Modeling and Simulation of HD 1.3 Thermal Initiation Tests on a Small Reinforced Concrete Storage Structure

Ming Liu, Michael Oesterle, and Robert Conway Naval Facilities Engineering and Expeditionary Warfare Center 1100 23rd Ave., Port Hueneme, CA93043-4370

Josephine Covino, Department of Defense Explosives Safety Board Suite 16E12 4800 Mark Center Drive Alexandria, VA, USA

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Abstract

This paper presents the efforts on modeling and simulation of HD 1.3 thermal initiation tests on a small reinforced concrete (RC) storage structure (called Kasun structure) with the finite element analysis software code LS-DYNA. Based on the detailed studies on mesh size, material model, erosion criterion, boundary condition, and modeling of lap splice, a full-scale half-symmetry numerical model was created to determine the worst-case loading scenario from the prescribed pressure load-time histories. In general, HD 1.3 explosive materials generate mass fire with little or no fragmentation. However, under a confined storage condition, HD 1.3 explosive materials can also generate a scenario that results in a pressure vessel like rupture of the RC structure. This scenario is called the "choked flow" and is heavily related to the flow rates. Thus, the prescribed pressure load-time histories in this study cover the choked flow with initial flow rate, choked flow with double flow rate, half-choked flow with initial flow rate, and the two other general loading curves called the P1 and P6 loading curves.

The simulation results demonstrated that the numerical model is capable of predicting failure modes observed in the live field test. These failure modes include the initial cracking at the corners of the walls, the heavy damage of the floor slab, the rupture of the roof, and the failure of the steel door. The simulation results also showed that the connection details between the steel doorframe and the concrete walls is an important factor that significantly affects the failure modes in both computer simulation and field tests. Based on the failure time and corresponding blast pressure at the time of the break-up of the Kasun structure, the worst-case loading scenario is determined to be the half-choked loading with double flow rate, followed by the half-choked loading with initial flow rate. The choked loading with initial flow rate and P1 loading curve also cause the break-up of Kasun structure, but the choked loading with initial flow rate and P6 loading curve did not cause any break-up at all. Recommendations for future studies have been made to improve qualities of computer simulations, such as sensitivity studies on the numerical erosion mechanism that represents the physical fracture mechanism as well as the coupled simulations that can consider the fluid-structure interaction (FSI) between the blast waves and the structure. Probabilistic approaches that can deal with the random process of heterogeneous material break-up are also recommended for future studies.

INTRODUCTION

Naval Air Warfare Center Weapons Division (NAWCWD) China Lake tasked Naval Facilities Engineering and Expeditionary Warfare Center (NAVFAC EXWC) to perform modeling and simulation for a series of confined blast experiments that used energetic materials with a 1.3 Hazard Division (*i.e.* HD 1.3) classification in Kasun structures. The purpose of the field tests was to determine the hazards associated with initiation of the HD 1.3 materials in a confined structure, specifically to demonstrate that for the "choked flow" conditions the debris hazards could exceed those specified in the Department of Defense (DoD) 6055.09-M [1]. Phase I report of this effort [2] contains the detailed descriptions of the field tests and the efforts on modeling and simulation with the LS-DYNA computer program. This paper documents the continuous efforts on modeling and simulation for the technical supports on determining the worst-case loading scenario for future tests.

Brief Descriptions of Kasun Structure

The Kasun structures constructed for blast tests were the reinforced concrete cubes with an overall exterior and interior dimensions of 7.5 feet and 6.5 feet, respectively. All reinforced concrete roof, walls and floor slab had a nominal thickness of 6.0 inches. The concrete of the Kasun structures had a specified 28-day minimum compressive strength equal to 7,000 psi. The typical steel reinforcement in the concrete (*i.e.* the rebar) was No. 4, A615 Grade 60 steel bars spacing at 4 in. on-center in both directions and on each face. A schematic of the Kasun structure cube with the door in place is shown in Figure 1 (a), and a cut away view of the structure cube that illustrates the doorframe is shown in Figure 1 (b).



