# Airblast Equivalent Weight and Yield Determinations Based on Measurements of Energy and Other Blast Wave Parameters 

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#### Abstract

The airblast Equivalent Weight (EW) behavior of various energetic materials is of primary importance in the fields of Protective Construction and Explosive Quantitative Risk Assessment (EQRA). The concept of EW/equivalent yield is reviewed and calculation methodologies are discussed. A new, simplified methodology is presented for determining the hemispherical TNT explosion yields based on measured values of incident pressure, time of arrival, positive phase duration, incident impulse, reflected pressure and reflected impulse.

The energy flux of the blast wave, based on the integral of the square of the overpressure versus time, is described. For explosions in air, its use has been problematic to evaluate as this integral must be divided by the characteristic impedance (product of the density and wave velocity) of the air. The characteristic (shock) impedance of air varies considerably with the shock pressure. The paper describes and demonstrates the calculation of energy flux measurements from conventional P-t curves taking into account the variation of characteristic impedance of air as a function of overpressure along the P-t curve. The energy flux for Modified Friedlander waveforms is also derived and the results applied to a compilation of ammonium nitrate/fuel oil (ANFO) airblast data.


## Introduction

The airblast characteristics of various explosives have been evaluated by conducting full-scale and modeled field tests since before World War II. Comparisons between explosives have been made by performing equivalent weight (EW) analyses, which are only comparative and do not provide the energy of the blast wave itself. The parameters usually observed (shockwave time of arrival, peak pressure, positive duration, positive phase impulse) are obtained from measurements of the overpressure of the blast wave in air versus time (P-t curves). The equivalent weight or equivalent yield can be determined from any of these airblast parameters. It should be noted, however, that the equivalent weight is not usually a constant value; rather, it will vary with the scaled distance (range divided by the cube root of the charge weight).

In addition to comparisons that are made using the usual airblast parameters, such as pressure and/or impulse, a parameter such as energy flux density could be computed and used. Energy flux is routinely computed for underwater explosions but has not generally been used for explosions in air. Later portions of this paper will describe a calculational methodology for explosions in air and then apply it to a sample problem.

## Conventional Equivalent Weight Determination

Conventional EW is determined by one of two procedures. For those variables such as pressure, which is not scaled by the cube root of the charge weight, the EW of a test explosive at a particular peak pressure level is calculated (Reference 1) as shown schematically in Figure 1 by using the following equation:

$$
\begin{equation*}
E W_{p}=\left(\lambda_{T} / \lambda_{S}\right)^{3} \tag{1}
\end{equation*}
$$

Where $\lambda_{\mathrm{T}}=$ scaled range of the test explosive and $\lambda_{\mathrm{S}}=$ scaled range of the standard explosive at a particular peak pressure.


Figure 1. Equivalent Weight Procedure-Pressure
In contrast to the above "conventional" procedure, the EW based on scaled parameters such as positive impulse, time of arrival and positive phase duration are determined in a different fashion as for pressure. The primary difference is that the values of scaled range $(\lambda)$ are taken from the test and standard curves at the intersections of a sloped line corresponding to the slope of the logarithmic cycles of the particular graph. Figure 2 shows this method schematically for analyzing the scaled parameters of impulse, time of arrival, and positive duration.


Figure 2. Equivalent Weight Procedure--Scaled Parameters
When the standard of comparison is a hemispherical TNT surface burst, as presented in References 2 and 3, a new relationship between the ratio of the scaled parameter divided by the scaled distance and the scaled distance has been developed. These relationships are shown graphically in Figure 3 through Figure 6. The ratio is obtained by taking values (from Reference 7) of the scaled parameters (time of arrival, duration, incident impulse, reflected impulse) divided by the scaled range and plotting the ratio as a function of scaled range. As can be seen in the figures, for some ratio values, the functions are multi-valued. However, outside of these regions, for every value of the scaled parameter ratio, there is a unique value for the scaled distance. For any particular set of values of scaled parameter, X, and range, R, one obtains a unique value of scaled range, $\left(\mathrm{R} / \mathrm{W}^{1 / 3}\right)$. Based upon the measured range, the value of $\mathrm{W}^{1 / 3}$ can be calculated. Thus, $\mathrm{W}_{\mathrm{I}}$ is the yield of the explosion based on scaled parameter at that particular range. To determine the EW based on that scaled measurement compared to TNT, one simply calculates the ratio of the yield, $\mathrm{W}_{\mathrm{I}}$, compared to that for $\mathrm{TNT}\left(\mathrm{W}_{\mathrm{I}-\mathrm{TNT}}\right)$ at the same scaled range.


