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Ballistic Trajectory Modeling for the Insensitive Munitions Type IV/V Hazardous Fragment Threshold

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2018 International Explosives Safety Symposium & Exposition
August 6-9, 2018, San Diego, CA

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OVERVIEW

- IM Testing and Requirements
- Current IM Hazardous Fragment Projection Criterion
- Aeroballistic Trajectory Modeling
- Hazard Criteria
- Computational Considerations
- Mass-Distance Curves, New Fragment Projection Criterion
- Calculation Procedure
- Results and Discussion
- Future Work

INSENSITIVE MUNITIONS REQUIREMENTS

Threats	**FUEL FIRE** Such as a truck or an aircraft on a flight deck	**NEARBY HEAT** Such as fire in adjacent magazine, store or vehicle.	**BULLETS** Such as small arms from terrorists or combat	**FRAGMENTS** Such as from bombs, artillery, or IEDs	**SYMPATHETIC REACTION** Such as detonation of adjacent stores	**SHAPED CHARGE JET** RPG, Bomblets, ATGMs: Combat or terrorists
						
Tests	Fast Cook-off	Slow Cook-off	Bullet Impact	Fragment Impact	Sympathetic Detonation	Shaped Charge Jet
	FCO	SCO	BI	FI	SD	SCJ
						

Reactions	Detonation/ Partial Detonation	Explosion	Deflagration/ Propulsion	Burn	No Sustained Reaction
	Type I/II	Type III	Type IV	Type V	Type VI
					

Response type determination

- Photographic evidence
- Blast overpressure
- **Debris thrown/recovered**
- Witness plate gouging

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Current Fragment Projection Curve

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- Fragment mapping is performed after tests, compared to 20J curve (79J curve also exists)
  - Criterion violated if allowable distance exceeded
  - Type IV if violated, Type V if not
  - Substantial efforts to obtain Type V
- Efforts to understand origin of curves
  - Modeling studies, some contact with originators, work done long ago
  - 20J curve corresponds to launch energy, could not reconstruct 79J curve
  - Means the maximum distance a fragment could travel if launched with 20J
- Ultimate outcome: 20J launch curve being replaced with 20J impact at 15m curve for several fragment densities

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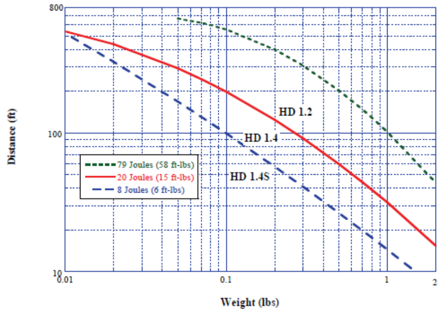

58 ft-lbs (79 J)

D in ft, M in lbs

$D = 101.65*M^{[-1.1061-0.15961*ln(M)]}$

$M = 4.24*D^{[1.8714-1.2433*(ln(D))+0.25442*(ln(D))^2-0.018948*(ln(D))^3]}$

D in m, M in kg

$D = 11.697*M^{[-1.35846-0.15961*ln(M)]}$

$M = 4.533*D^{[0.132633-0.49695*(ln(D))+0.16437*(ln(D))^2-0.018948*(ln(D))^3]}$

15 ft-lb (20 J)

D in ft, M in lbs

$D = 31.49*M^{(-0.98-0.0788*ln(M))}$

$M = 0.000006151*D^{[14.8843-6.0304*((ln(D))+1.0077*((ln(D))^2-0.06313*((ln(D))^3]}$

D in m, M in kg

$D = 4.212*M^{(-1.103-0.0788*ln(M))}$

$M = 0.1283*D^{[4.399-2.973*((ln(D))+0.7077*((ln(D))^2-0.06313*((ln(D))^3]}$

6 ft-lb (8 J)

D in ft, M in lbs

$D = 14.41*M^{(-0.896-0.0252*ln(M))}$

$M = 7.2157*D^{[0.6007-0.9509*((ln(D))+0.2155*((ln(D))^2-0.01758*((ln(D))^3]}$

D in m, M in kg

$D = 2.13*M^{(-0.935-0.0252*ln(M))}$

$M = 2.4193*D^{[-0.8642-0.3317*((ln(D))+0.1319*((ln(D))^2-0.01758*((ln(D))^3]}$

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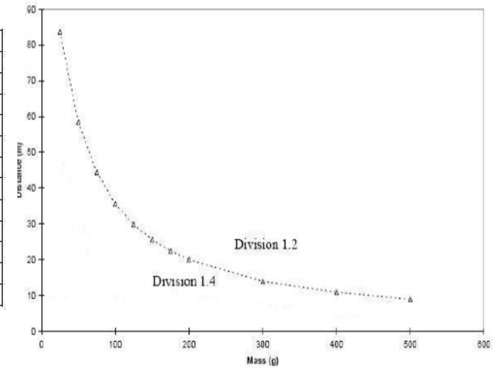
Mass	Projection distance (m)	
(g)	20 J	8 J
25	83.6	46.8
50	58.4	28.7
75	44.4	20.6
100	35.6	16.2
125	29.8	13.3
150	25.6	11.4
175	22.43	10
200	20	8.8
300	13.9	6.3
400	10.9	4.9
500	8.9	4.1



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Existing curve corresponds to 20J launch energy!

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Ballistic Trajectory Modeling

Equations of Motion (Vector Form)

$$\frac{d\mathbf{V}}{dt} = -\frac{\rho S C_D}{2m}|\mathbf{V}|\mathbf{V} + \mathbf{g}$$

$$\frac{d\mathbf{x}}{dt} = \mathbf{V}$$

F=ma with air drag

Equations of Motion (Scalar Form)

$$\frac{dV_x}{dt} = -\frac{\rho S C_D}{2m} V V_x = -\frac{\rho S C_D}{2m} V_x \sqrt{V_x^2 + V_y^2}$$

$$\frac{dV_y}{dt} = -\frac{\rho S C_D}{2m} V V_y - g = -\frac{\rho S C_D}{2m} V_y \sqrt{V_x^2 + V_y^2} - g$$

$$\frac{dx}{dt} = V_x$$

$$\frac{dy}{dt} = V_y$$

(Modeling approximations)

Shape factor data

$$m = kS^{3/2}$$

If the fragments from a given weapon are assumed to be geometrically similar, the mass m and presented area A are related by M = kA$^{3/2}$. Values of k, called a shape factor or ballistic density, may be determined from weight and presented area measurements on fragments recovered from tests of particular weapons. Although the value of k differs from one weapon to another, for forged steel projectiles and fragmentation bombs the average value of 660 grains/in.^{3} (2.60 g/cm^3) has been recommended, while for demolition bombs the value 590 grains/in.^{3} (2.33 g/cm^3) has been applied.

(from Zaker, DDESB-TP-12)

(2.60 g/cm^3)

Drag Coefficient Data

TABLE I DRAG COEFFICIENTS FOR IRREGULAR FRAGMENTS

MACH NUMBER	DRAG COEFFICIENT
0	0.80
0.75	0.88
0.90	1.09
1.15	1.26
2.00	1.14
4.00	1.08
>4.00	1.08

Note: Drag coefficient varies linearly between Mach Number entries

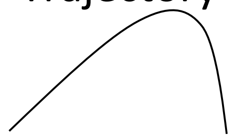
In Design Considerations for Toxic Chemical and Explosives Facilities; Scott, Ralph A., et al.; ACS Symposium Series; American Chemical Society: Washington, DC, 1987.