Figure 1: Illustration of Kasun Structure

The front wall of the Kasun structure had a centralized doorway opening of 5.0 feet in height and 3.0 feet in width as shown in Figure 1 (b), which was framed with A36 steel plates. All of the steel doorframe plates were ¼ -in. thick except for those on the exterior face of the wall, which were ½ -in. thick. The exterior and interior doorframe plates were 6.0 inches wide, and were connected to each other with 2-in. diameter steel tube sections. All adjacent doorframe components, including the steel plates and the tube sections, were welded together. A total of 16 tube sections along the perimeter of the doorframe were used to tighten a ¾-in. thick steel doorplate to the doorframe via

1.5-in. diameter bolts that passed through the tube sections. The steel doorplate was 6.0 feet in height and 4.0 feet in width and had a centralized 15.3-in. diameter hole that could create the "choked flow" conditions for blast tests.

Phase I report of this effort [2] provides the detailed information regarding the Kasun structure.

Two Kasun structures were constructed at NAWCWD, China Lake, California between March and July, 2014. NAVFAC EXWC personnel conducted two field inspections to ensure the match between the actual structures and the computer models, where the steel reinforcement details were the focus. Figure 2 presents some photos taken during the construction process.





Figure 1: Construction of Kasun Structure

Finite Element Modeling

The Kasun HD 1.3 field tests were simulated using finite element methods via the LS-DYNA computer program [3], where nonlinear explicit dynamic analyses with a second order central difference scheme were performed for large structural deformations under blast pressure loads. Only half of the Kasun structure was modeled with symmetry in the XZ-plane to improve computational efficiency. Based on the mesh convergence study in Phase I of this effort [2], the eight-node, single point integration solid cube elements with a typical size of $\frac{1}{2}$ in. were used to model the concrete roof, walls and floor slab as well as the steel door system. The beam elements with a typical length of $\frac{1}{2}$ in. were used to model the steel reinforcement in the concrete. As a result, there were over one million finite elements in the LS-DYNA model for the Kasun structure, including 1,031,632 solid elements and 65,938 beam elements.

The steel reinforcement in the concrete (*i.e.* the rebar) was modeled using Hughes-Liu beam elements with 2 x 2 Gauss quadrature cross sectional integration [3]. A perfect bond between the rebar and the concrete was assumed because the beam rebar elements were directly merged with the solid concrete elements in the LS-DYNA models herein. Phase I report of this effort [2] had proven that this perfect bond modeling provided very good estimates of overall dynamic responses for reinforced concrete structures under blast and impact loads. However, the report also recognized that the perfect bond modeling had limitations in simulating discrete discontinuous behaviors, such as concrete cracking and blast fragment performance, due to lack of explicit representation of the bond-slip relations.

According to Phase I report of this effort [2], the lap splices among the rebar were carefully modeled to include one or two layers of solid concrete elements between the beam rebar elements to capture the pullout failure mechanisms observed in the previous Kasun field tests. Since this explicit lap splice modeling assumes the loads transferred

among the rebar are controlled by the concrete strength only, and ignores the friction among the rebar or the tire wires in rebar cages, a lower bound lap splice strength is obtained from this approach. Figure 3 (a) and (b) shows the beam rebar elements and the explicit lap splices in the LS-DYNA models herein, respectively.



The steel door system, including the doorplate, doorframe and bolts, was modeled with the solid steel elements (instead of the shell elements in Phase I of this effort). This modification aimed to avoid potential computational problems in the LS-DYNA's contact algorithms under large deformations caused by blast pressure loads. The surface-to-surface contact algorithms for solid-to-solid elements may work better than those for solid-to-shell elements in LS-DYNA when large blast loads are applied. Only one layer of solid elements was used to model the thin steel plates in the door system in order to maximize the required minimum time-step internal that is usually proportional to the smallest element size in a LS-DYNA model. The 2-in. diameter steel tube sections and 1.5-in. diameter steel bolts were also simplified as the solid bars that had a square cross section of 1.5 inches. It is recognized that the dynamic behaviors of the steel door system under blast pressure loads were not accurately captured due to these simplifications. However, the previous Kasun field tests indicated that the doorframe failed along the joints in the concrete at the edges of the steel plates and little structural deformations occurred in the steel door system itself. In addition, the purpose of the current modeling and simulation effort was to provide the technical basis on determining the worst-case loading scenario for future tests. Therefore, the steel door system modeling herein was determined to be acceptable.