Scaled Time of Arrival-Scaled Range Ratio (ms/ft)
Figure 3. Scaled Time of Arrival-Scaled Range Ratio-Hemispherical TNT Surface Burst


Figure 4. Scaled Positive Duration-Scaled Range Ratio-Hemispherical TNT Surface Burst


Figure 5. Scaled Positive Impulse-Scaled Range Ratio-Hemispherical TNT Surface Burst


Scaled Reflected Impulse-Scaled Range Ratio (psi-ms/ft)
Figure 6. Scaled Reflected Impulse-Scaled Range Ratio-Hemispherical TNT Surface Burst

As can be seen in Figure 4 and Figure 5, both the scaled duration-scaled range ratios and the scaled impulse-scaled range ratios become multi-valued at certain ranges; i.e., for a given value of the ratio, there is more than one value of scaled distance. In these regions, this methodology cannot and should not be used.

Consider the following sample calculation: An unknown material weighing 500 lbs detonates. At a range of 90 ft , the time of arrival of the shockwave is 40.0 ms , the duration of the positive phase is 21.0 ms , and the positive phase impulse is $60 \mathrm{psi}-\mathrm{ms}$. Based on this information, what is the effective TNT hemispherical weight of the material and what is its TNT equivalent weight?

- Scaled range $=90 / 500^{1 / 3}=11.34 \mathrm{ft} / \mathrm{lb}^{1 / 3}$
- Scaled time of arrival $=40.0 / 500^{1 / 3}=5.04 \mathrm{~ms} / \mathrm{lb}^{1 / 3}$
- Scaled positive duration $=21.0 / 500^{1 / 3}=2.65 \mathrm{~ms} / \mathrm{lb}^{1 / 3}$
- Scaled positive impulse $=60.0 / 500^{1 / 3}=7.56 \mathrm{psi}-\mathrm{ms} / \mathrm{lb}^{1 / 3}$
- Scaled time of arrival-scaled range ratio $=5.04 / 11.34=0.44 \mathrm{~ms} / \mathrm{ft}$
- Scaled positive duration-scaled range ratio $=2.65 / 11.34=0.23 \mathrm{~ms} / \mathrm{ft}$
- Scaled positive impulse-scaled range ratio $=7.56 / 11.34=0.67 \mathrm{psi}-\mathrm{ms} / \mathrm{ft}$

From Figure 3, a scaled time of arrival-scaled range ratio of 0.44 corresponds to a scaled range of $10.05 \mathrm{ft} / \mathrm{lb}^{1 / 3}$. From Figure 4, a scaled positive duration-scaled range ratio of 0.23 gives a scaled range that is multi-valued. From Figure 5, a scaled positive impulse-scaled range ratio of 0.67 corresponds to a scaled range of $11.10 \mathrm{ft} / \mathrm{b}^{1 / 3}$. These values of effective scaled distance along with the actual range at which the data were collected gives the following yields:

- Time of arrival
- Effective TNT hemispherical yield $=(90 / 10.05)^{3}=718 \mathrm{lbs}$
- $\mathrm{EW}=718 / 500=1.44$
- Positive duration
- Effective TNT hemispherical yield = indeterminate
- $\mathrm{EW}=718 / 500=$ indeterminate
- Positive impulse
- Effective TNT hemispherical yield $=(90 / 11.10)^{3}=533 \mathrm{lbs}$
- $\mathrm{EW}=533 / 500=1.07$

An Excel software tool based on this methodology has been developed and is available from the authors that can calculate hemispherical TNT equivalence based on most common airblast parameters. The tool is called the Hemispherical Equivalence Calculator.

Misty Picture (Reference 4) was a 4684.7 ton, hemispherical Ammonium Nitrate/Fuel Oil (ANFO) event. The airblast data recorded from this event were compiled by the authors and
evaluated using the methodology just described to obtain the impulse equivalent weight relative to hemispherical TNT. The results are shown in Figure 7, which presents a curve of equivalent weight based on positive impulse as a function of scaled range for this event.


Figure 7. Misty Picture Hemispherical TNT Impulse Equivalence

## Shockwave Energy

The energy flux of blast waves has been described theoretically for underwater explosions by Cole in Reference 5. This theoretical description applies to air as well. The energy flux, E, is based on the following basic equation:

$$
\begin{equation*}
E=\frac{1}{\rho U} \int P_{S}^{2} d t \tag{2}
\end{equation*}
$$

where
$\rho=$ Air density
$\mathrm{U}=$ Wave velocity
$\mathrm{P}_{\mathrm{s}}=$ Overpressure
$\mathrm{t}=$ Time
For underwater explosions, the characteristic impedance, $\rho \mathrm{U}$, is nearly constant except very close to the explosion point. However, this is not the case in air. The product $\rho \mathrm{U}$, (sometimes called the characteristic impedance), in real air is a strong function of $\gamma$, the ratio of the specific heats. Gamma, in turn, is a function of the shock strength. The variation of $\rho U$ for air as a function of overpressure, taken from Reference 6, is shown graphically in Figure 8.