(from Swisdak, "Fragmentation Effects: An Overview")

Numerical Solver

- MATLAB ode45 (variable-timestep Runge-Kutta)
- Validate with flat fire solutions / ensure same results obtained for very small constant timestep
- Launch height usually zero
- Stop integration when trajectory ordinate becomes negative

Trajectory



Curve Generation / Other Analysis

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Ballistic Trajectory Modeling (Cont'd)

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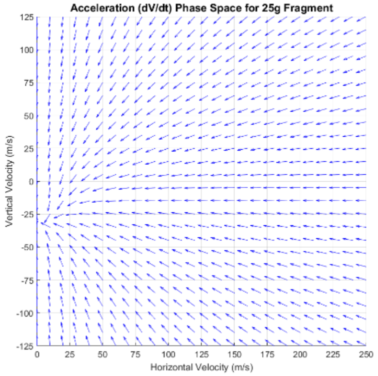
Equations of Motion (Vector Form)

$$\frac{d\mathbf{V}}{dt} = -\frac{\rho S C_D}{2m}|\mathbf{V}|\mathbf{V} + \mathbf{g}$$

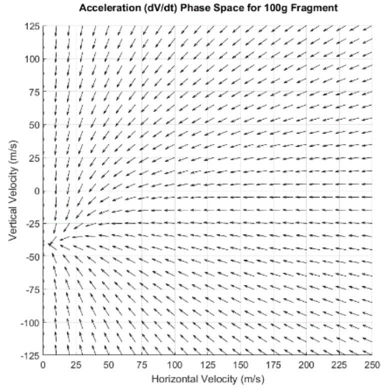
$$\frac{d\mathbf{x}}{dt} = \mathbf{V}$$

F=ma with air drag

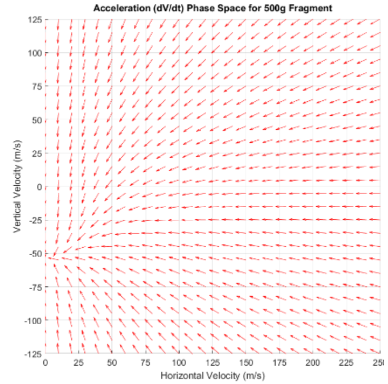
25g



100g



500g



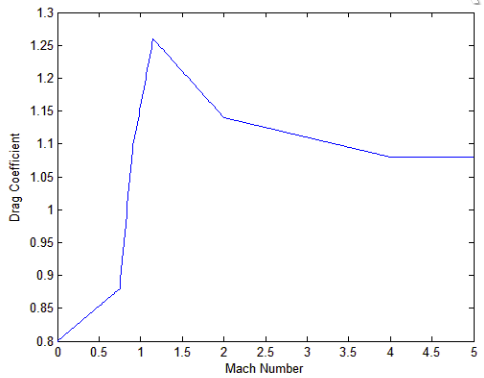
(Sample direction fields for steel fragments)

- System of first-order ODEs; Nonlinear, autonomous (right side a function of **V** only)
- Time-independent direction field in phase space
  - Each initial condition has a unique trajectory
- Critical point where RHS is zero (i.e., terminal velocity)
  - Eigenvalues of Jacobian indicate asymptotically stable
  - All trajectories terminate at critical point

Can run calculations forward or backward in time, except backward from terminal velocity

Ballistic Trajectory Modeling (Cont'd)

Drag Coefficient



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4.00	1.08
>4.00	1.08

Note: Drag coefficient varies linearly between Mach Number entries

$S = Sexact$ - Presented area for a projectile with a specific orientation (no shape factor needed)

$S_{avg} = AS/4$ - Average presented area for randomly tumbling well-defined convex object is ¼ of its surface area

Shape Factor

$m = kS_{avg}{}^{3/2}$

- Shape factor *k* relates mass to average presented area of tumbling chunky fragment
- Measured using icosahedron gauge (*k* = 2600 kg/m^3 for warhead fragments, K = 0.33)

$$m = kS^{3/2} \Rightarrow \rho V_{frag} = kS_{avg}^{3/2} \Rightarrow k = \frac{V_{frag}}{S_{avg}^{3/2}} \rho \equiv K\rho$$

- For a given fragment size and shape, *k* proportional to density

Steel warhead fragment data, simple modification for density changes!

May not be appropriate for debris other than steel warhead fragments!

Hazard Criteria

- IM concerned with warfighter survivability in a combat scenario, but safety benefits as well
- Some kind of optimal acceptable hazard for IM design requirements
  - Safer weapons at the cost of logistics operations and performance → Decrease in survivability
  - Munitions allowed to react violently enough to cause damage and casualties which generate additional logistical burdens and IM hazards → Decrease in survivability
- 20J is presumed to be based on a personnel hazard criterion
  - But results in low velocities for larger fragments
- Injury severity – AIS 2? (moderate injury, 1-2% probability of death)
- Intuitively, energy/area more realistic injury criterion
  - 7.9 J/cm^2 for skin penetration found in open literature
  - But several difficulties, and not conservative for large masses [9]
- Lethality modeling currently being performed for masses of interest

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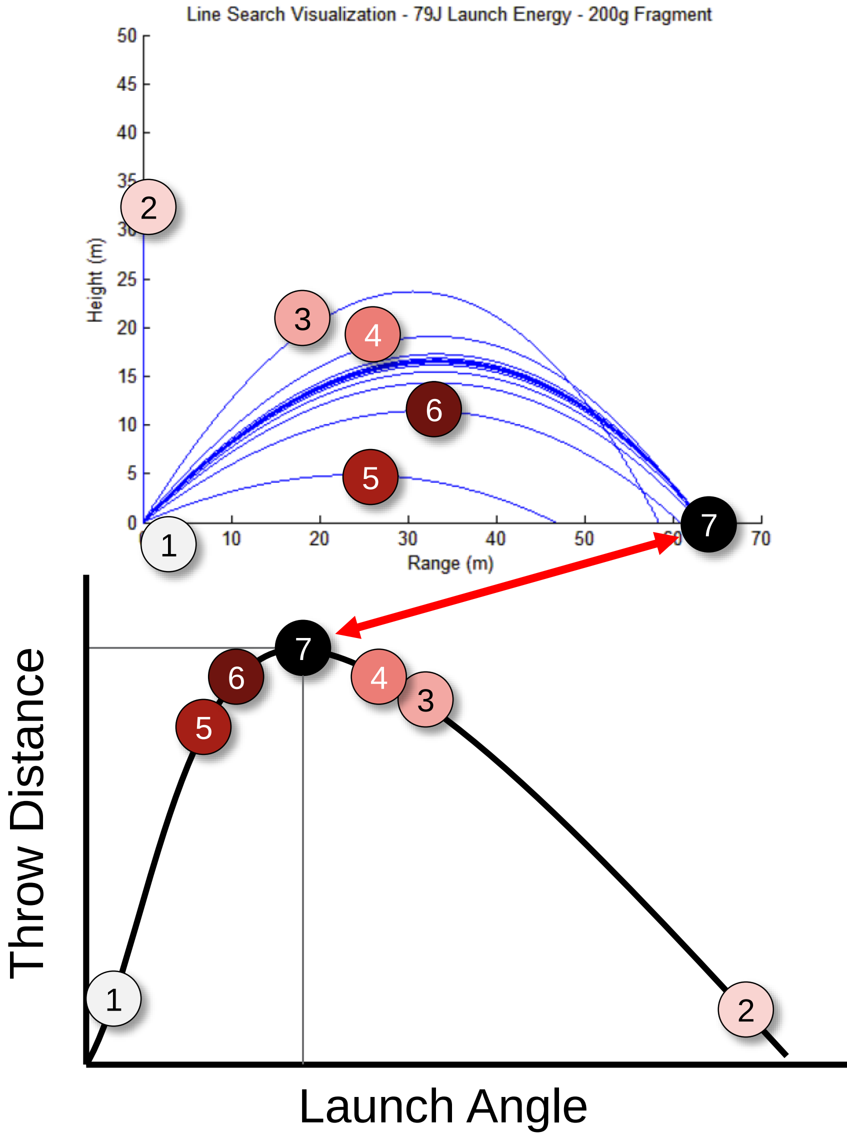
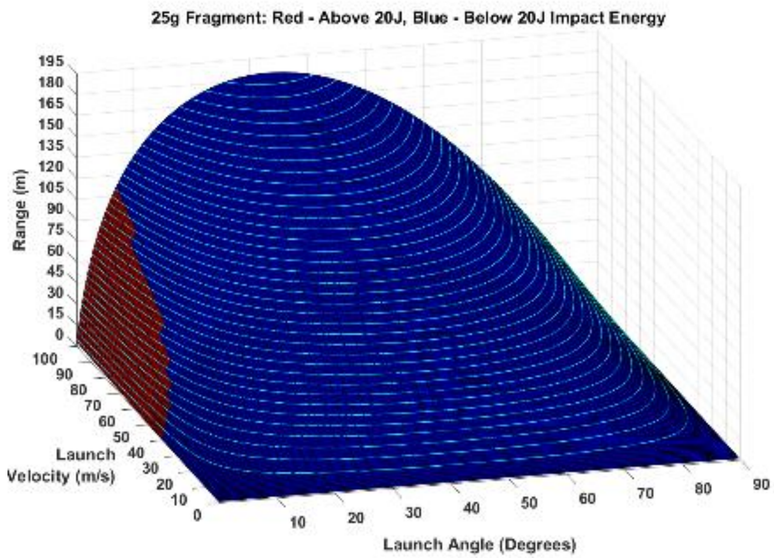
Line Search Algorithms

