Uncertainty Modeling Enhancement Concepts in Quantitative Risk Assessment Methodology

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Abstract

In modeling future explosives risk, many uncertainties must be considered. Since 2002, the Department of Defense Explosives Safety Board (DDESB) has been developing and refining models incorporating uncertainty estimation into its approved explosives risk methodology. Various reviews of the methodology have identified quantitative risk assessment modeling enhancements with potential to improve the estimation of explosives risk uncertainties. The paper reports the findings of investigations into the feasibility of these enhancements to improve analytical models used to estimate explosives risks and the uncertainties associated with these estimates.

Introduction

Since 1997, the DDESB has developed and continuously improved its approved methodology for estimating the risks posed by assembly and storage of bulk explosives and explosive munitions. This risk-based methodology has been used by the DDESB to approve explosives safety site plans in scenarios that cannot comply with Quantity Distance (QD) requirements.

In 2003, a second-generation risk uncertainty model was developed and integrated into the DDESB methodology. This methodology was reported at the 2004 DDESB Explosive Safety Conference (Ref. 1, 2) and was fully documented in DDESB Technical Paper (TP) 14 (Ref. 3).

During 15 years of use, characterization, evolution, and internal/external reviews, a few features of the original uncertainty methodology have been identified as candidates for improvement. These features include:

- Introducing large uncertainties can push the calculated Expected Fatality (Risk) estimate unreasonably high.
- Modeling as lognormal element distributions that represent probabilities (e.g., P_e, Δt, $P_{f|e}$) produces distributions that extend beyond the acceptable range of [0,1]. (Ref. 4)
- Assigning the point estimates generated by the science algorithms as the medians of elemental distributions is counter to the intent of some algorithm developers. In these cases, the risk estimate is increased without cause.

In 2017, the DDESB sponsored a Risk Assessment Program Team-led effort to explore the viability of changes needed to implement the improvements recommended by various reviews.

This paper reports the approach and findings (Ref. 5) of the recently completed effort.

Background

Current Methodology

The primary function of the risk-based methodology is to estimate the risk associated with use of potentially explosive material at a variety of Department of Defense (DoD)-controlled sites. As described in Ref. 3, the Safety Assessment for Explosives Risk (SAFER) uncertainty model incorporates aleatory uncertainty (random variation inherent in real life events) and epistemic uncertainty (knowledge uncertainty inherent in the model of the world and associated scientific algorithms). The uncertainty model leverages the risk equation in the original methodology to formulate the risk estimator for the expected number of fatalities (F) per year:

$$F = \Delta t * S * \lambda(NEW,E) * P_{f|e}(NEW, yield, effects) * E$$

where the factors are taken to be lognormally distributed random variables addressing various effects on the expected risk:

Δt addresses the fraction of the time that explosives are present that exposures are also present,

λ addresses the probability of an explosive event during a given year based on the type of explosives present and the activity performed at the explosives site,

S incorporates a scaling factor that increases the probability of an event based on extenuating circumstances at the site – such as operations in a remote area or under combat conditions,

$P_{f||e}$ addresses the probability of fatality given an explosive event – this factor aggregates the consequences of four fatality mechanisms: overpressure, debris, building collapse, and glass hazards, and

E addresses the exposure of personnel to an explosive event based on the number of people present in a facility during the year and the number of hours the exposed site is occupied.

Table 1 illustrates the random variables used to calculate risk in the presence of uncertainty in the current methodology. As noted previously, all elemental random variables are modeled as lognormally distributed.

The methodology described in Ref. 1 uses an analytical, equation-based approach to compute the parameters of the Risk (expected fatality) distribution directly from the parameters of the elemental distributions that model the risk equation factors.

Another common method for combining the elemental distributions would be to run a Monte Carlo experiment to compile the resulting Risk distribution from the results of thousands of runs. Ref. 2 describes the use of a Monte Carlo approach to verify that this analytical approach to computing Risk distribution characteristics produces reasonable results.

Table 1. Random Variables in Uncertainty Methodology

Original Input Description	Paper Symbol	Rndm Var Ref.	Original Distribution
Median Value of Delta t	Δt_o	RV1	Lognormal
Std Dev of Delta t	$\sigma_{\Delta t}$		
Median Value of Scale Factor	S_o	RV2	Lognormal
Std Dev of SF	σ_S		
Median Value of Lambda	λ_o	RV3	Lognormal
Std Dev of Lambda	σ_λ		
Ep Median Daily Exposure	E_{oo}	RV4	Lognormal
Rand Var Std Dev Exposure	σ_e		
Ep Std Dev of Exposure	σ_{Eo}	RV5	Lognormal
Ep Median Pf|e Overpressure	$P_{f\|1oo}$	RV6	Lognormal
Ep Std Dev for Overpressure	σ_{1o}		
Std Dev for Variation in O/P	σ_1	RV7	Lognormal
Ep Median Pf|e Bldg Collapse	$P_{f\|2oo}$	RV8	Lognormal
Ep Std Dev for Bldg Damage	σ_{2o}		
Std Dev for Variation in B/C	σ_2	RV9	Lognormal
Ep Median Pf|e Debris	$P_{f\|3oo}$	RV10	Lognormal
Ep Std Dev for Debris	σ_{3o}		
Std Dev for Variation in Debris	σ_3	RV11	Lognormal
Ep Median Pf|e Glass	$P_{f\|4oo}$	RV12	Lognormal
Ep Std Dev for Glass	σ_{4o}		
Std Dev for Variation in Glass	σ_4	RV13	Lognormal
Std Dev Pf|e Due to Yield	σ_y	RV14	Lognormal
Ep Std Dev Pf|e Due to Yield	σ_{yo}	RV15	Lognormal
Std Dev Pf|e due to NEW	σ_{NEW}	RV16	Lognormal

On-going Reviews

At the time the original uncertainty model was incorporated into the TP-14 methodology in 2004, a similar task was undertaken culminating in the paper, *Uncertainty as Modeled in SAFER 3.0* (Ref 2). That task was to implement the original *Analytical* uncertainty model in spreadsheet form and to use it to estimate the risk distributions for four diverse test cases. These results were then compared to the results of a Monte Carlo experiment, which by the nature of the method did not include the assumption that the resulting risk distribution would be lognormal. The four test cases explored the corners of the risk space: low risk with low uncertainty, low risk with high uncertainty, high risk with low uncertainty, and high risk with high uncertainty. These test cases have been retained as the standard cases for testing modifications to the methodology in TP-14.

The original experiment tested whether the resultant risk distribution could be properly assumed to be lognormal when all input distributions are modeled as lognormal. The Analytical results and Monte Carlo experiment results matched within 2% for all risk distribution parameters across the four test cases. This was taken as adequate confirmation that it is reasonable to model the risk as a lognormal distribution when all input parameters are lognormal. In addition, the results of this task confirmed that the TP-14 Analytical model could be used to represent uncertainty in SAFER (version 2.0 through version 3.1).

In 2015, an independent assessment of the SAFER version 3.1 Uncertainty Model was conducted, resulting in the white paper, *Uncertainty Treatment in Risk-Based Explosives Safety Modeling* (Ref 6). This effort provided the technical rationale for future TP-14 uncertainty models to use the mean as the point estimate of the elemental models as opposed to the median.

Since 2003, the science models incorporated in the TP-14 methodology have evolved and improved based on insights gained via continued explosives testing. These insights have renewed questions relating to whether some of the input variables would be better modeled using different statistical distributions.