The solid elements representing the steel door system merged together wherever they met each other, but the interaction between the steel doorframe and the front wall concrete was modeled with *AUTOMATIC_SURFACE_to_SURFACE_CONTACT in LS-DYNA [3]. Both static and dynamic coefficients of friction were assigned as 0.5 for the contact surface. The assigned contact surface allowed element separation or relative sliding, but not passing through to each other. In addition, the front wall concrete was merged with the solid bars representing the steel tube sections and the steel bolts wherever they met together.

A layer of solid concrete elements was artificially added below the Kasun LS-DYNA model to provide a fixed boundary condition. The interaction between these added solid elements and the Kasun model was simulated with *AUTOMATIC_SURFACE_to_SURFACE_CONTACT, where both static and dynamic coefficients of friction

were assigned as 0.9 in this report. Since only half of the Kasun structure that was symmetrical to the XZ-plane was modeled, the fixed boundary in the Y-direction was provided.

Material Models

The concrete material model used herein was *MAT_CONCRETE_DAMAGE_REL3 (also known as *MAT_72_R3) in LS-DYNA [3]. This constitutive material model adopts a plasticity formulation that computes the current time-step shear failure surface from the three shear yield surfaces and a damage parameter that is a function of the effective plastic strain [4]. The damage parameter represents the current time-step shear yield surface coincides with the limits of the elastic, the maximum yield and the residual yield surfaces when its value equals to 0, 1 and 2, respectively. This material model also includes the strain rate effects. An equation-of-state (EOS) associated with *MAT_72_R3 was used to represent the relation between pressure and volumetric strain.

The steel material model in this report was the piecewise linear plasticity model (*i.e.* *MAT_24) in LS-DYNA. The material properties of the A615 Grade 60 and A36 steel were assigned for the rebar and the steel door system, including the doorframe, doorplate and bolts, respectively. The strain rate effects were also considered herein.

Based on the Phase I report of this effort [2], the erosion criteria were set as the maximum principal strain of 0.5 for the concrete material and the maximum plastic strains of 0.4 and 0.12 for the A615 Grade 60 and A36 steel, respectively. When the calculated strains at the current time-step reached these maximum values, the material failure or erosion occurred. As a result, the associated elements were deleted from the next time-step calculations.

Blast Pressure Load Curves

NAWCWD provided eight pressure load-time histories as shown in Figure 4, based on their detailed blast analyses. Four of them were from the assumed choked conditions, while another two were from the assumed half-choked conditions. The P6 and P1 loading curves were the real data from the previous experiments. The flow rates in the assumed conditions were independent of the generated blast pressure, but varied as labeled as "initial fr" and "double fr" in Figure 4. For the assumed choked conditions, the Kasun structure was either half-occupied or quarter-occupied (*i.e.* half or quarter volume inside the structure was taken up by non-responsive mass), however, for the assumed half-choked conditions, only quarter-occupied structure was studied. In fact, the free spaces in the structure did not make much difference in the loading curves, where the assumed half-choked conditions produced much higher peak values and longer durations of the blast pressures than the others. In addition, when the flow rate was doubled, the resulting peak pressures were increased.

These loading curves were uniformly applied to all interior surfaces of the Kasun structure, including the roof, walls and floor slab, with *LOAD_SEGMENT command in LS-DYNA [3]. It was recognized that the actual blast pressures could vary spatially, however when considering the purpose of this effort, these derivations would not affect the conclusions of this study. Therefore, the blast pressure load applications herein were determined to be acceptable.



Figure 4: Blast Pressure Load Curves from NAWCWD

Gravity loading was applied in this study with the LS-DYNA command *LOAD_BODY_Z [3], where the gravity acceleration was applied as a time history that had an initial ramp period of 0.1 seconds when increasing from 0 to 386.4 in./sec² before holding constant. The blast pressure loading started at 0.25 second as shown in Figure 4. Thus, there was 0.15 seconds quite time between the gravity and blast pressure loadings in order to avoid the artificial vibration of the computer model due to the sudden application of the gravity of 386.4 in./sec².