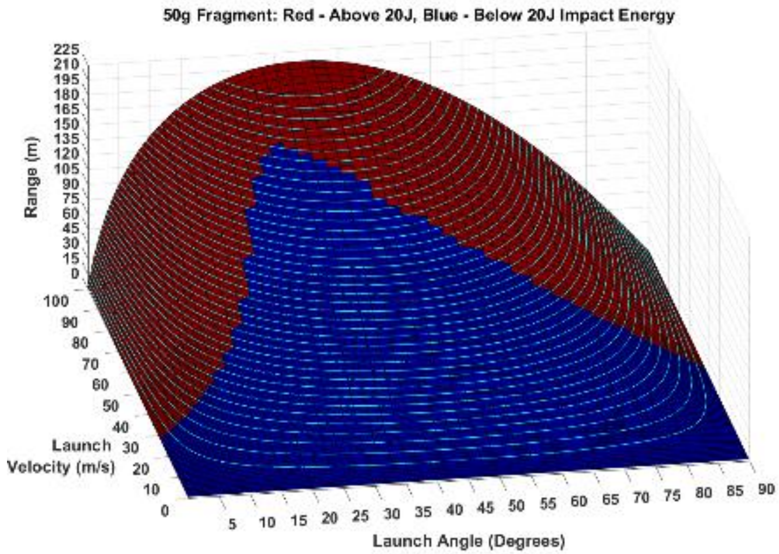
- During curve calculation, working with numerical inputs and outputs
- Maximum or minimum output quantities required
  - Min launch velocity to hit 15m with 20J, max distance for a given launch velocity
- Extremely tedious to adjust input parameters manually
  - Especially if unable to run backwards in time
- Variety of line search algorithms can find max/min quantities automatically if output function known unimodal
  - Reduce search interval until optimum bracketed
  - Used in optimization codes



Mass-Distance Curves

- Direct search plots: Throw distance (z) as a function of launch angle (x) and launch velocity (y)
  - Color coded by impact energy
    - Red >= 20J, Blue < 20J
- For larger masses, there is a distance above which impacts are guaranteed to be hazardous
  - Can use mass-distance curve, but distances are larger
- For smaller masses, only low (and sometimes high) trajectories can produce hazardous impacts
  - Small masses drag down faster
  - Can travel arbitrarily far without hazardous impact
  - Mass-distance curve unbounded

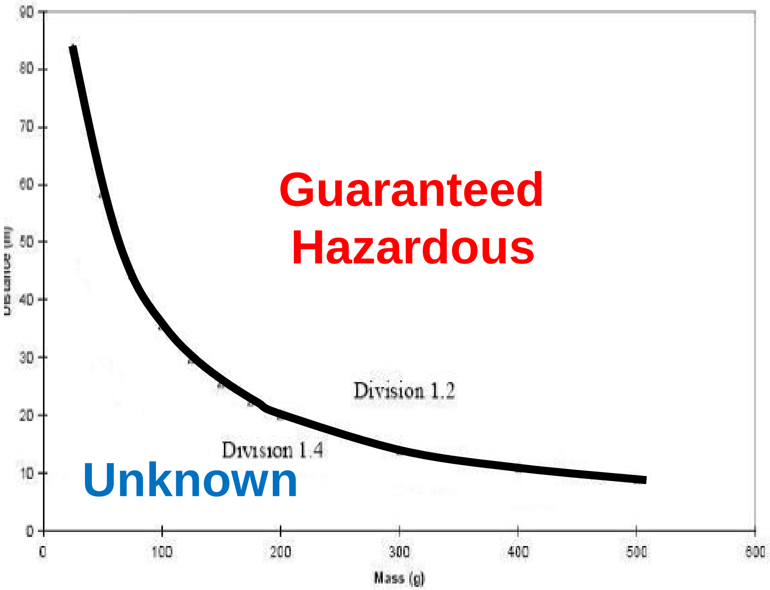




Mass-Distance Curves (Cont'd)

- Important issue with mass-distance curves:
  - Curve represents maximum distance a fragment could travel before being guaranteed hazardous
  - Nothing can be said about fragments which pass the criterion
    - Fragment launched straight up in the air or directly at the ground *always* passes
- As the hazard criterion is made less conservative, the ability to detect hazardous fragments is reduced
  - Driven toward more conservative criterion
  - Errors in the modeling assumptions also have more of an influence on the answers