The current effort uses an approach similar to that used in 2003: Model the inputs, determine an Analytical model to compute the risk distribution parameters, and compare results of the Analytical model to those produced by a related Monte Carlo experiment. For the current task, correlations in the test cases were not used, consistent with the original study, as detailed in *Uncertainty as Modeled in SAFER 3.0* (Ref 1).

Current Effort

Overview

The 2017 Uncertainty Task endeavored to answer two important questions regarding the feasibility of two important changes under consideration to improve the modeling of uncertainty in the approved methodology:

- Can the lognormal distributions used to model elemental random variables be replaced with statistical distributions that more accurately represent each variable without invalidating the closed-form, analytical model?
- Can these more accurate distributions be combined with an approach that assigns the point estimates from science algorithms as Expected Values rather than Medians?

To investigate these questions, the task was divided into four major parts that incrementally make desired changes individually before combining them. Figure 1 provides a graphical overview of the process and general differences between each part.

Part	Distribution	Point Estimate	Completion References
1	Lognormal	Median	CE1-17-00300
2	Lognormal	Mean	CE1-17-00300
3a	Triangular	Mode	CE1-17-00301
3b	Normal	Median	CE1-17-00301
3c	Mixed*	Median / Mode	CE1-17-00302
4	Mixed*	Mean / Mode	CE1-17-00302

* Mixed Distribution Selections detailed in Figure 3

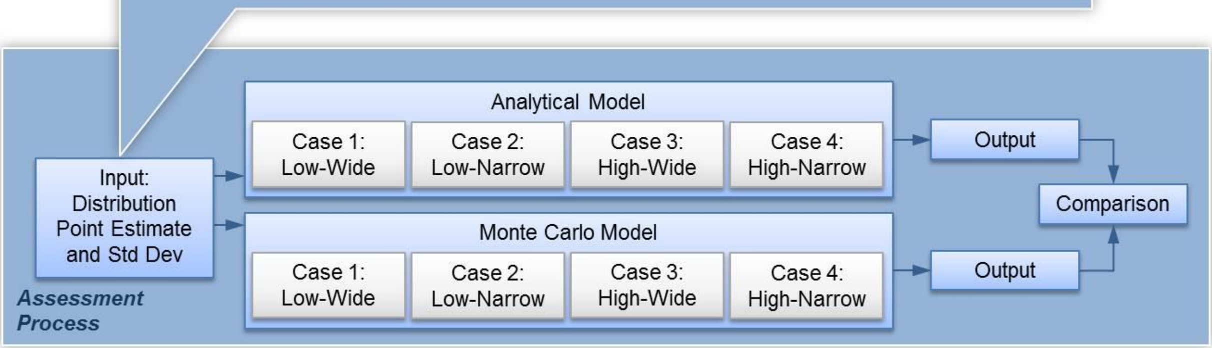

Figure 1. Analysis Process Overview

Part 1 was to access the tools developed during the 2003 uncertainty task, re-run the four test cases (Low-Wide, Low-Narrow, High-Wide, and High-Narrow), and verify the Analytical and Monte-Carlo toolsets produced the same results as those observed in 2003. This is best described as anchoring the analysis to a previous effort the community has recognized as successful.

Part 2 of this study replicated Part 1. However, the input parameters were changed so that the point estimate of the elemental models taken was used as the mean of the distribution as opposed to the median. This is consistent with the Risk-Based Explosives Safety Criteria Team (RBESCT) decision to use the mean as the point estimate in the next version of TP-14, which will be released as Revision 5. The same four test cases were run, with results compared to the Part 1 results. This part assessed whether the current analytical model would produce similar results as the Monte Carlo should the point estimate of the elemental models be used as the mean rather than the median. Analytical results were compared to Monte Carlo results.

Part 3 was subdivided into three subtasks. Part 3a was designed to closely mimic Part 1, however skewed, triangular distributions were used for each elemental distribution rather than lognormal. Part 3b was also designed to closely mimic Part 1, however the input distributions were to be symmetric (normal) as opposed to lognormal. Part 3c was designed to mimic Part 1, however a mixture of input distributions was used (triangular, normal, lognormal). Part 3 was intended to assess whether input distributions other than lognormal could be used in the Analytical model; this was to be accomplished by checking the Analytical model results against Monte Carlo model results. It should be noted that the distributions and associated parameters were not chosen to realistically model real-world phenomena. Instead, these distributions were chosen to test dependencies on symmetry or the exclusive use of lognormal distributions.

Part 4 was a single trial combining the efforts of Part 2 and Part 3c. Part 4 included the same mixed input distributions as Part 3c, however the point estimate of the elemental models was used as the mean, where possible, similar to Part 2. Part 4 assessed whether combining both of those changes in the Analytical model would produce comparable results using the Monte Carlo methodology. The sections that follow report the methodology and results for these parts.

Part 1

Part 1 of this uncertainty task was to access the tools developed during the 2003 uncertainty task, re-run the four test cases (Low-Wide, Low-Narrow, High-Wide, and High-Narrow), and verify the Analytical and Monte Carlo toolsets produced the same results as those observed in 2003. This could best be described as anchoring the analysis to a previous effort that has been recognized as successful within the community.

For the Part 1 Analytical study, the original spreadsheet model from 2003 was retrieved. Inputs were verified to match the inputs used for the Monte Carlo model. It should be noted that the uncertainty model was changed after the 2003 study to correct an error in the original Mensing paper (Ref 3). The coefficients used in computing σ values for higher order terms of $P_{f|e}$ were corrected to be the order of the term, rather than the order squared. The spreadsheet was modified to reflect this change and inputs for the four test cases were re-entered. Results were compiled and observed to match the 2003 results to the third significant digit.

For the Part 1 Monte Carlo study, the original model was retrieved. Figure 2 depicts the Monte Carlo risk model process. This model uses a two-loop approach to partition epistemic and aleatory uncertainty as described in the Mensing paper (Ref 3). For each replication of the Monte Carlo model, the outer loop is run. During a run of the outer loop, variables associated with epistemic uncertainty are sampled and then passed to the inner loop. The inner loop uses variables sampled in the outer loop, samples variables associated with aleatory uncertainty, and then determines the number of expected fatalities for a given year. For each run of the outer loop, the inner loop is run for Y years. After the inner loop is run for Y years, the mean expected annual fatalities, EF, is calculated. An observation of EF is made for each replication, K, of the Monte Carlo model. Experimental parameters used in Part 1 and throughout this study are the same as those used in the 2003 study. A separate experiment is conducted for each of the four cases where the Monte Carlo model is run for K = 50,000 replications (the outer loop is run for 50,000 replications). For each run of the outer loop, the inner loop is run for Y = 50,000 years. For each case, this experiment produces 50,000 observations of EF. The same random number seed used in the 2003 study was used in this study.

