## ATRBLAST EQUIVALENT WEIGHT AND YIELD DETERMINATIONS BASED ON MEASUREMENT OF ENERGY AND OTHER BLAST WAVE PARAMETERS

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## OVERVIEW

- Classic Equivalent Weight Defined
- New Methodology Description
- Sample Problem
- Shockwave Energy Definition
- Shockwave Energy Application to Pressure-Time Waveform
- Shockwave Energy Closed Form Solution for Modified Friedlander Waveform
- Summary an employee-owned company


## EQUIVALENT WEIGHT ANALYSIS FOR PRESSURE

## (1) <br> Scaled Distance ( $\mathbf{( t / / b}{ }^{1 / 3}$ )

## EQUIVALENT WEIGHT ANALYSIS FOR SCALED PARAMETERS, X (TIME OF ARRIVAL, DURATION, IMPULSE)


$E W=\left(\lambda_{T} / \lambda_{S}\right)^{3}$

Values of scaled range ( $\lambda$ ) are taken from the test and standard curves at the intersections of a sloped line corresponding to the slope of the logarithmic cycles of the graph.

## AN ALTERNATIVE CALCULATIONAL METHODOLOGY

- Assume that the "Standard" Explosive is a hemispherical TNT surface burst as defined by Kingery and Bulmash (ARBRL-TR-02555)
- For each scaled parameter, X (time of arrival, duration, impulse), consider the ratio of the scaled parameter to the scaled distance

$$
\left(\mathrm{X} / \mathrm{W}^{1 / 3}\right) /\left(\mathrm{R} / \mathrm{W}^{1 / 3}\right)=\mathrm{X} / \mathrm{R}
$$

> (note: relation works in both Imperial and SI units)

- Next consider the scaled distance as a function of this parameter ratio for each scaled parameter of interest
- To calculate a yield based on a given scaled parameter (X/W ${ }^{1 / 3}$ ) at the measurement range $R$, perform the following steps:
- Calculate the ratio of the parameter to the range (X/R)
- Enter the appropriate graph and determine the hemispherical TNT scaled distance corresponding to that parameter ratio
- Effective weight of TNT = (actual range R/effective TNT scaled distance) ${ }^{3}$
- Equivalent weight = Actual weight/Effective Weight an employee-owned company


## SCALED TIME OF ARRIVAL/ SCALED DISTANCE RATIO



Scaled Time of Arrival-Scaled Range Ratio (ms/ft)

## SCALED DURATION/ SCALED DISTANCE RATIO



## SCALED POSITIVE IMPULSE/ SCALED DISTANCE RATIO



## SCALED REFLECTED IMPULSE/ SCALED DISTANCE RATIO



## SAMPLE CALCULATION

- An unknown material weighing 500 lb detonates. At a range of 90 ft , the following parameters are measured:
- Time of arrival $=40.0 \mathrm{~ms}$
- Positive duration $=21.0 \mathrm{~ms}$
- Positive impulse $=60.0$ psi-ms
- Based on these values, what is the effective TNT hemispherical weight and the effective hemispherical TNT equivalence?


## SAMPLE CALCULATION (CONTINUED)

- Time of Arrival
- TOA ratio $=40.0 / 90.0=0.44 \mathrm{~ms} / \mathrm{ft}$
- Effective hemispherical scaled distance $=10.05 \mathrm{ft} / \mathrm{bb}^{1 / 3}$
- Effective TNT yield $=(90 / 10.05)^{3}=718 \mathrm{lb}$
- Effective TNT Equivalence $=(718 / 500)=1.44$
- Duration
- Duration ratio $=21.0 / 90.0=0.23 \mathrm{~ms} / \mathrm{ft}$
- Effective hemispherical scaled distance = indeterminate
- Effective TNT yield = $(90 / \text { ??? })^{3}=$ indeterminate
- Effective TNT Equivalence $=(718 / 500)=$ indeterminate
- Positive Impulse
- Impulse ratio $=60.0 / 90.0=0.67 \mathrm{psi}-\mathrm{ms} / \mathrm{ft}$
- Effective hemispherical scaled distance $=11.10 \mathrm{ft} / \mathrm{lb}{ }^{1 / 3}$
- Effective TNT yield $=(90 / 11.10)^{3}=533 \mathrm{lb}$
- Effective TNT Equivalence $=(533 / 500)=1.07$


## HEMISPHERICAL EQUIVALENCE CALCULATOR

## Hemispherical

Equivalence
Calculator

- Tool developed to automate calculation process
- Tool available from author
- mswisdak@apt-research.com


## SHOCKWAVE ENERGY

- The shockwave energy flux is defined as:

$$
\begin{aligned}
& \boldsymbol{E}=\frac{1}{\rho U} \int P_{S}^{2} d t \\
& \rho=\text { Air density } \\
& \text { U = Wave velocity } \\
& \rho U=\text { Characteristic Impedance } \\
& \mathrm{P}_{\mathrm{s}}=\text { Overpressure } \\
& \mathrm{t}=\text { Time }
\end{aligned}
$$

For underwater explosions, the characteristic impedance, $\rho \mathrm{U}$, is nearly constant except very close to the explosion point. However, this is not the case in air. The product $\rho \mathrm{U}$ in real air is a strong function of $\gamma$, the ratio of the specific heats.