AOP-39 Mass-Distance Curve



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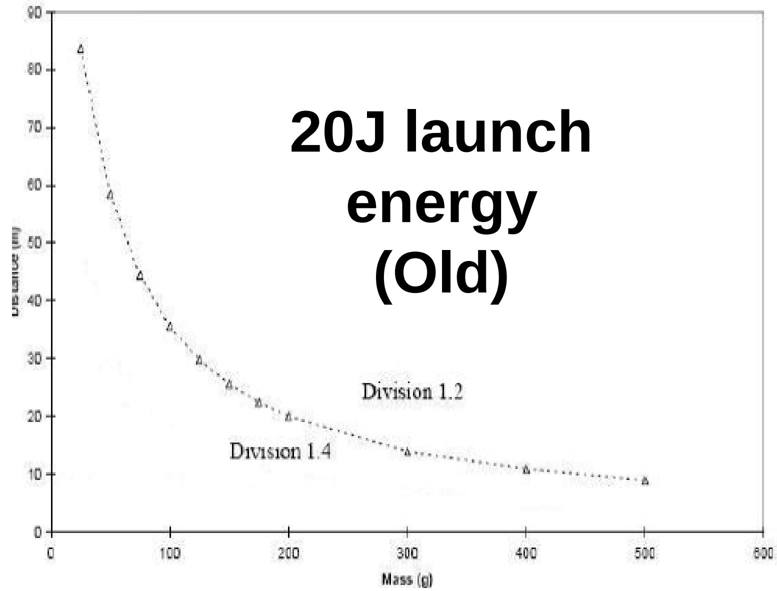
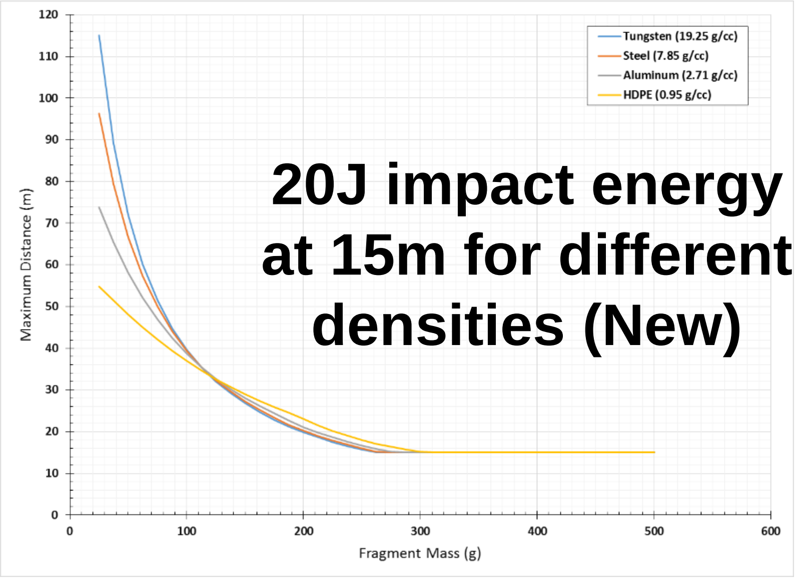
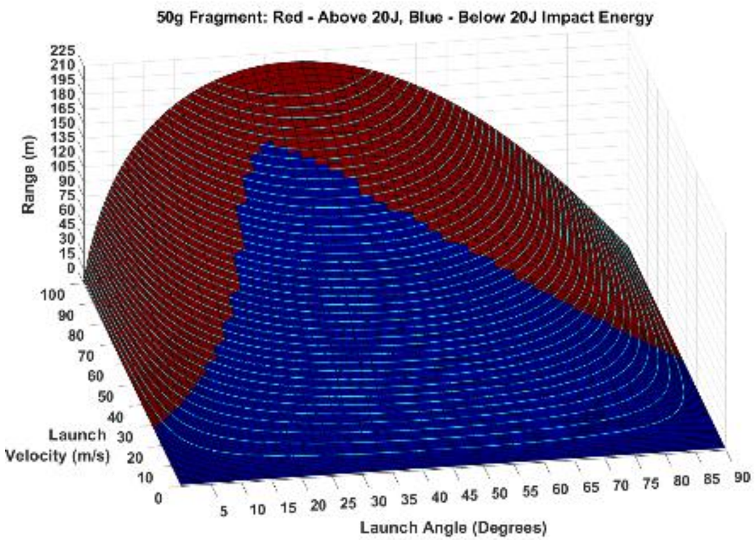
New Fragment Projection Criterion

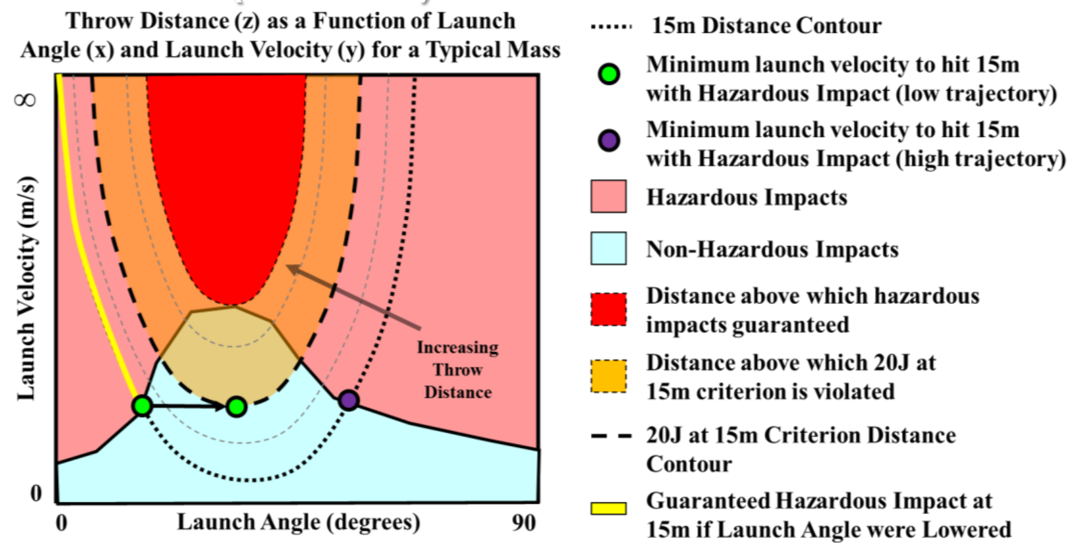
- New fragment projection criterion is 20J impact at 15m [9] for several different fragment densities
- Bumps up the launch velocity to the minimum required to produce a ≥20J impact on the ground at 15m
  - Otherwise maximum distance calculated the same way as before
- A few interesting properties
  - Does not guarantee actual impact was hazardous, but does guarantee it would have been at 15m if the trajectory were lowered
  - Cutoff above ~300g since large fragments impact with at least 20J by virtue of traveling 15m





New Fragment Projection Criterion (Cont'd)





- Distance output calculated in 2 steps for a given mass:
  1) Find the minimum launch velocity to reach 15m with 20J
  2) Using that launch velocity, find the maximum distance the fragment could travel
- Only 2 trajectories impact 15m at 20J; choose the lower launch energy
- Graphically, a fixed launch velocity touches some distance contour at a single point, i.e., the maximum distance achievable
  – Fragments found above that distance guaranteed hazardous at 15m contour for low trajectories

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CALCULATION PROCEDURES

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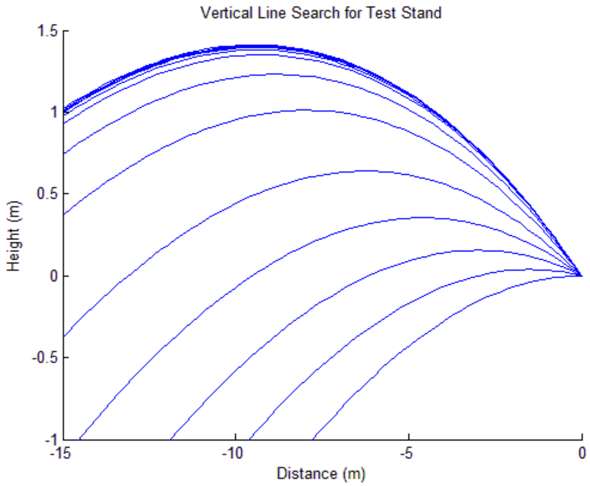
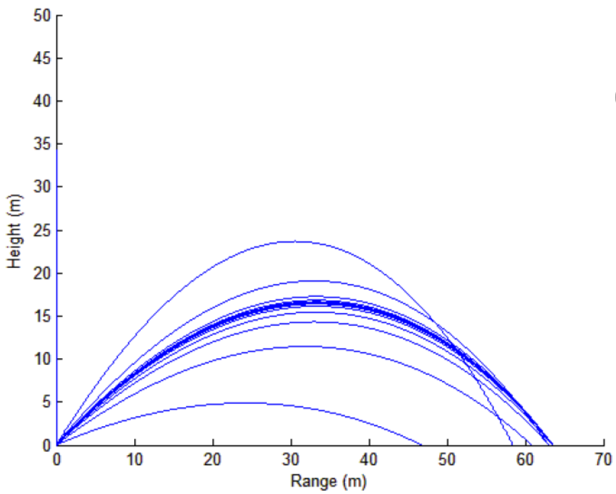
For each fragment mass and density:

1



- Run trajectories backward in time from 20J impact at y=0 at 15m
- Perform line search in vertical direction for trajectory which crosses x=-15m at y=1m (generally 2 possible trajectories)
- If no solution, output is 15m cutoff
- Velocity at (-15,1) is the appropriate launch velocity to use in step 2

2



- Run trajectories forward in time from 1m test stand using launch velocity calculated in Step 1
- Perform line search in horizontal direction to find maximum throw distance
- This max throw distance is a single output point on the mass-distance curve

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RESULTS AND DISCUSSION

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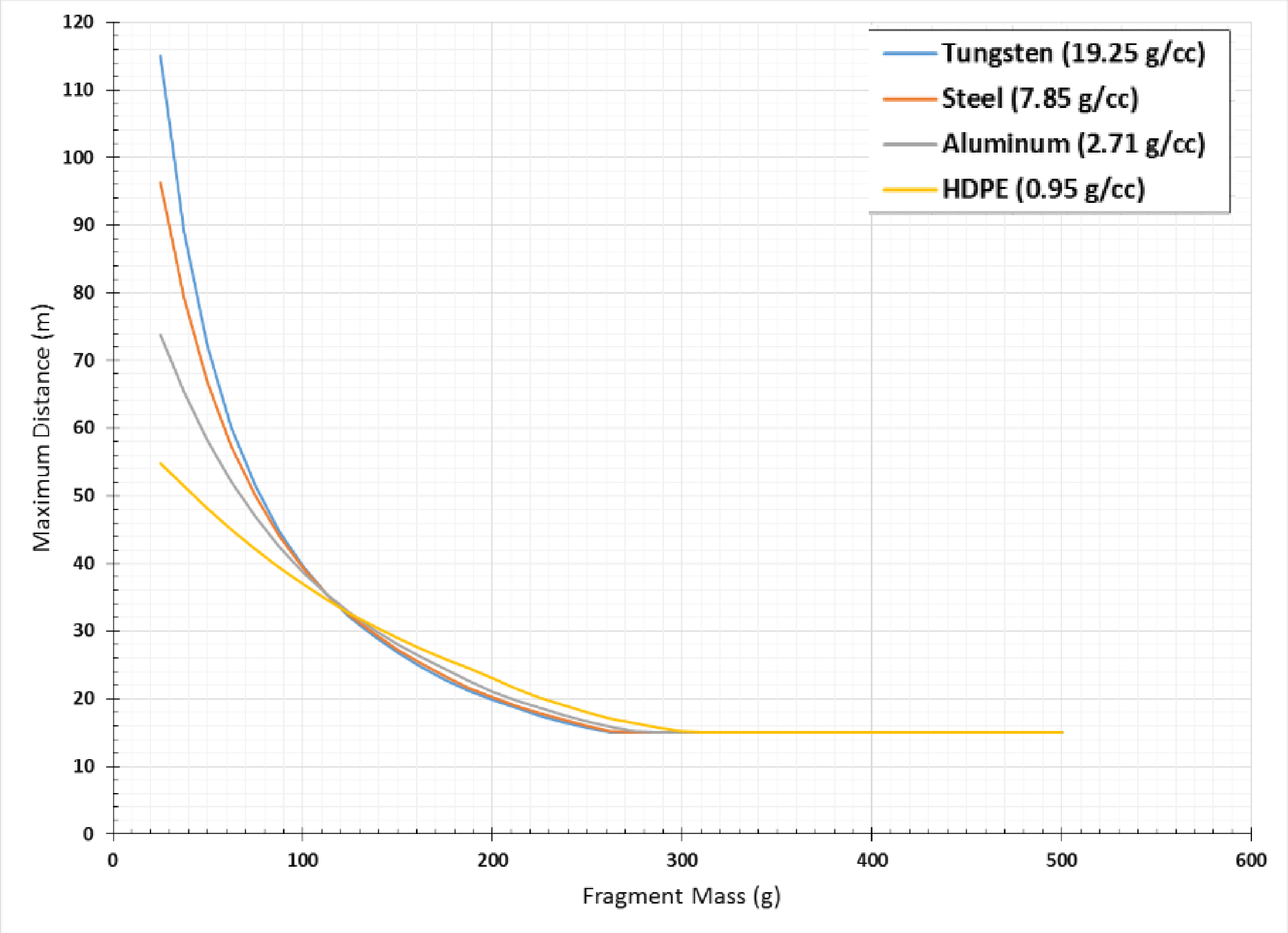

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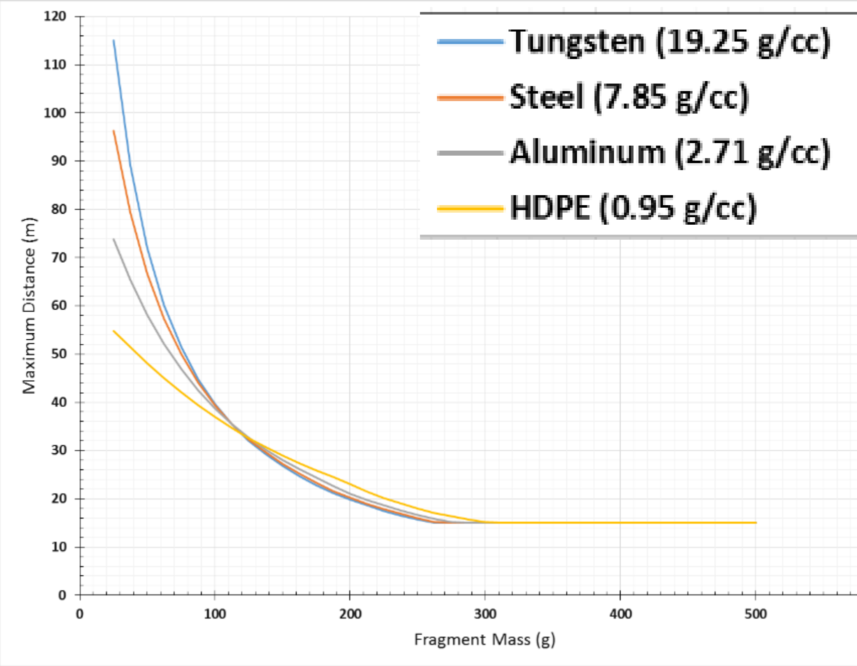
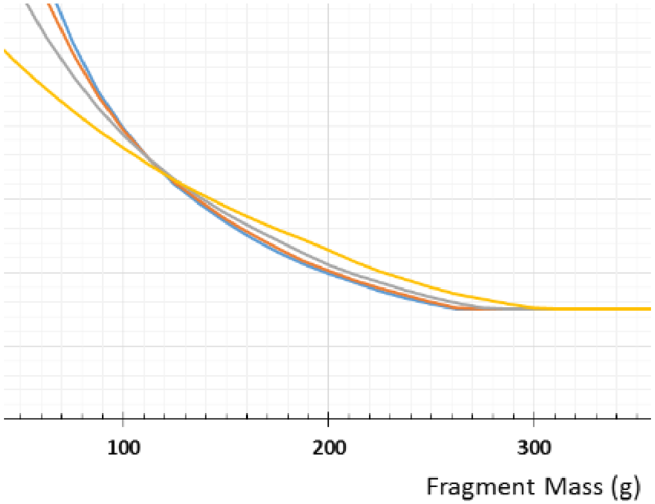
Results and Discussion