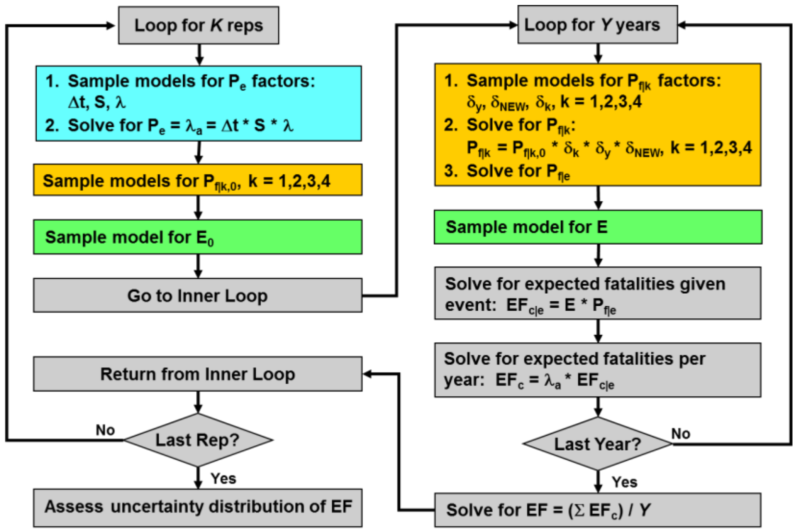

Figure 2. Monte Carlo Risk Model

In the Part 1 Monte Carlo experiment, the four original test cases were run using the same input variables and experimental parameters used in the 2003 study. Results of EF were identical to those obtained by the Monte Carlo experiment conducted in the 2003 study.

For all four test cases, the Analytical and Monte Carlo models produced the same results in 2017 for estimated risk (expected annual fatalities) and epistemic uncertainty (standard deviation and 95th percentile) as observed during the 2003 uncertainty task. Table 2 shows the results of the models that were used for this comparison.

Table 2. Part 1 Results Comparison

Solution Method	Part 1, Case 1 (Low-Wide)			Part 1, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method (2003)	5.26E-15	9.29E-15	1.83E-14	2.54E-17	1.93E-17	6.14E-17
Analytical Method (2017)	5.26E-15	9.29E-15	1.83E-14	2.54E-17	1.93E-17	6.14E-17
Monte Carlo (2003)	5.28E-15	9.39E-15	1.81E-14	2.55E-17	1.91E-17	6.08E-17
Monte Carlo (2017)	5.28E-15	9.39E-15	1.81E-14	2.55E-17	1.91E-17	6.08E-17

Solution Method	Part 1, Case 3 (High-Wide)			Part 1, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method (2003)	9.00E-05	2.20E-04	3.37E-04	4.85E-04	8.42E-04	1.68E-03
Analytical Method (2017)	9.00E-05	2.20E-04	3.37E-04	4.85E-04	8.42E-04	1.68E-03
Monte Carlo (2003)	9.01E-05	2.18E-04	3.38E-04	4.87E-04	8.45E-04	1.68E-03
Monte Carlo (2017)	9.01E-05	2.18E-04	3.38E-04	4.87E-04	8.45E-04	1.68E-03

The original tools were successfully located, updated, and utilized to match the results produced in 2003. The successful completion of Part 1 adequately anchored the current study to previously accepted analysis. The primary result of Part 1 is demonstrating that the Analytical approach and Monte Carlo experiment match when all inputs and the output risk distribution are assumed to be lognormal and the point estimates are set to the median of the elemental distributions.

Part 2

This portion replicated Part 1 with one modification: the input parameters were changed so that the point estimates of the elemental models were set to the mean of the distribution. The same four test cases were run for Part 2. The goals of Part 2 were to assess whether the change from median to mean for the point estimate would impact the agreement observed between the Analytical approach and the Monte Carlo experiment while still using lognormal distributions.

The objective of Part 2 was to assess whether the current analytical model would produce similar results as the Monte Carlo should the point estimate be used as the mean rather than the median. Given the same inputs and running the same four test cases (Low-Wide, Low-Narrow, High-Wide, and High-Narrow), the goal was to compare Analytical and Monte Carlo model results.

Part 2 model inputs for both the Analytical and Monte Carlo approaches needed to be derived as a function of changing the point estimate for the elemental models from the median to the mean. This required a series of calculations to determine the base input set for both models. To calculate the new input distributions, the point estimates from TP-14 Rev 4a (Ref 4) science modules were treated as the mean of each input distribution. Then, for each input distribution (still lognormal distributions, as in Part 1), the point estimate (assigned as the mean) was used to calculate the median using the following equation:

$$Median = Mean \;/\; e^{\frac{1}{2}\sigma^2}$$

where σ includes all the uncertainty contributions for that lognormal distribution (Ref 3). This approach allowed the calculation of adjusted input sets for each of the four test cases. The medians were decreased by the shift of the mean using a scale of $1/e^{1/2\ \sigma\wedge 2}$ (it should be noted that this shift was not applied to the “delta distributions” that modify original distributions with the variation due to epistemic uncertainty). All σ values remained unchanged as they were calculated using the point estimate and an upper bound or upper-bound multiplier, as described in TP-14 Rev 4a (Ref 4). Part 2 model inputs for both the Analytical and Monte Carlo approaches were developed for all four test cases.

The execution of the Monte Carlo experiment was the same as that used for the Part 1 Monte Carlo methodology. The results were compared to those from the Analytical model to verify that the two methodologies would produce similar results. The Part 2 Monte Carlo results match closely with the Part 2 Analytical results, as shown in Table 3.

Table 3. Part 2 Results Comparison

Solution Method	Part 2, Case 1 (Low-Wide)			Part 2, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	3.25E-16	5.67E-16	1.12E-15	1.48E-17	1.13E-17	3.60E-17
Experimental (Monte Carlo)	3.26E-16	5.71E-16	1.13E-15	1.49E-17	1.13E-17	3.58E-17
Δ%	0.31%	0.71%	0.89%	0.68%	0.00%	0.56%

Solution Method	Part 2, Case 3 (High-Wide)			Part 2, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	1.74E-05	4.27E-05	6.54E-05	2.29E-04	3.97E-04	7.93E-04
Experimental (Monte Carlo)	1.74E-05	4.23E-05	6.54E-05	2.30E-04	3.98E-04	7.90E-04
Δ%	0.00%	0.94%	0.00%	0.44%	0.25%	0.38%

Table 4 presents a comparison of the differences in Analytical and Monte Carlo results in Parts 1 and 2. This demonstrates the two solution methods produce similar results regardless of the point estimate assignment (median vs. mean).

Table 4. Part 1 and 2 Analytical Δ% Comparison

Study Part	Case 1 (Low-Wide)			Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Part 1: Δ% Analytical-Experimental	0.38%	1.08%	1.09%	0.39%	1.04%	0.98%
Part 2: Δ% Analytical-Experimental	0.31%	0.71%	0.89%	0.68%	0.00%	0.56%

Study Part	Case 3 (High-Wide)			Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Part 1: Δ% Analytical-Experimental	0.11%	0.91%	0.30%	0.41%	0.36%	0.00%
Part 2: Δ% Analytical-Experimental	0.00%	0.94%	0.00%	0.44%	0.25%	0.38%

The Part 2 analytical results demonstrate a reduction in all values from the Part 1 results as expected, consistent with the findings of *Uncertainty Treatment in Risk-Based Explosives Safety Modeling* (Ref 2). This is depicted in Table 5, where the reduction varies from 41.37% to 93.90% for the four selected test cases.

