	

A mixture of Normal, Lognormal, and Triangular distributions was used for study Parts 3c and 4.

PART 3C: MIXTURE OF DISTRIBUTIONS, POINT ESTIMATES USED AS MEDIAN

- Part 3c explored the validity of using a mixture of elemental distributions. In Part 3c, element models used lognormal, normal, or triangular distributions.
- The Analytical and Monte Carlo approaches produced comparable values for Expected Value, Standard Deviation, and 95th percentiles.

Solution Method	Part 3c, Case 1 (Low-Wide)			Part 3c, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	5.51E-15	4.57E-15	1.39E-14	1.30E-16	8.35E-17	2.88E-16
Experimental (Monte Carlo)	5.80E-15	6.30E-15	1.68E-14	1.38E-16	1.02E-16	3.30E-16
Δ%	5.27%	37.78%	20.57%	5.90%	22.49%	14.48%

Solution Method	Part 3c, Case 3 (High-Wide)			Part 3c, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	3.66E-04	4.87E-04	1.16E-03	3.23E-04	3.70E-04	9.57E-04
Experimental (Monte Carlo)	3.85E-04	5.53E-04	1.30E-03	3.38E-04	4.26E-04	1.04E-03
Δ%	5.26%	13.51%	11.94%	4.63%	15.21%	8.50%

PART 4: MIXTURE OF DISTRIBUTIONS, POINT ESTIMATES USED AS MEAN

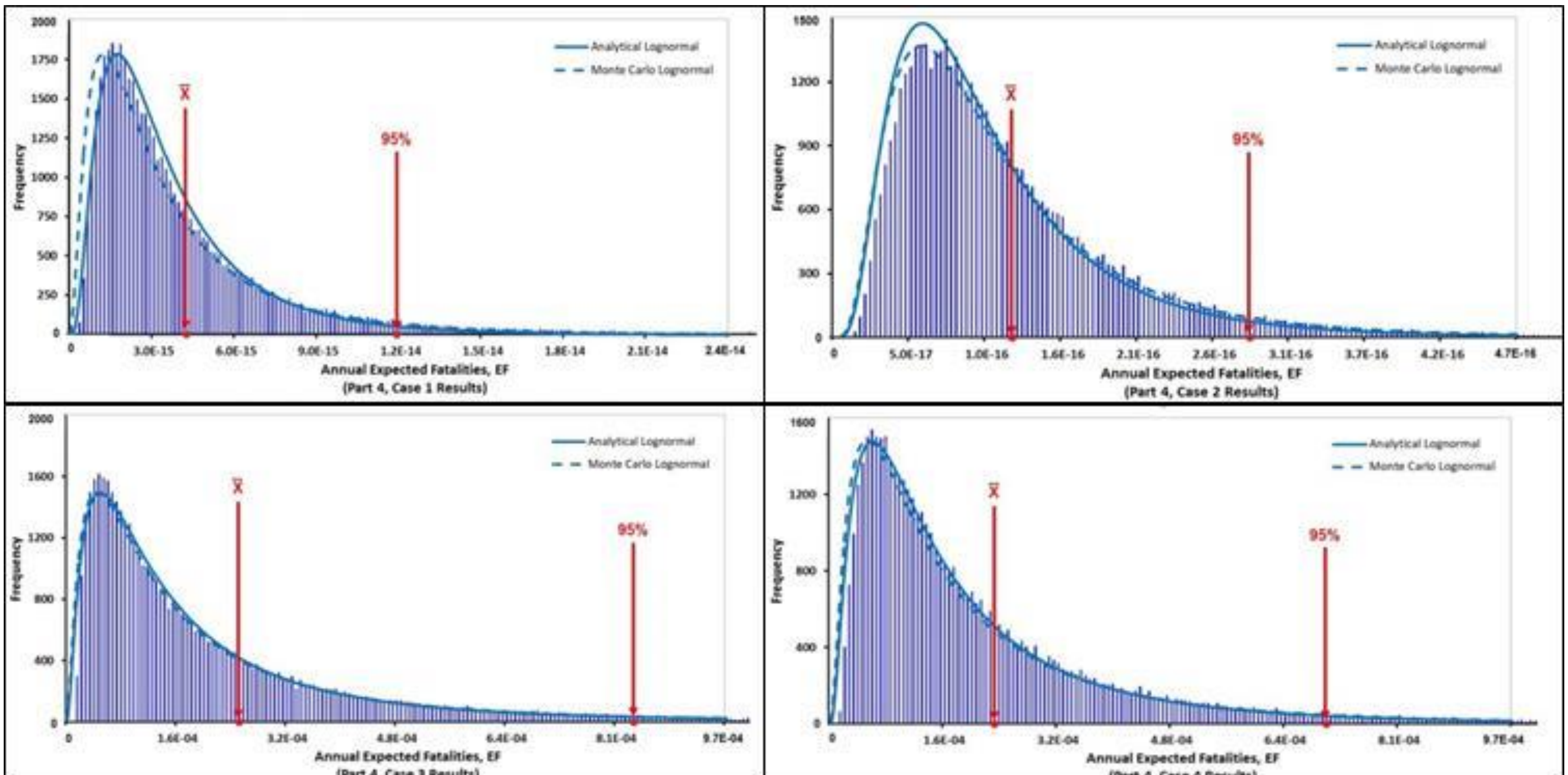
- Part 4 validated that the two changes could be successfully used together. In this part, point estimates were used as means for a mixture of distributions.
- The Analytical and Monte Carlo approaches produced comparable values for Expected Value, Standard Deviation, and 95th percentiles.