- Crossover in curves
  - Small masses: large velocity required to impact 15m at 20J
    - Angle raised in second step, denser frags fly further
  - Large masses: lower velocities involved, higher first step velocity for less dense frags wins out
- Knee in curves, most noticeable for HDPE, is where the high trajectory begins producing a lower minimum launch velocity to hit 15m with 20J
  - Difference is within 1m
- Cutoff at 15m – large masses impact with ≥ 20J by virtue of traveling 15m
- Launch energy slightly less than impact energy for some larger masses due to the 1m test stand





Summary, Conclusions, Future Work

- Ballistic trajectory modeling and hazard metric
- Characteristics of mass-distance curves
  - Lose accuracy and ability to detect hazardous fragments as curves become less conservative
  - Pure impact curves becomes unbounded, and result in very large distances
- Legacy and newly updated hazardous fragment projection curves, and their calculation procedures and results, discussed
  - Old: 20J launch curve
  - New: 20J impact at 15m curve for various densities
- Variety of improvements which could be made
  - Higher fidelity modeling, uncertainty quantification and reduction
  - Consensus on the appropriate injury level, hazard metric, and implementation of more elaborate criteria
- Projection criteria currently being used in various IM programs
  - Better understanding of assumptions → improved interpretation and development of IM standards

QUESTIONS

Backup Slides

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RESULTS AND DISCUSSION (CONT'D)

Curve Results				Backward Search for Minimum Launch Velocity to Hit 15m with 20J (0m Impact Height)								Forward Search for Maximum Range using Same Launch Velocity (1m Launch Height)						
Mass	Mass (g)	Distance		Launch angle	Launch Velocity	Launch Energy	Impact Angle	Impact Velocity	Impact Energy	Range		Launch angle	Launch Velocity	Launch Energy	Impact Angle	Impact Velocity	Impact Energy	Range
0.0250	25.0000	114.9833		-1.4048	42.8505	22.9521	-6.3340	40.0000	20.0000	15.0000		40.9622	42.8505	22.9521	-53.1818	27.7967	9.6582	114.9833
0.0375	37.5000	89.0619		-0.1305	34.5466	22.3776	-7.6289	32.6599	20.0000	15.0000		42.2027	34.5466	22.3776	-50.8082	25.9192	12.5963	89.0619
0.0500	50.0000	72.0147		1.1720	29.6459	21.9720	-8.9400	28.2843	20.0000	15.0000		42.8299	29.6459	21.9720	-49.4732	24.1387	14.5670	72.0147
0.0625	62.5000	60.1222		2.5039	26.3197	21.6477	-10.2686	25.2982	20.0000	15.0000		43.1706	26.3197	21.6477	-48.6490	22.5752	15.9263	60.1222
0.0750	75.0000	51.4186		3.8663	23.8718	21.3698	-11.6192	23.0940	20.0000	15.0000		43.3681	23.8718	21.3698	-48.1181	21.2256	16.8947	51.4186
0.0875	87.5000	44.8027		5.2649	21.9723	21.1216	-12.9932	21.3809	20.0000	15.0000		43.4779	21.9723	21.1216	-47.7604	20.0600	17.6052	44.8027
0.1000	100.0000	39.6180		6.7028	20.4419	20.8935	-14.3965	20.0000	20.0000	15.0000		43.5325	20.4419	20.8935	-47.5146	19.0472	18.1397	39.6180
0.1125	112.5000	35.4532		8.1863	19.1739	20.6798	-15.8335	18.8562	20.0000	15.0000		43.5541	19.1739	20.6798	-47.3481	18.1600	18.5505	35.4532
0.1250	125.0000	32.0382		9.7214	18.1003	20.4763	-17.3115	17.8885	20.0000	15.0000		43.5541	18.1003	20.4763	-47.2391	17.3766	18.8715	32.0382
0.1375	137.5000	29.1909		11.3191	17.1757	20.2816	-18.8350	17.0561	20.0000	15.0000		43.5325	17.1757	20.2816	-47.1663	16.6794	19.1265	29.1909
0.1500	150.0000	26.7808		12.9875	16.3676	20.0924	-20.4170	16.3299	20.0000	15.0000		43.4994	16.3676	20.0924	-47.1252	16.0543	19.3306	26.7808
0.1625	162.5000	24.6794		70.8544	15.6406	19.8761	-72.3322	15.6893	20.0000	15.0000		43.4558	15.6406	19.8761	-47.1083	15.4798	19.4695	24.6794
0.1750	175.0000	22.8227		68.9382	14.9779	19.6294	-70.5691	15.1186	20.0000	15.0000		43.4009	14.9779	19.6294	-47.1096	14.9468	19.5482	22.8227
0.1875	187.5000	21.2200		66.8939	14.3877	19.4069	-68.7014	14.6059	20.0000	15.0000		43.3462	14.3877	19.4069	-47.1321	14.4659	19.6185	21.2200
0.2000	200.0000	19.8230		64.6846	13.8578	19.2039	-66.6971	14.1421	20.0000	15.0000		43.2692	13.8578	19.2039	-47.1507	14.0294	19.6824	19.8230
0.2125	212.5000	18.5946		62.2529	13.3783	19.0166	-64.5063	13.7199	20.0000	15.0000		43.2033	13.3783	19.0166	-47.1930	13.6310	19.7416	18.5946
0.2250	225.0000	17.5074		59.5033	12.9422	18.8437	-62.0454	13.3333	20.0000	15.0000		43.1268	12.9422	18.8437	-47.2365	13.2660	19.7984	17.5074
0.2375	237.5000	16.5397		56.2437	12.5436	18.6844	-59.1467	12.9777	20.0000	15.0000		43.0496	12.5436	18.6844	-47.2882	12.9305	19.8548	16.5397
0.2500	250.0000	15.6763		51.9570	12.1790	18.5409	-55.3580	12.6491	20.0000	15.0000		42.9724	12.1790	18.5409	-47.3465	12.6222	19.9151	15.6763
0.2625	262.5000	15.0000		NaN	NaN	NaN	NaN	NaN	NaN	15.0000		NaN	NaN	NaN	NaN	NaN	NaN	15.0000

Output data for tungsten (19.25 g/cc)