Table 5. Part 1 and 2 Analytical Results Comparison

Analytical Solution Method	Case 1 (Low-Wide)			Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Part 1: Point Estimate = Median	5.26E-15	9.29E-15	1.83E-14	2.54E-17	1.93E-17	6.14E-17
Part 2: Point Estimate = Mean	3.25E-16	5.67E-16	1.12E-15	1.48E-17	1.13E-17	3.60E-17
% Reduction	93.82%	93.90%	93.88%	41.73%	41.45%	41.37%

Analytical Solution Method	Case 3 (High-Wide)			Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Part 1: Point Estimate = Median	9.00E-05	2.20E-04	3.37E-04	4.85E-04	8.42E-04	1.68E-03
Part 2: Point Estimate = Mean	1.74E-05	4.27E-05	6.54E-05	2.29E-04	3.97E-04	7.93E-04
% Reduction	80.67%	80.59%	80.59%	52.78%	52.85%	52.80%

In all test cases, estimates of risk and associated uncertainty produced by the Analytical and Monte Carlo models agreed within 1% difference, as shown in Table 6. These results indicate the Analytical and Monte Carlo models produce very similar estimates of risk and associated uncertainty. As expected, for all test cases, estimates of risk and associated uncertainty were lower when using the point estimate of the elemental models as the mean (Part 2) versus using the point estimate as the median (Part 1), as shown in Table 7. These results confirm that a point estimate set as the mean will consistently lower risk distribution parameters.

Part 3

Part 3 investigated the replacement of lognormal elemental distributions with a variety of alternative distributions. Part 3a of this uncertainty task was to replicate Part 1 using triangularly distributed input parameters. The same four test cases were run for Part 3a using triangular input distributions developed for this part.

The goal of Part 3a was to investigate the effect non-lognormal, skewed uncertainty distributions would have on the predicted risk distributions. Given the same inputs in the Analytical and Monte Carlo models, and running the same four test cases (Low-Wide, Low-Narrow, High-Wide, and High-Narrow), the goal was to compare the results of both approaches when using triangular uncertainty distributions for the input parameters.

For Part 3a, non-symmetric, triangular input distributions were developed to replace the lognormal inputs used in Parts 1 and 2. The input distributions were selected with the intent to approximate the characteristics of the lognormal distribution that each was to replace. The goal was to adhere to the central measure and upper bound of each distribution to the degree possible.

Unlike Parts 1 and 2, both the Analytical and the Monte Carlo models had to be modified to meet the requirements of Part 3a. This is also true of Parts 3b and 3c. The sections that follow describe the changes made and their impact on the comparisons to be performed.

The original Analytical methodology was developed using only lognormal distributions to model the individual input variables. This approach led to a set of complex equations for calculating the output risk distribution expected value and variance that are specific to the assumption that all input variable distributions are lognormal. These equations are presented in Ref 3.

The first challenge in this task was to convert these equations to a simpler form that could be expressed in terms of the expected value and variance of each input variable without assuming that the variable had any specific distribution type. Such a construct could then be used to calculate the Analytical results required by Parts 3a, 3b, 3c, and 4. This process produced equations to calculate the output risk distribution expected value in a straightforward manner.

The equations Mensing used to calculate the risk distribution variance, however, were quite entangled. This is primarily due to the varied application of aleatory and epistemic uncertainties and to the higher order terms required to completely address the effects of multiple fatality mechanisms. Reasonably reformulating these equations into an approach that is independent of distribution type required omission of the higher order $P_{f|e}$ terms and incomplete application of some features of the full Mensing model. These changes could not be fully addressed by the Monte Carlo formulation, as it computes only the EF result for each set of draws and the output variance and standard deviation are computed separately from the EF population generated by the Monte Carlo runs. With these simplifications, a set of equations was generated to analytically calculate the expected value and variance of the output risk distribution for each case in Part 3a. In this part, all input variable distributions are taken to be triangular.

It was also necessary to modify the Monte Carlo code in order to run the experiments for Part 3a. For the Part 3a experiment, the draws for each input variable were made using a Triangular Distribution Process Generator to calculate a value drawn from the input triangular distribution. The execution of the Monte Carlo experiment was the same as described in Parts 1 and 2. For each case, this experiment produces 50,000 observations of EF. The switch to triangular distributions also affected the manner in which the aleatory and epistemic uncertainties were combined in the inner loop. Part 3a Analytical and Monte Carlo results are compared in Table 6.

Table 6. Part 3a Analytical and Monte Carlo Results Comparison

Solution Method	Part 3a, Case 1 (Low-Wide)			Part 3a, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	6.00E-11	6.46E-11	1.73E-10	2.29E-15	2.16E-15	6.19E-15
Experimental (Monte Carlo)	5.96E-11	7.08E-11	1.99E-10	2.28E-15	2.44E-15	7.15E-15
Δ%	0.72%	9.58%	14.85%	0.43%	13.20%	15.47%

Solution Method	Part 3a, Case 3 (High-Wide)			Part 3a, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	1.01E-02	1.13E-02	2.97E-02	7.42E-04	7.48E-04	2.07E-03
Experimental (Monte Carlo)	1.01E-02	1.29E-02	3.41E-02	7.37E-04	9.31E-04	2.58E-03
Δ%	0.74%	13.72%	14.93%	0.72%	24.41%	24.46%

Figure 3 presents histograms of the Monte Carlo results. These histograms show the distribution of the 50,000 values of EF generated for each case (these 50,000 values of EF were generated by running the Monte Carlo model for K = 50,000 replications). Lognormal curves using the characteristics of the Analytical and Monte Carlo results from Table 9 (expected value and standard deviation) were overlaid on the histograms for comparison. Bin sizes used in the histograms vary from case to case. In each case, the lognormal plots incorporate both the bin size and the 50,000 observations of EF to determine the frequency expected for each bin. Expected value and 95th percent arrows from the Monte Carlo results are also indicated on each of the histograms.