Solution Method	Part 4, Case 1 (Low-Wide)			Part 4, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	3.92E-15	3.24E-15	9.90E-15	1.12E-16	7.22E-17	2.49E-16
Experimental (Monte Carlo)	4.13E-15	4.44E-15	1.19E-14	1.19E-16	8.85E-17	2.85E-16
Δ%	5.34%	36.92%	19.78%	5.93%	22.62%	14.37%

Solution Method	Part 4, Case 3 (High-Wide)			Part 4, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	2.35E-04	3.12E-04	7.42E-04	2.19E-04	2.51E-04	6.49E-04
Experimental (Monte Carlo)	2.47E-04	3.54E-04	8.32E-04	2.29E-04	2.89E-04	7.04E-04
Δ%	5.01%	13.50%	12.11%	4.70%	15.22%	8.55%

SHOULD THE RESULTING DISTRIBUTION BE MODELED AS LOGNORMAL?

- The final risk distribution is modeled as lognormal because Risk is the product of the element distributions [Central Limit theory].
- Histograms of the Monte Carlo experiments for the four test cases in Part 4 are comparable to lognormal curves computed using the mean and standard deviation computed by both the Analytical and Monte Carlo approaches.



CONCLUSIONS

1. Use of non-lognormal distributions to represent input variables does not “break” the TP-14 Analytical model of uncertainty which assumes the output risk results in a lognormal distribution.
   - Monte Carlo experiment does not assume a form for the output distribution.
   - Histograms demonstrate that the results follow a lognormal distribution.
2. Assigning elemental model point estimates as the mean rather than the median of the input distributions does not “break” the TP-14 Analytical model of uncertainty.
   - Agreement of Analytical and Monte Carlo results for Part 2 compares well to that of Part 1.
     - Risk estimates are reduced when Point Estimates are applied as the Mean.
   - Agreement between the two methods for Part 4 compares well to that of Part 3c.
   - Part 4 to Part 3c comparison demonstrates that the shift from median to mean does not depend on use of lognormally distributed element variables.
3. Agreement between Analytical and Monte Carlo results for Expected Value was significantly better than that observed for the standard deviation and 95th percentile.
   - This is due to the more straightforward modeling of expected values in the Analytical method. Variance equations are much more extensive, requiring additional study.
4. Use of a mix of distributions, including non-lognormal distributions, to model input variables does not “break” the ability to develop an analytical model to directly calculate parameters of the output risk distribution.
   - Results showed that Analytical results compare well with Monte Carlo experiments using a variety of distribution types.

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