## CHARACTERISTIC IMPEDANCE ( $\rho \mathrm{U}$ ) OF AIR



$$
\begin{aligned}
& P_{\mathrm{s}}<=0.6 \text { bars: } \rho \mathrm{U}=421.43 \mathrm{e}^{0.918 \mathrm{P}_{\mathrm{s}}} \\
& 0.6<P_{\mathrm{s}}<1.2 \text { bars: } \rho \mathrm{U}=504.47 \mathrm{e}^{0.6 P_{\mathrm{s}}} \\
& \mathrm{P}_{\mathrm{s}}>=1.2 \text { bars: } \rho \mathrm{U}=868.86 \mathrm{P}_{\mathrm{s}}^{0.763}
\end{aligned}
$$

## APPLICATION

- For digitized pressure-time wave-forms, application can be accomplished in the same manner as the positive impulse-by numerically integrating the wave form
- Impulse—numerically integrate $\mathrm{p}(\mathrm{t}) \mathrm{dt}$ from 0 to the $\tau$, the positive phase duration
- Energy—numerically integrate $[\mathrm{P}(\mathrm{t})]^{2} \mathrm{dt}$ from 0 to the $\tau$, the positive phase duration AN EMPLOYEE-OWNED COMPANY


## SAMPLE WAVEFORM



- 0.5 kg TNT
- Detonated 1m above ground
- Range $=7 \mathrm{~m}$


## AIRBLAST WAVEFORMS

- Assume that a blast wave can be described by a Modified Friedlander Wave Form:

$$
\begin{aligned}
& \mathbf{P}(t)=\mathbf{P}_{s} *(1-t / \tau) * e^{-a t} \\
& P_{s} \quad=\text { Measured peak overpressure } \\
& \tau \quad=\text { Positive phase duration } \\
& \mathrm{t} \quad=\text { Time (in same units as } \tau \text { ) } \\
& " \mathrm{a} "=\text { Modified Friedlander parameter }
\end{aligned}
$$

## MODIFIED FRIEDLANDER IMPULSE AND ENERGY

- Impulse
- If the Modified Friedlander wave form is integrated between 0 and $\tau$, a value for the Positive Phase Impulse, I, is obtained

$$
\begin{equation*}
\mathrm{I}=\mathrm{P}_{\mathrm{s}}^{*}\left(\mathrm{a} \tau+\mathrm{e}^{-\mathrm{at}}-1\right) /\left(\tau^{*} \mathrm{a}^{2}\right) \tag{1}
\end{equation*}
$$

- Energy
- If the integral of $[\mathrm{P}(\mathrm{t})]^{2}$ between 0 and $\tau$ is taken, a value for $\mathrm{E}(\rho \mathrm{O})$ will be obtained:
$\mathrm{E}^{*}(\rho \mathrm{U})=\left\{\left[\left(2 * \mathrm{a}^{*} \tau^{*}\left(\mathrm{a}^{*} \tau-1\right)-\mathrm{e}^{-\left(2^{*} \mathrm{a}^{*} \tau\right)}+1\right] /\left(4 * \tau^{2} * \mathrm{a}^{3}\right)\right\} * \mathrm{P}_{\mathrm{s}}{ }^{2}\right.$
Measured values of $\mathrm{P}_{\mathrm{s}}, \mathrm{I}$, and $\tau$ can be used with Equation 1 to calculate a for each ( $\left.\mathrm{P}_{\mathrm{s}}, \mathrm{I}, \tau\right)$ combination

Each combination of ( $\mathrm{P}_{\mathrm{s}}, \mathrm{I}, \tau, \mathrm{a}$ ) can be used with Equation 2 to calculate $\mathrm{E}^{*}(\rho \mathrm{U})$

## MISTY PICTURE EXAMPLE

- 4,684.7 ton hemisphere of Ammonium Nitrate/Fuel Oil
- Use airblast data compiled on the event to estimate the hemispherical TNT equivalent weight based on incident pressure, positive phase impulse, and energy flux


## MISTY PICTURE



## SUMMARY

- Conventional methodologies for calculating equivalent weight have been reviewed
- A new calculation methodology was described and its use explained through the solution of a sample problem
- A new tool implementing the procedure was introduced
- The concept of shockwave energy as a metric for explosions in air was presented and implementation methodologies discussed
- The use of a Modified Friedlander waveform description for the observed pressure-time wave form was described and a closed form solution of the shockwave energy derived