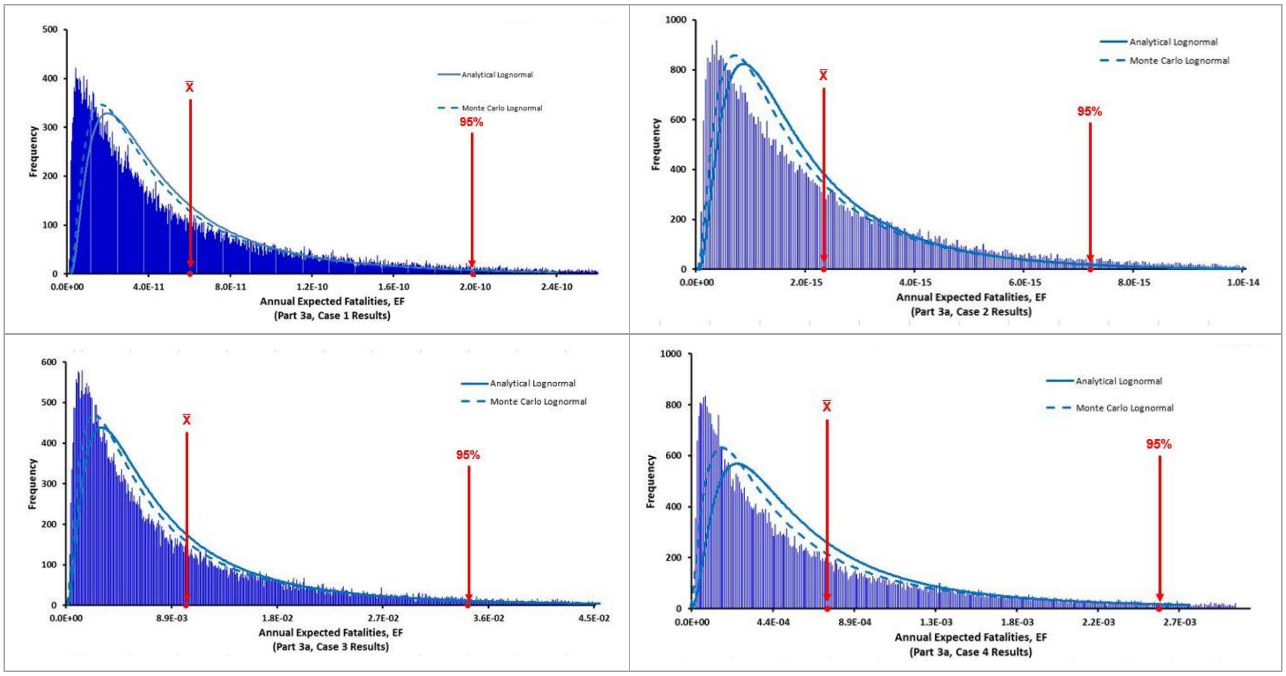

Figure 3. Monte Carlo and Analytical results with plotted lognormal risk distributions

The goal of Part 3a was to investigate the effect non-lognormal, skewed uncertainty distributions would have on the predicted risk distributions. In all test cases, estimates for all parameters of risk and associated uncertainty produced by the Analytical and Monte Carlo models were similar, deviating by no more than 25%, as shown in Table 9. The primary comparison of this study was the expected value, which deviated by no more than 1%, also shown in Table 9.

It was anticipated that these results would have greater disparities than those found in Parts 1 and 2, where a complete and fully vetted Analytical Model was compared to a Monte Carlo experiment designed to the same risk model construction.

These results suggest use of non-lognormal, skewed distributions in both the Analytical and Monte Carlo methods is feasible.

Part 3b of this uncertainty task was to replicate Part 1 with one modification. The uncertainty distributions of the input parameters were assumed to be normal as opposed to lognormal (as was the case for Part 1). The same four test cases were run for Part 3b using normal input distributions developed for this part.

The goal of Part 3b was to investigate the effect symmetric, non-lognormal uncertainty distributions would have on the predicted risk distributions produced by the Analytical and Monte Carlo methods. Given the same inputs in both models, and running the same four test cases (Low-Wide, Low-Narrow, High-Wide, and High-Narrow), the goal was to compare the results of the Analytical and Monte Carlo models when using normal uncertainty distributions for the input parameters.

For Part 3b, normal input distributions were developed, and equations to compute the expected value and variance of the risk distribution based on expected value and variance of each input distribution were developed. Part 3b model inputs for both the Analytical and Monte Carlo approaches were the same for all four test cases.

The approach used for Part 3b mirrored that used for Part 3a, except that input expected values and variances were computed assuming normal distributions.

It was also necessary to modify the Monte Carlo code in order to run the experiments for Part 3b. For the Part 3b experiment, the draws for each input variable were made using a Normal Distribution Process Generator to calculate a value drawn from the input normal distribution. The execution of the Monte Carlo experiment was the same as described in the Part 1 Monte Carlo Methodology

The Part 3b Analytical and Monte Carlo results demonstrate similar estimates of risk and epistemic uncertainty, as depicted in Table 7.

Table 7. Part 3b Analytical and Monte Carlo Results Comparison

Solution Method	Part 3b, Case 1 (Low-Wide)			Part 3b, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	3.25E-16	1.21E-16	5.51E-16	1.48E-17	5.61E-18	2.53E-17
Experimental (Monte Carlo)	3.25E-16	1.22E-16	5.47E-16	1.48E-17	5.00E-18	2.40E-17
Δ%	0.10%	0.58%	0.64%	0.31%	10.81%	5.23%

Solution Method	Part 3b, Case 3 (High-Wide)			Part 3b, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	1.75E-05	6.69E-06	3.00E-05	2.28E-04	8.76E-05	3.92E-04
Experimental (Monte Carlo)	1.75E-05	6.02E-06	2.85E-05	2.29E-04	7.84E-05	3.72E-04
Δ%	0.18%	10.00%	5.07%	0.45%	10.46%	5.09%

The figures that follow present histograms of the Monte Carlo results. These histograms show the distribution of the 50,000 values of EF generated for each case (these 50,000 values of EF were generated by running the Monte Carlo model for K = 50,000 replications). Lognormal curves using the characteristics of the Analytical and Monte Carlo results from Table 11 (expected value and standard deviation) were overlaid on the histograms for comparison. Bin sizes used in the histograms vary from case to case. In each case, the lognormal plots incorporate both the bin size and the 50,000 observations of EF to determine the frequency expected for each bin. Expected value and 95th percent arrows from the Monte Carlo results are also indicated on each of the histograms. Results are shown in Figure 4.

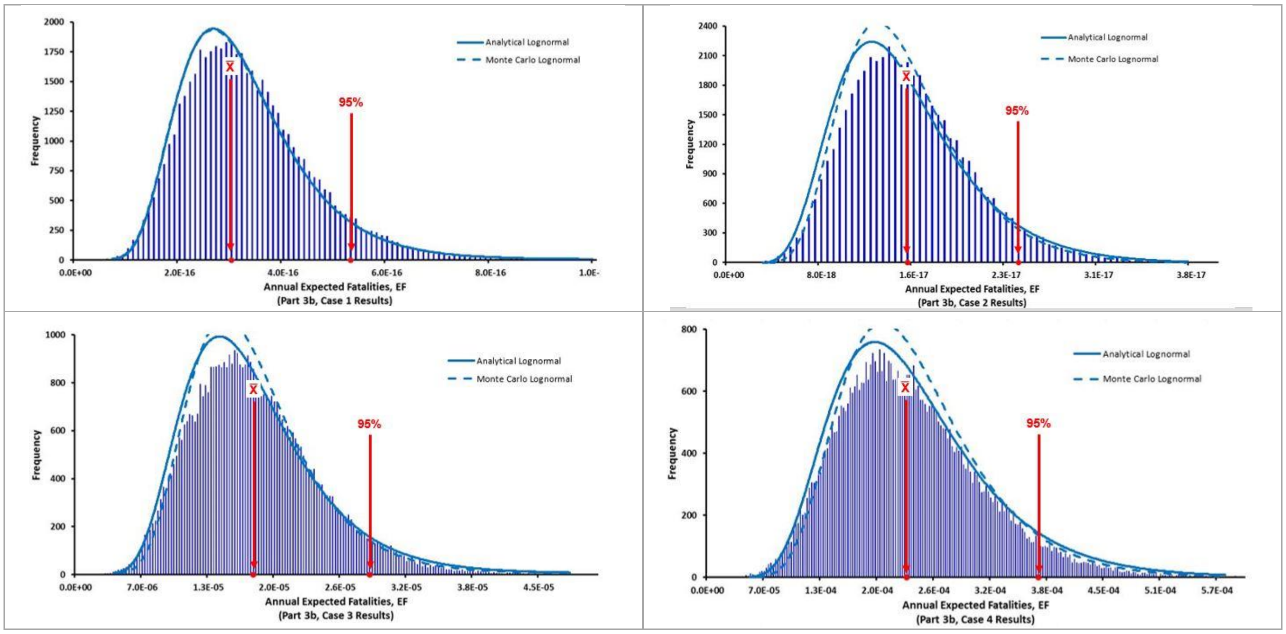

Figure 4. Monte Carlo and Analytical results with plotted lognormal risk distributions

The goal of Part 3b was to investigate the effect symmetric, non-lognormal uncertainty distributions would have on the predicted risk distributions. In all test cases, estimates for all parameters of risk and associated uncertainty produced by the Analytical and Monte Carlo models were similar, deviating by no more than 11%, as shown in Table 7. The primary comparison of this study was the expected value, which deviated by no more than 1%, also shown in Table 7.