EXPERIMENTAL VALIDATION of LS-DYNA MODELS

The LS-DYNA models of Kasun structure were validated by comparing the failure modes from the computer simulations with those from the experiments. It was found that the method to model the connections between the steel reinforcement and doorframe significantly affected the failure modes obtained from the computer simulations. Thus, the effects of the connection modeling on failure modes were studied and presented herein.

Kasun HD 1.3 Test 6 was conducted at NAWCWD, China Lake, California on August 13, 2014. This field test was intended to re-evaluate HD 1.3 hazards under Test 4 condition, but with minor structural modification that tied the steel doorframe to the rebar in the concrete. Figure 5 compares the failure modes from Test 6 with that from LS-DYNA simulation, where both doors failed outward, rotating about the roof-to-front wall joints. The rebar connecting to the top doorframes remained attached or partially failed after the complete failure of the rebar connecting to the bottom doorframes. This failure mode was believed due to the original structural design that had only one layer of the rebar in the floor slab connecting to the bottom doorframe, but two layers of the rebar in the

roof connecting to the top doorframe. The LS-DYNA models also correctly predicted that (a) the concrete cracking initiated at the wall-to-wall joints; (b) the side and back walls as well as the roof kept almost intact after initiation of the HD 1.3 material; and (c) the floor slab suffered much more damage than the roof. It should be noted that the failure mode in Test 6 was much different from that in Test 4 where the debris were ejected from the roof as shown in Figure 6. This indicated that the minor structural modification that tied the doorframe to the rebar in the concrete significantly changed the failure modes. Further analytical and experimental investigations shall be conducted to better understand the effects of the connection details and modeling on potential failure modes.



(a) Test 6



(b) LS-DYNA Simulation





(a) Test 4 (Back View)
(b) Test 6 (Front View)
Figure 6: Comparison of Failure Mode in Test 4 and Test 6

WORST-CASE LOADING SCENARIO

The worst-case loading scenario was determined from the eight pressure load-time histories provided by NAWCWD. As shown in Figure 4, the occupation status (*i.e.* half or quarter occupied) has little effects on the resulting load curves. Thus, the efforts to determine the worst-case loading scenario focused on the quarter-occupied cases. The initial analyses indicated that the choked loading scenario with initial flow rate as well as P6 loading curve did not cause the break-ups of Kasun structure. Therefore, the LS-DYNA runs with the full connection detail modeling were conducted only for the choked loading with double flow rate, the half-choked loading with both initial and double flow rates, as well as P1 loading curve.

Figure 7 presents an example of the break-up time determination with the resultant velocities. The resultant velocities for the four nodes randomly selected in a full connection model accelerated greatly starting at 0.708 second when P1 loading curve was applied. Since the nodal velocities are proportional to the blast energy absorbed by the Kasun structure, Time = 0.708 second was determined as the break-up time for this case.



Figure 7: Break-up Time Determination with Resultant Velocities

Choked Loading with Initial Flow Rate and Half-Occupied

This loading curve of the choked loading with initial flow rate and half-occupied was not applied to the full connection models because the loading curve did not cause the break-ups of Kasun structure. Figure 8 presents the loading curve with a peak pressure of 16.5 psi and the damage indices contour at 1.25 second for scenario (i) not connected. The damage indices are a function of plastic strains of *MAT_CONCRETE_DAMAGE_REL3 in LS-DYNA [3].

Choked Loading with Initial Flow Rate and Quarter-Occupied

Similarly, this loading curve of the choked loading with initial flow rate and quarter-occupied was not applied to the full connection models because the loading curve did not cause the break-ups of Kasun structure. Figure 9 presents the loading curve with a peak pressure of 16.5 psi and the damage indices contours at 1.25 seconds for scenario (i) not connected.







Figure 9: Choked Loading with Initial Flow Rate and Quarter-Occupied (scenario (i) not connected)

Choked Loading with Double Flow Rate and Half-Occupied

This loading curve of the choked loading with double flow rate and half-occupied was not applied to the full connection models because the efforts to determine the worst-case loading scenario focused on the quarter-occupied cases. Figure 10 presents the loading curve with a peak pressure of 38.0 psi for the model with partial connections, and the corresponding damage indices contours at 0.32895, 0.47895, and 0.51089 second, respectively. The break-up of Kasun structure was determined to occur at 0.48 second, based on the method in Figure 7.