Curve Results				Backward Search for Minimum Launch Velocity to Hit 15m with 20J (0m Impact Height)								Forward Search for Maximum Range using Same Launch Velocity (1m Launch Height)						
Mass	Mass (g)	Distance		Launch angle	Launch Velocity	Launch Energy	Impact Angle	Impact Velocity	Impact Energy	Range		Launch angle	Launch Velocity	Launch Energy	Impact Angle	Impact Velocity	Impact Energy	Range
0.0250	25.0000	96.3297		-1.5911	45.5661	25.9534	-6.2359	40.0000	20.0000	15.0000		38.6882	45.5661	25.9534	-56.8152	23.4733	6.8874	96.3297
0.0375	37.5000	79.3393		-0.3822	36.4488	24.9097	-7.4985	32.6599	20.0000	15.0000		40.4682	36.4488	24.9097	-53.9709	22.7713	9.7225	79.3393
0.0500	50.0000	66.7740		0.8603	31.1246	24.2185	-8.7788	28.2843	20.0000	15.0000		41.4674	31.1246	24.2185	-52.1408	21.8435	11.9285	66.7740
0.0625	62.5000	57.2818		2.1344	27.5359	23.6945	-10.0781	25.2982	20.0000	15.0000		42.0827	27.5359	23.6945	-50.9074	20.8847	13.6304	57.2818
0.0750	75.0000	49.9333		3.4405	24.9080	23.2653	-11.3994	23.0940	20.0000	15.0000		42.4674	24.9080	23.2653	-50.0311	19.9656	14.9484	49.9333
0.0875	87.5000	44.1128		4.7812	22.8765	22.8958	-12.7456	21.3809	20.0000	15.0000		42.7310	22.8765	22.8958	-49.4087	19.1115	15.9797	44.1128
0.1000	100.0000	39.4090		6.1625	21.2449	22.5673	-14.1182	20.0000	20.0000	15.0000		42.9063	21.2449	22.5673	-48.9500	18.3276	16.7951	39.4090
0.1125	112.5000	35.5398		7.5879	19.8966	22.2678	-15.5230	18.8562	20.0000	15.0000		43.0167	19.8966	22.2678	-48.6028	17.6115	17.4468	35.5398
0.1250	125.0000	32.3070		9.0620	18.7570	21.9892	-16.9673	17.8885	20.0000	15.0000		43.0937	18.7570	21.9892	-48.3491	16.9577	17.9727	32.3070
0.1375	137.5000	29.5708		10.5954	17.7773	21.7273	-18.4541	17.0561	20.0000	15.0000		43.1268	17.7773	21.7273	-48.1469	16.3600	18.4009	29.5708
0.1500	150.0000	27.2264		12.1952	16.9223	21.4774	-19.9951	16.3299	20.0000	15.0000		43.1483	16.9223	21.4774	-48.0048	15.8122	18.7520	27.2264
0.1625	162.5000	25.1967		13.8738	16.1671	21.2368	-21.6006	15.6893	20.0000	15.0000		43.1483	16.1671	21.2368	-47.8977	15.3085	19.0410	25.1967
0.1750	175.0000	23.2821		69.4092	15.4402	20.8600	-71.5517	15.1186	20.0000	15.0000		43.1268	15.4402	20.8600	-47.8150	14.8045	19.1777	23.2821
0.1875	187.5000	21.6336		67.4154	14.7990	20.5322	-69.7267	14.6059	20.0000	15.0000		43.0937	14.7990	20.5322	-47.7608	14.3476	19.2987	21.6336
0.2000	200.0000	20.2020		65.2765	14.2285	20.2450	-67.7828	14.1421	20.0000	15.0000		43.0496	14.2285	20.2450	-47.7272	13.9317	19.4093	20.2020
0.2125	212.5000	18.9479		62.9462	13.7168	19.9910	-65.6795	13.7199	20.0000	15.0000		43.0059	13.7168	19.9910	-47.7184	13.5516	19.5125	18.9479
0.2250	225.0000	17.8410		60.3497	13.2546	19.7646	-63.3514	13.3333	20.0000	15.0000		42.9507	13.2546	19.7646	-47.7188	13.2029	19.6105	17.8410
0.2375	237.5000	16.8592		57.3509	12.8354	19.5637	-60.6792	12.9777	20.0000	15.0000		42.8948	12.8354	19.5637	-47.7344	12.8823	19.7069	16.8592
0.2500	250.0000	15.9864		53.6392	12.4545	19.3893	-57.3906	12.6491	20.0000	15.0000		42.8299	12.4545	19.3893	-47.7542	12.5877	19.8064	15.9864
0.2625	262.5000	15.2187		47.9885	12.1123	19.2553	-52.4081	12.3443	20.0000	15.0000		42.7640	12.1123	19.2553	-47.7813	12.3208	19.9240	15.2187
0.2750	275.0000	15.0000		NaN	NaN	NaN	NaN	NaN	NaN	15.0000		NaN	NaN	NaN	NaN	NaN	NaN	15.0000

Output data for steel (7.85 g/cc)

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RESULTS AND DISCUSSION (CONT'D)

Curve Results				Backward Search for Minimum Launch Velocity to Hit 15m with 20J (0m Impact Height)								Forward Search for Maximum Range using Same Launch Velocity (1m Launch Height)						
Mass	Mass (g)	Distance		Launch angle	Launch Velocity	Launch Energy	Impact Angle	Impact Velocity	Impact Energy	Range		Launch angle	Launch Velocity	Launch Energy	Impact Angle	Impact Velocity	Impact Energy	Range
0.0250	25.0000	73.8584		-1.9605	52.4708	34.4148	-6.0322	40.0000	20.0000	15.0000		34.8771	52.4708	34.4148	-61.7570	18.4122	4.2376	73.8584
0.0375	37.5000	65.4265		-0.8881	41.2201	31.8580	-7.2232	32.6599	20.0000	15.0000		37.2060	41.2201	31.8580	-58.8720	18.5594	6.4585	65.4265
0.0500	50.0000	58.1843		0.2260	34.8028	30.2809	-8.4375	28.2843	20.0000	15.0000		38.6995	34.8028	30.2809	-56.7284	18.4051	8.4687	58.1843
0.0625	62.5000	52.0592		1.3761	30.5438	29.1538	-9.6738	25.2982	20.0000	15.0000		39.7204	30.5438	29.1538	-55.0882	18.1020	10.2401	52.0592
0.0750	75.0000	46.8803		2.5612	27.4600	28.2769	-10.9321	23.0940	20.0000	15.0000		40.4573	27.4600	28.2769	-53.8203	17.7232	11.7791	46.8803
0.0875	87.5000	42.4808		3.7824	25.0969	27.5560	-12.2139	21.3809	20.0000	15.0000		40.9956	25.0969	27.5560	-52.8167	17.3074	13.1052	42.4808
0.1000	100.0000	38.7185		5.0415	23.2119	26.9397	-13.5220	20.0000	20.0000	15.0000		41.4018	23.2119	26.9397	-52.0186	16.8781	14.2435	38.7185
0.1125	112.5000	35.4782		6.3420	21.6630	26.3973	-14.8594	18.8562	20.0000	15.0000		41.6986	21.6630	26.3973	-51.3644	16.4483	15.2183	35.4782
0.1250	125.0000	32.6669		7.6884	20.3605	25.9093	-16.2305	17.8885	20.0000	15.0000		41.9289	20.3605	25.9093	-50.8368	16.0268	16.0537	32.6669
0.1375	137.5000	30.2098		9.0846	19.2444	25.4615	-17.6426	17.0561	20.0000	15.0000		42.1042	19.2444	25.4615	-50.4049	15.6178	16.7693	30.2098
0.1500	150.0000	28.0488		10.5406	18.2741	25.0458	-19.0986	16.3299	20.0000	15.0000		42.2360	18.2741	25.0458	-50.0490	15.2241	17.3830	28.0488
0.1625	162.5000	26.1353		12.0639	17.4195	24.6544	-20.6089	15.6893	20.0000	15.0000		42.3354	17.4195	24.6544	-49.7566	14.8466	17.9092	26.1353
0.1750	175.0000	24.3991		70.4445	16.6453	24.2432	-73.7288	15.1186	20.0000	15.0000		42.4124	16.6453	24.2432	-49.5166	14.4782	18.3416	24.3991
0.1875	187.5000	22.6206		68.5398	15.8539	23.5636	-71.9785	14.6059	20.0000	15.0000		42.4674	15.8539	23.5636	-49.3019	14.0702	18.5597	22.6206
0.2000	200.0000	21.0911		66.5250	15.1661	23.0012	-70.1412	14.1421	20.0000	15.0000		42.4892	15.1661	23.0012	-49.1248	13.6968	18.7602	21.0911
0.2125	212.5000	19.7625		64.3691	14.5615	22.5290	-68.1894	13.7199	20.0000	15.0000		42.5001	14.5615	22.5290	-48.9916	13.3541	18.9477	19.7625
0.2250	225.0000	18.5986		62.0265	14.0250	22.1287	-66.0827	13.3333	20.0000	15.0000		42.5001	14.0250	22.1287	-48.8913	13.0387	19.1258	18.5986
0.2375	237.5000	17.5733		59.4247	13.5458	21.7894	-63.7569	12.9777	20.0000	15.0000		42.4784	13.5458	21.7894	-48.8059	12.7480	19.2982	17.5733
0.2500	250.0000	16.6672		56.4322	13.1164	21.5051	-61.0955	12.6491	20.0000	15.0000		42.4561	13.1164	21.5051	-48.7479	12.4805	19.4705	16.6672
0.2625	262.5000	15.8683		52.7509	12.7324	21.2774	-57.8351	12.3443	20.0000	15.0000		42.4235	12.7324	21.2774	-48.7030	12.2360	19.6506	15.8683
0.2750	275.0000	15.1830		47.2415	12.3981	21.1354	-52.9670	12.0605	20.0000	15.0000		42.3904	12.3981	21.1354	-48.6737	12.0196	19.8646	15.1830
0.2875	287.5000	15.0000		NaN	NaN	NaN	NaN	NaN	NaN	15.0000		NaN	NaN	NaN	NaN	NaN	NaN	15.0000