It was anticipated that the results would have greater disparities than those found in Parts 1 and 2, where a complete and fully vetted analytical model was compared to a Monte Carlo experiment designed to represent the same risk methodology. These results suggest use of symmetric, non-lognormal distributions in both the Analytical and Monte Carlo methods is feasible.

Part 3c of this uncertainty task was to replicate Part 1 with a mixture of uncertainty distributions (normal, lognormal, and triangular), as opposed to all lognormal (as was the case for Part 1). The same four test cases were run as for all previous parts.

The goal of Part 3c was to investigate the effect mixed uncertainty distributions would have on the agreement between the Analytical and Monte Carlo results. Given the same inputs and running the same four test cases (Low-Wide, Low-Narrow, High-Wide, and High-Narrow), the goal was to compare the results of the Analytical and Monte Carlo models when using a mix of uncertainty distributions for the input parameters.

For Part 3c, a mix of input distributions was developed, and equations to compute the expected value and variance of the risk distribution based on expected value and variance of each input distribution were used for the analytical approach. The mix of input distributions for Parts 3c and 4 is provided in Table 8.

Table 8. Parts 3c and 4 "Mixed" Input Distributions

INPUT VARIABLES	
median value of delta t	Δt_o
std dev of delta t	$\sigma_{\Delta t}$
median value of Scale Factor	S_o
std dev of Scale Factor	σ_S
median value of λ_O	λ_{oo}
std dev of λ_O	$\sigma_{\lambda o}$
Ep Median Daily Exposure	E_{oo}
Rand Var std dev Exposure	σ_e
Ep std dev of Exposure	σ_{Eo}
Ep Median Pf|e blast	$P_{f\|1oo}$
Ep std dev for blast	σ_{1o}
std dev for variation in blast	σ_1
Ep Median Pf|e bldg damage	$P_{f\|2oo}$
Ep std dev for bldg damage	σ_{2o}
std dev for variation in bldg damage	σ_2
Ep Median Pf|e debris	$P_{f\|3oo}$
Ep std dev for debris	σ_{3o}
std dev for variation in debris	σ_3
Ep Median Pf|e glass	$P_{f\|4oo}$
Ep std dev for glass	σ_{4o}
std dev for variation in glass	σ_4
Ep std dev Pf|e due to Yield	σ_{yo}
Std Dev Pf|e due to Yield	σ_y
Std Dev Rnd Var λ due to NEW	σ_{NEW1}

Input Distribution	Variable	Normal	Lognormal	Triangular
Delta t	Median of delta t	X		
	Std dev of delta t			
Scale Factor	Median of Scale Factor			X
	Std dev of Scale Factor			
Lambda	Median of lambda		X	
	Std dev of lambda			
Daily Exposure	Ep Median Daily Exposure	X		
	Ep std dev of Exposure			
	Rand Var std dev Exposure			X
Blast	Ep Median Pf|e blast		X	
	Ep std dev for blast			
	Std dev for variation in blast	X		
Building Collapse	Ep Median Pf|e bldg collapse		X	
	Ep std dev for bldg collapse			
	Std dev for variation in bldg collapse	X		
Debris	Ep Median Pf|e debris		X	
	Ep std dev for debris			
	Std dev for variation in debris	X		
Glass	Ep Median Pf|e glass		X	
	Ep std dev for glass			
	Std dev for variation in glass	X		
Yield	Ep std dev Pf|e due to Yield		X	
	Std dev Pf|e due to Yield	X		
NEW	St dev Pfe due to NEW		X	

X = Distribution for subject input variable

The methodology used for Part 3c mirrored that used for Parts 3a and 3b except that input expected values and variances for the various input distributions were computed based on the assigned distribution type for that variable.

It was also necessary to modify the Monte Carlo code in order to run the experiments for Part 3c. For the Part 3c experiment, the draws for each input variable were made using either the Triangular, Normal, or Lognormal Distribution Process Generators to calculate a value drawn from the mixed distribution specified by the input data. The execution of the Monte Carlo experiment was the same as described in the Part 1 Monte Carlo Methodology.

The switch to mixed distributions also affected the manner in which the aleatory and epistemic uncertainties were combined in the inner loop. To address this change, the inputs for two variables were swapped to ensure that the central value of the exposure distribution was properly addressed in the inner loop. This approach ensured that the central value of the exposure distribution was properly addressed in the Monte Carlo experiment.

Part 3c Analytical and Monte Carlo results are shown in Table 9.

Table 9. Part 3c Analytical and Monte Carlo Results Comparison

Solution Method	Part 3c, Case 1 (Low-Wide)			Part 3c, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	5.51E-15	4.57E-15	1.39E-14	1.30E-16	8.35E-17	2.88E-16
Experimental (Monte Carlo)	5.80E-15	6.30E-15	1.68E-14	1.38E-16	1.02E-16	3.30E-16
Δ%	5.27%	37.78%	20.57%	5.90%	22.49%	14.48%

Solution Method	Part 3c, Case 3 (High-Wide)			Part 3c, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	3.66E-04	4.87E-04	1.16E-03	3.23E-04	3.70E-04	9.57E-04
Experimental (Monte Carlo)	3.85E-04	5.53E-04	1.30E-03	3.38E-04	4.26E-04	1.04E-03
Δ%	5.26%	13.51%	11.94%	4.63%	15.21%	8.50%

Figure 5 presents histograms of the Monte Carlo results. These histograms show the distribution of the 50,000 values of EF generated for each case (these 50,000 values of EF were generated by running the Monte Carlo model for K = 50,000 replications). Lognormal curves using the characteristics of the Analytical and Monte Carlo results from Table 9 (expected value and standard deviation) were overlaid on the histograms for comparison. Bin sizes used in the histograms vary from case to case. In each case, the lognormal plots incorporate both the bin size and the 50,000 observations of EF to determine the frequency expected for each bin. Expected value and 95th percent arrows from the Monte Carlo results are also indicated on each of the histograms.