(a) Loading Curve

(b) Damage Indices Contour @ 0.32895 sec.



(c) Damage Indices Contour @ 0.47895 sec. (d) Damage Indices Contour @ 0.51089 sec.



Choked Loading with Double Flow Rate and Quarter-Occupied

This loading curve of the choked loading with double flow rate and quarter-occupied was applied to the full connection models. Figure 11 presents the loading curve with a peak pressure of 39.5 psi, and the corresponding damage indices contours at 0.54707, 0.55663, and 0.59663 second, respectively. The break-up of Kasun structure was determined to occur at 0.545 second when the blast pressure in Figure 11(a) reached the peak pressure of 39.5 psi.



Damage Indices Contour @ 0.55663 sec. (d) Damage Indices Contour @ 0.59663 sec.



Half-Choked Loading with Initial Flow Rate and Quarter-Occupied

The loading curve of the half-choked loading with initial flow rate and quarter-occupied was applied to the full connection models. Figure 12 presents the loading curve with a peak pressure up to 91.5 psi, and the corresponding damage indices contours at 0.54645, 0.57395, and 0.58895 second, respectively. The break-up of Kasun structure

was determined to occur at 0.545 second when the blast pressure in Figure 12(a) reached 52.4 psi. It should be noted that the loading curve in Figure 12(a) was developed with the assumptions that all components of Kasun structure was made of rigid materials with infinite strength. As a result, the peak pressure shown as 91.5 psi in Figure 12(a) was never reached in the LS-DYNA computer simulations where the nonlinear and inelastic materials of Kasun structure were modeled.



(a) Loading Curve

(b) Damage Indices Contour @ 0.54645 sec.



(c) Damage Indices Contour @ 0.57395 sec. (d) Damage Indices Contour @ 0.58895 sec.



Half-Choked Loading with Double Flow Rate and Quarter-Occupied

The loading curve of the half-choked loading with double flow rate and quarter-occupied was applied to the full connection models. Figure 13 presents the loading curve with a peak pressure up to 187.5 psi, and the corresponding damage indices contours at 0.42395, 0.44645, and 0.45000 second, respectively. The break-up of Kasun structure was determined to occur at 0.420 second when the blast pressure in Figure 13(a) reached 60.0 psi. The loading curve in Figure 13(a) was developed with the assumptions that all components of Kasun structure was made of rigid materials with infinite strength. As a result, the peak pressure up to 187.5 psi in Figure 13(a) was never reached in the LS-DYNA computer simulations where the nonlinear and inelastic materials of Kasun structure were modeled.





•PrePost



(b) Damage Indices Contour @ 0.42395 sec.



(c) Damage Indices Contour @ 0.44645 sec. (d) Damage Indices Contour @ 0.45000 sec.



P1 Loading Curve

The P1 loading curve as shown in Figure 14(a) was applied to full connection models. Figure 14 also presents the corresponding damage indices contours at 0.85801, 0.88162, and 0.89951 second, respectively. The break-up of Kasun structure was determined to occur at 0.708 second when the blast pressure in Figure 14(a) reached the peak pressure of 47.0 psi. It should be noted that P1 loading curve was developed from the real experimental data. As a result, the Kasun structure in the LS-DYNA computer simulations broke up at the peak pressure of 47.0 psi that matched the experimental results.



(c) Damage Indices Contour @ 0.88162 sec. (d) Damage Indices Contour @ 0.89951 sec. Figure 14: P1 Loading (scenario (iii) full connection)

P6 Loading Curve

The P6 loading curve was not applied to the full connection models because the loading curve did not cause the break-ups of Kasun structure. Figure 15 presents the loading curve with a peak pressure of 42.7 psi and the damage indices contour at 0.90 second for the model with partial connections.



Figure 15: P6 Loading Curve (scenario (ii) partial connection)

Determination of the Worst-Case Loading Scenario

Based on the LS-DYNA computer simulations presented above, the worst-case loading scenario was determined to be the half-choked loading with double flow rate, followed by the half-choked loading with initial flow rate. The choked loading with double flow rate and P1 loading curve also caused the break-up of Kasun structure. Table 1 summarizes the failure time and blast pressure at the time of break-up for the full connection models.