Output data for aluminum (2.71 g/cc)

Curve Results				Backward Search for Minimum Launch Velocity to Hit 15m with 20J (0m Impact Height)								Forward Search for Maximum Range using Same Launch Velocity (1m Launch Height)						
Mass	Mass (g)	Distance		Launch angle	Launch Velocity	Launch Energy	Impact Angle	Impact Velocity	Impact Energy	Range		Launch angle	Launch Velocity	Launch Energy	Impact Angle	Impact Velocity	Impact Energy	Range
0.0250	25.0000	54.7585		-2.5052	69.5653	60.4916	-5.6983	40.0000	20.0000	15.0000		29.9562	69.5653	60.4916	-66.8254	14.0326	2.4614	54.7585
0.0375	37.5000	51.4468		-1.6624	52.6909	52.0563	-6.7632	32.6599	20.0000	15.0000		32.4822	52.6909	52.0563	-64.3955	14.5080	3.9466	51.4468
0.0500	50.0000	48.1366		-0.7653	43.4888	47.2820	-7.8589	28.2843	20.0000	15.0000		34.3057	43.4888	47.2820	-62.4308	14.7405	5.4321	48.1366
0.0625	62.5000	45.0016		0.1743	37.5597	44.0853	-8.9810	25.2982	20.0000	15.0000		35.6896	37.5597	44.0853	-60.7792	14.8342	6.8767	45.0016
0.0750	75.0000	42.0890		1.1525	33.3587	41.7301	-10.1265	23.0940	20.0000	15.0000		36.7774	33.3587	41.7301	-59.3711	14.8390	8.2573	42.0890
0.0875	87.5000	39.4049		2.1671	30.1931	39.8836	-11.2954	21.3809	20.0000	15.0000		37.6563	30.1931	39.8836	-58.1644	14.7831	9.5611	39.4049
0.1000	100.0000	36.9404		3.2174	27.7022	38.3705	-12.4893	20.0000	20.0000	15.0000		38.3703	27.7022	38.3705	-57.1161	14.6836	10.7804	36.9404
0.1125	112.5000	34.6805		4.3046	25.6781	37.0892	-13.7095	18.8562	20.0000	15.0000		38.9524	25.6781	37.0892	-56.1969	14.5523	11.9120	34.6805
0.1250	125.0000	32.6084		5.4319	23.9921	35.9763	-14.9575	17.8885	20.0000	15.0000		39.4350	23.9921	35.9763	-55.3925	14.3981	12.9566	32.6084
0.1375	137.5000	30.7069		6.6010	22.5596	34.9892	-16.2378	17.0561	20.0000	15.0000		39.8423	22.5596	34.9892	-54.6910	14.2274	13.9164	30.7069
0.1500	150.0000	28.9594		7.8160	21.3224	34.0983	-17.5547	16.3299	20.0000	15.0000		40.1828	21.3224	34.0983	-54.0725	14.0446	14.7939	28.9594
0.1625	162.5000	27.3513		9.0837	20.2396	33.2834	-18.9111	15.6893	20.0000	15.0000		40.4573	20.2396	33.2834	-53.5156	13.8531	15.5925	27.3513
0.1750	175.0000	25.8683		10.4099	19.2808	32.5282	-20.3145	15.1186	20.0000	15.0000		40.6985	19.2808	32.5282	-53.0337	13.6562	16.3181	25.8683
0.1875	187.5000	24.4974		11.8025	18.4231	31.8199	-21.7735	14.6059	20.0000	15.0000		40.9076	18.4231	31.8199	-52.6128	13.4558	16.9743	24.4974
0.2000	200.0000	23.0595		68.7496	17.5578	30.8277	-74.4551	14.1421	20.0000	15.0000		41.0835	17.5578	30.8277	-52.1947	13.2199	17.4766	23.0595
0.2125	212.5000	21.4975		66.7883	16.6482	29.4485	-72.6763	13.7199	20.0000	15.0000		41.2486	16.6482	29.4485	-51.7893	12.9339	17.7741	21.4975
0.2250	225.0000	20.1578		64.7250	15.8752	28.3525	-70.8180	13.3333	20.0000	15.0000		41.3802	15.8752	28.3525	-51.4603	12.6701	18.0597	20.1578
0.2375	237.5000	18.9974		62.5289	15.2092	27.4694	-68.8521	12.9777	20.0000	15.0000		41.4674	15.2092	27.4694	-51.1755	12.4260	18.3355	18.9974
0.2500	250.0000	17.9853		60.1565	14.6298	26.7540	-66.7385	12.6491	20.0000	15.0000		41.5336	14.6298	26.7540	-50.9400	12.2005	18.6066	17.9853
0.2625	262.5000	17.0989		57.5370	14.1224	26.1767	-64.4139	12.3443	20.0000	15.0000		41.5889	14.1224	26.1767	-50.7504	11.9929	18.8776	17.0989
0.2750	275.0000	16.3237		54.5449	13.6776	25.7232	-61.7648	12.0605	20.0000	15.0000		41.6217	13.6776	25.7232	-50.5864	11.8028	19.1546	16.3237
0.2875	287.5000	15.6535		50.8974	13.2916	25.3960	-58.5373	11.7954	20.0000	15.0000		41.6436	13.2916	25.3960	-50.4510	11.6322	19.4504	15.6535
0.3000	300.0000	15.1114		45.5463	12.9766	25.2588	-53.7870	11.5470	20.0000	15.0000		41.6652	12.9766	25.2588	-50.3490	11.4907	19.8055	15.1114
0.3125	312.5000	15.0000		NaN	NaN	NaN	NaN	NaN	NaN	15.0000		NaN	NaN	NaN	NaN	NaN	NaN	15.0000

Output data for high density polyethylene (0.95 g/cc)