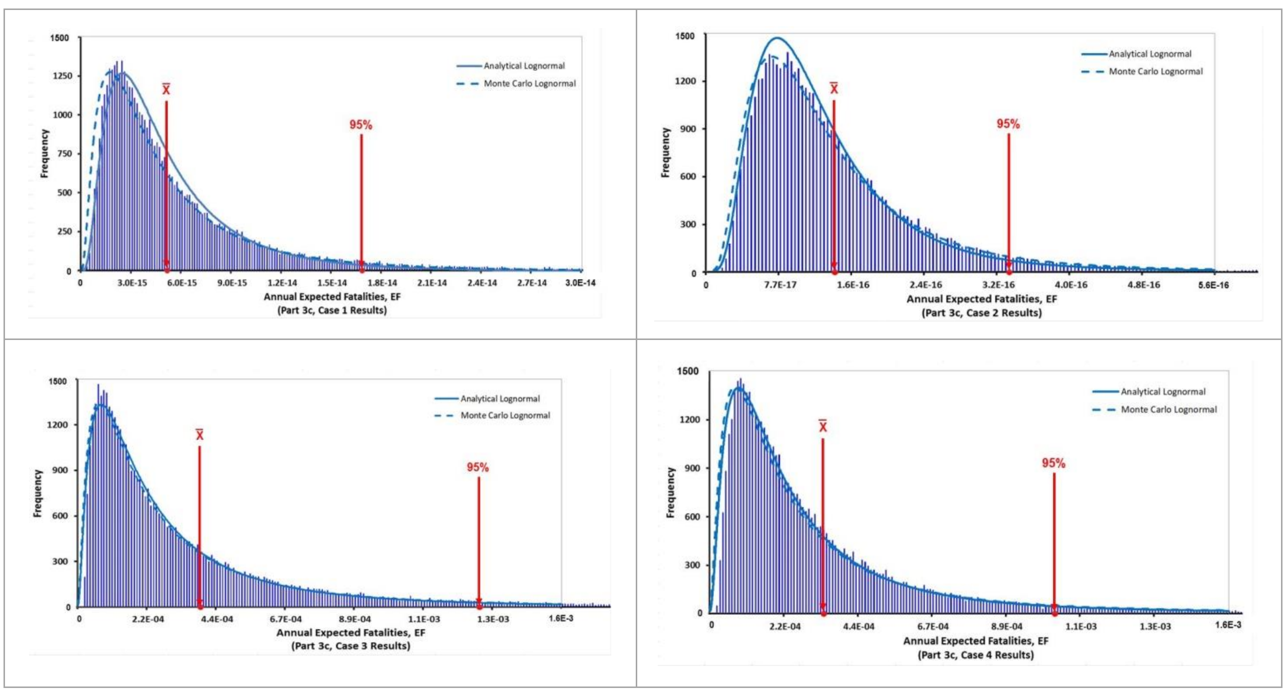

Figure 5. Monte Carlo and Analytical results with plotted lognormal risk distributions

In all test cases, estimates for all parameters of risk and associated uncertainty produced by the Analytical and Monte Carlo models were similar, deviating by no more than 38%, as shown in Table 9. The primary comparison of this study was the expected value, which deviated by no more than 6%, also shown in Table 9.

The differences observed between the Analytical and Monte Carlo results were greater for Part 3c than for any previous part. When considering whether a given level of differences should be considered “similar,” “reasonable,” or “acceptable,” it is important to view these differences objectively and within the larger context of the phenomena being estimated by the risk model. The following are a few considerations to inform the process of drawing conclusions from the differences observed:

1. These comparisons do not represent differences between a model and truth, but between two methods of modeling a common unknown. As such, the delta percentages provide insight into the level of inconsistency between the models, rather than providing a measure of the error between either model’s result and the “real” answer.

2. The differences observed between the two modeling approaches appear insignificant when compared to the acknowledged uncertainties in the submodels, as described in TP-14 Rev 4a (Ref 4). Subject Matter Experts (SMEs) set the upper-bound multipliers used for major components of the uncertainty model. These SMEs set factors between nine (overpressure) and 20 (glass) for differences between the elemental model point estimates and the upper limit of the potential $P_{f|e}$ produced by fatality mechanisms. Depending on the activity modeled, a factor between three and 10 was assigned to the potential difference between the point estimate and the upper limit of the unscaled probability of event.

3. Inconsistencies in the treatment of variables and distributions would be expected to generate differences in the results. In an agile environment, where both methods must be modified repeatedly to adjust to differences in distribution types, these inconsistencies are likely. In the course of the task, some inconsistencies between the Analytical and Monte Carlo models were identified and corrected, while others either could not be isolated or were inherent to the modeling approaches used. If the TP-14 risk estimation and uncertainty methodologies are to be updated with distribution types deemed more appropriate, then the Analytical model will be developed with attention to every detail and significant oversight. Likewise, the Monte Carlo experiment used to verify the Analytical model will be crafted from the ground up to match the intent of the risk model. Elimination of inconsistencies in treatment of parameters can be expected to significantly reduce the differences observed between the two modeling approaches.

In this context, it is believed that the results produced by the modified Analytical Method and modified Monte Carlo experiment are certainly close enough to be considered “similar.” The differences observed, however, are not believed to be indicative of the best achievable for a model designed to incorporate varied statistical distributions.

These results suggest use of mixed distributions in both the methods is feasible.

Part 4

Part 4 of this Uncertainty Task was to replicate Part 3c with one modification: the point estimate of the elemental models would be taken as the mean as opposed to the median (as was the case for Part 3c). The same four test cases were run for all previous parts.

The goal of Part 4 was to investigate the effect mixed uncertainty distributions combined with a point estimate of the elemental models set to the mean would have on the predicted risk distributions. Given the same inputs and running the same four test cases (Low-Wide, Low-Narrow, High-Wide, and High-Narrow), the goal was to compare the results of the Analytical and Monte Carlo models when using a mix of uncertainty distributions and a point estimate set to the mean.

For Part 4, the same mix of input distributions used in Part 3c was developed with one change. The point estimates from the science models were treated as the mean of each distribution instead of the median. The input distribution parameters were shifted from those used in Part 3c in a manner similar to that used to develop Part 2 inputs to replace those used in Part 1.

Part 4 model inputs for both the Analytical and Monte Carlo approaches were the same for all four test cases.

The Analytical methodology required for Part 4 is the same as that used in Parts 3a, 3b, and 3c. No modification to the equations used in Part 3c were required to perform Part 4. The change for Part 4 was use of modified inputs to set point estimates to the mean. The inputs for variables x4 and x10 were again swapped to maintain consistency of modeling between the Analytical methodology and the Monte Carlo experiment for the reasons described in Part 3a.

For the Part 4 Monte Carlo study, the inputs were run through the same Monte Carlo model used to perform Part 3c. For each case, this experiment produces 50,000 observations of *EF*. The results were compared to those from the Analytical model to verify either methodology would produce similar results.

The Part 4 Analytical and Monte Carlo results are shown in Table 10.

Table 10. Part 4 Analytical and Monte Carlo Results Comparison

Solution Method	Part 4, Case 1 (Low-Wide)			Part 4, Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	3.92E-15	3.24E-15	9.90E-15	1.12E-16	7.22E-17	2.49E-16
Experimental (Monte Carlo)	4.13E-15	4.44E-15	1.19E-14	1.19E-16	8.85E-17	2.85E-16
Δ%	5.34%	36.92%	19.78%	5.93%	22.62%	14.37%

Solution Method	Part 4, Case 3 (High-Wide)			Part 4, Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Analytical Method	2.35E-04	3.12E-04	7.42E-04	2.19E-04	2.51E-04	6.49E-04
Experimental (Monte Carlo)	2.47E-04	3.54E-04	8.32E-04	2.29E-04	2.89E-04	7.04E-04
Δ%	5.01%	13.50%	12.11%	4.70%	15.22%	8.55%

The differences between Analytical and Monte Carlo results for Part 4 follow the same trends and magnitudes observed in Part 3c. This is to be expected since the models used for Part 3c were used without modification. Part 4 changed only the inputs used.

Figure 6 presents histograms of the Monte Carlo results. Lognormal curves using the characteristics of the Analytical and Monte Carlo results from Table 10 (expected value and standard deviation) were overlaid on the histograms for comparison.