Loading Curve	Flow Rate	Occupied	Failure Time (second)	Blast Pressure (psi)
Half Choked	double	quarter	0.420	60.0
Half Choked	initial	quarter	0.545	52.4
Choked	double	quarter	0.545	39.5
P1	N/A		0.858	47.0

Table 1: Break-U	o Time and Blast	Pressure for Full	Connection Models
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Conclusions and Recommendations

The LS-DYNA computer simulations for HD 1.3 tests with Kasun structure were performed to determine the worstcase loading scenario from the eight pressure load-time histories provided by NAWCWD. Based on the previous studies on mesh size, material model, erosion criterion, boundary condition, and modeling of lap splice, a full-scale half-symmetry LS-DYNA model was built to collaborate changes in the new Kasun structures constructed for future tests. This model was validated by comparing the failure modes from the computer simulations with those from the experimental results. The connection detail modeling between the steel doorframe and the concrete walls was identified as an important factor that significantly affects the failure modes in both computer simulation and field tests. This supports the previous conclusion [2] that "the strength of the doorframe might be important to the strength and failure mode of the cubicle If the doorframe remains intact for a longer duration, the failure mechanisms change because the internal stress of the structural components will increase". The studies presented herein also support the previous recommendation that the well-documented details of steel reinforcement including lap splice details and locations improve the quality of the computer simulations.

No effort was made to predict the exact failure time for the eight pressure load-time histories provided by NAWCWD, but the failure time from the computer simulations were used as an important factor to determine the worst-case loading scenario, along with the blast pressure at the time of the break-up of Kasun structure. Ideally, the launch velocities and mass distribution of the debris ejected from the structure should be used to determine the worst-case loading scenario. However, the previous studies indicated that the accuracy of these two important parameters was significantly affected by Fluid (i.e. blast energy waves) -Structure (i.e. Kasun structure) Interaction (FSI), material erosion criteria, and detail modeling of the steel doorframe [2]. In the current method, FSI is uncoupled meaning that the structural response is completely independent of the fluid response. This may lead to inaccuracies in predicting the launch velocities, because in reality as the structure fails, the blast pressure will vent instead of continuing to load and accelerate the resulting debris in the computer simulations. The material erosion criteria also produce uncertainty in predicting the launch velocities as the eroding timing determines the accelerating duration of the debris masses [2]. As stated previously, the doorframe detail modeling, particularly modeling the connection details between the steel doorframe and the surrounding concrete, significantly affects the failure modes of the Kasun structure. In addition, the current modeling techniques, particularly in modeling the bonds between the steel reinforcement and the surrounding concrete, appear lack of capacity in correctly predicting the break-up locations and corresponding debris mass distribution. This is because the current methods are based on continuum mechanics formulations that are not suited to simulate discontinuous behavior of materials [2]. Probabilistic approaches that can deal with random process of the break-up of materials may provide the solutions on correctly predicting debris mass distribution as well as launch velocities.

As a result, based on the failure time and corresponding blast pressure at the time of the break-up of Kasun structure, the worst-case loading scenario is determined to be the half-choked loading with double flow rate, followed by the half-choked loading with initial flow rate. The choked loading with double flow rate and P1 loading curve also cause the break-up of Kasun structure.

The recommendations for future HD 1.3 field tests with Kasun structure and associated modeling efforts are presented as follows:

- Perform non-uniform loading simulations to investigate the spatial discrepancy of the blast pressures on the potential failure modes of Kasun structure. For example, in Run 2 of the previous studies [2], the blast pressure applied to the roof was 20% greater than that applied to the walls and the bottom floor slab;
- Select and/or develop more rigorous modeling techniques for fluid structure interaction;
- Quantify the effects of erosion criteria on launch velocities;
- Refine the connection detail modeling with spot welding of the doorframe;

- Quantify the effects of the door failure on the breakup of Kasun structure;
- Refine the lap splice modeling with constrained beam element option in LS-DYNA;
- Select and/or develop more rigorous modeling techniques for bonding between the steel reinforcement and the surrounding concrete;
- Adopt probabilistic approaches to predict the launch velocities and mass distribution of the debris ejected from Kasun structure.

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