Figure 8. Characteristic Impedance of Air ( $\rho U$ ) Versus Pressure, $P_{s}$
For purposes of curve fitting, the curve shown in Figure 8 can be divided into three regions of $\rho U$ versus overpressure:

$$
\begin{gather*}
P_{\mathrm{s}}<=0.6 \text { bars: } \rho \mathrm{U}=421.43 \mathrm{e}^{0.918 \mathrm{Ps}}  \tag{3}\\
0.6<P_{\mathrm{s}}<1.2 \text { bars: } \rho \mathrm{U}=504.47 \mathrm{e}^{0.6 \mathrm{Ps}}  \tag{4}\\
P_{\mathrm{s}}>=1.2 \text { bars: } \rho \mathrm{U}=868.86 \text { Ps }^{0.763} \tag{5}
\end{gather*}
$$

where $P_{s}$ is in bars and $\rho U$ is in $\mathrm{kg} / \mathrm{m}^{2}$-s
If a digitized pressure-time waveform is available, the procedure for calculating the energy flux density is similar to that done for the positive phase impulse-numerically integrating the waveform. The waveform shown in Figure 9 was recorded at 7 meters from a 0.5 kg cast TNT charge detonated at a height of 1 m . Note that the values of total impulse in units of KPa-ms designated as I, and the energy flux density in units of $\mathrm{kg}-\mathrm{m} / \mathrm{m}^{2}$, are shown at the top of this figure.


Figure 9. Recorded Waveform-- 0.5 Kg Cast TNT Charge, Range - 7 meters
Assume that recorded airblast pressure-time waveforms have the form of a Modified Friedlander wave, as described by Dewey in Reference 7. This means that a pressure-time waveform can be represented by an equation of the form:

$$
\begin{equation*}
\mathrm{P}(\mathrm{t})=\mathrm{P}_{\mathrm{s}} *(1-\mathrm{t} / \tau) * \mathrm{e}^{-\mathrm{at}} \tag{6}
\end{equation*}
$$

where
$\mathrm{P}_{\text {s }} \quad=$ Measured peak overpressure
$\tau \quad=$ Positive phase duration
$\mathrm{t} \quad=$ Time (in same units as $\tau$ )
"a" = Modified Friedlander parameter
If Equation 6 is integrated between 0 and $\tau$, the following expression for the incident impulse is obtained:

$$
\begin{equation*}
\mathrm{I}=\mathrm{P}_{\mathrm{s}}^{*}\left(\mathrm{a} \tau+\mathrm{e}^{-\mathrm{a} \tau}-1\right) /\left(\tau^{*} \mathrm{a}^{2}\right) \tag{7}
\end{equation*}
$$

By integrating $\mathrm{P}(\mathrm{t})^{2}$ between 0 and $\tau$, an expression for $\mathrm{E}^{*}(\rho \mathrm{U})$ can be obtained:

$$
\begin{equation*}
\mathrm{E}^{*}(\rho \mathrm{U})=\left\{\left[\left(2^{*} \mathrm{a}^{*} \tau^{*}\left(\mathrm{a}^{*} \tau-1\right)-\mathrm{e}^{-\left(2^{*} \mathrm{a}^{*} \tau\right)}+1\right] /\left(4^{*} \tau^{2 *} \mathrm{a}^{3}\right)\right\} * \mathrm{P}_{\mathrm{s}}^{2} \ldots \ldots \ldots .\right. \tag{8}
\end{equation*}
$$

Equation 7, along with the measured values for $\mathrm{I}, \mathrm{P}_{\mathrm{s}}$, and $\tau$ for each waveform, can be used to calculate the value of the Modified Friedlander parameter "a". Once a value of "a" is obtained, Equation 8 can be used to calculate $E^{*}(\rho U)$ for each waveform. Equations (3), (4), and (5) (relationship between $P_{s}$ and $(\rho U)$ ) given previously can be used to calculate ( $\rho \mathrm{U}$ ). Once the value of $(\rho \mathrm{U})$ is known, the energy flux can be obtained.

As an example of this technique, consider the airblast recorded on the Misty Picture event (Reference 4). Using the compiled airblast data scaled to 1 kg and Equations (7) and (8), values of $E^{*}(\rho U)$ as a function of scaled distance can be obtained. Figure 10 presents a plot of energy flux vs. scaled distance for Misty Picture as well as for a hemispherical TNT charge (data taken from Reference 2). The information used to generate Figure 10 can also be used to generate an equivalence comparison based on energy flux. Figure 11 takes this information and presents a plot of average $\mathrm{EW}_{\text {Energy }}$ for Misty Picture relative to hemispherical TNT.



Figure 11. Average Energy Flux Equivalence for Misty Picture Event

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