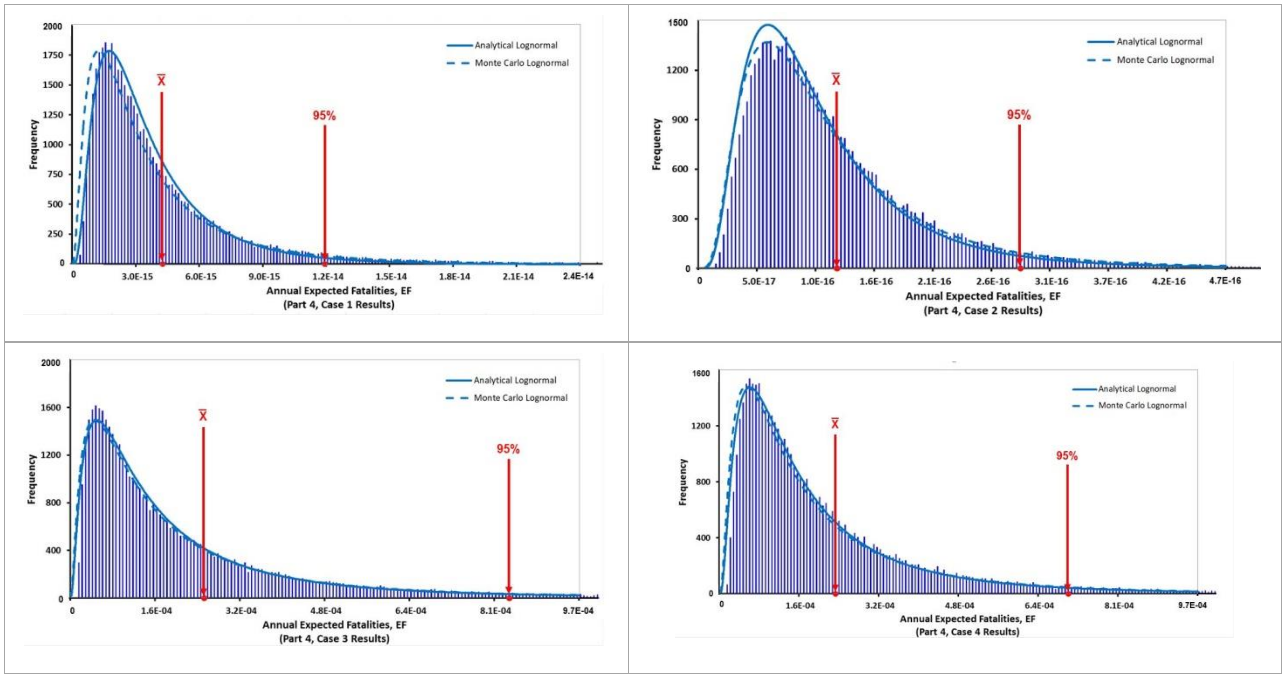

Figure 6. Monte Carlo and Analytical results with plotted lognormal risk distributions

In all test cases, estimates for all parameters of risk and epistemic uncertainty produced by the Analytical and Monte Carlo models were similar, deviating by no more than 37%, as shown in Table 10. The primary comparison of this study was the expected value, which deviated by no more than 6%, also shown in Table 10. Similar to the discussion of Part 3c results, these results indicate the Analytical and Monte Carlo models produce similar estimates of risk and epistemic uncertainty when input variables are sampled from a mix of distributions and the point estimate of the elemental models is set to the mean, as shown in Table 11.

These results suggest use of mixed distributions in both the Analytical and Monte Carlo methods is feasible.

Table 11. Part 3c and 4 Analytical and Monte Carlo Results Comparison

Case		Analytical		Monte Carlo	
		Part 3c	Part 4	Part 3c	Part 4
1	Expected Value	5.51E-15	3.92E-15	5.80E-15	4.13E-15
	Standard Deviation	4.57E-15	3.24E-15	6.30E-15	4.44E-15
	95 Percentile	1.39E-14	9.90E-15	1.68E-14	1.19E-14
2	Expected Value	1.30E-16	1.12E-16	1.38E-16	1.19E-16
	Standard Deviation	8.35E-17	7.22E-17	1.02E-16	8.85E-17
	95 Percentile	2.88E-16	2.49E-16	3.30E-16	2.85E-16
3	Expected Value	3.66E-4	2.35E-4	3.85E-4	2.47E-4
	Standard Deviation	4.87E-4	3.12E-4	5.53E-4	3.54E-4
	95 Percentile	1.16E-3	7.42E-4	1.30E-3	8.32E-4
4	Expected Value	3.23E-4	2.19E-4	3.38E-4	2.29E-4
	Standard Deviation	3.70E-4	2.51E-4	4.26E-4	2.89E-4
	95 Percentile	9.57E-4	6.49E-4	1.04E-3	7.04E-4

Similar trends in delta percentages between Part 3c Analytical vs. Monte Carlo were also observed in Part 4, as shown in Table 12.

Table 12. Part 3c and 4 Analytical Δ% Comparison

Study Part	Case 1 (Low-Wide)			Case 2 (Low-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Part 3c: Δ% Analytical-Experimental	**5.27%**	**37.78%**	**20.57%**	**5.90%**	**22.49%**	**14.48%**
Part 4: Δ% Analytical-Experimental	**5.34%**	**36.92%**	**19.78%**	**5.93%**	**22.62%**	**14.37%**

Study Part	Case 3 (High-Wide)			Case 4 (High-Narrow)		
	Expect Val	Std Dev	95th %	Expect Val	Std Dev	95th %
Part 3c: Δ% Analytical-Experimental	**5.26%**	**13.51%**	**11.94%**	**4.63%**	**15.21%**	**8.50%**
Part 4: Δ% Analytical-Experimental	**5.01%**	**13.50%**	**12.11%**	**4.70%**	**15.22%**	**8.55%**

Conclusions

The following conclusions are drawn from the results, discussions, and explanations provided in the previous sections addressing each part of the task:

1. **Use of non-lognormal distributions to represent input variables does not "break" the TP-14 Analytical model of uncertainty which assumes the output risk results in a lognormal distribution.** The Monte Carlo experiment does not assume any form for the output distribution, but the histograms produced by the experiments are visually lognormal in nature. The lognormal plots shown with the histograms of Parts 3a, 3b, 3c, and 4 demonstrate that the results follow a lognormal distribution.

2. **Assigning elemental model point estimates as the mean rather than the median of the input distributions does not "break" the TP-14 Analytical model of uncertainty.** The agreement between the Analytical and Monte Carlo approaches for Part 2 compares well

with the agreement observed in Part 1. The agreement between the two methods for Part 4 also compares well with that observed in Part 3c. The Part 4 to Part 3c comparison also shows that the shift from median to mean does not depend on lognormally distributed variables.

3. **The agreement between Analytical and Monte Carlo results for the expected value was significantly better than the agreement observed between the standard deviation and 95th percentile.** This is believed to be due to the more straightforward modeling of expected values in the original Analytical method. Since the distribution variance equations in the original model are much more convoluted than those used for the expected value calculation, there is significantly higher potential for modeling inconsistencies between the newly developed approaches (Analytical and Monte Carlo) used to calculate these results.

4. **The use of a mix of distributions, including non-lognormal distributions, to model input variables does not "break" the ability to develop an analytical model to directly calculate the parameters of the output risk distribution.** The results showed that the two methods produce similar results using a variety of distribution types.

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