Headquarters U.S. Air Force

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Model-Based Engineering (MBE) Structures Viewed Through Axiomatic Investigation in Abstract Algebra & Application Implications

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Mr. Jeffrey D Havlicek
SAF/AQR

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Axiomatic Review of MBE Structures -- Overview

- U.S. Office of the Secretary of Defense Digital Engineering Strategy
- How Do We MBE?
- ABCs of Axioms
- Group Theory Requirements
- Testing Axiom Fit-for-Use
- Empowering MBE by Using Category Theory Tools
- Summary

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U.S. Office of the Secretary of Defense Digital Engineering Strategy

This strategy describes the "what" necessary to foster the use of digital engineering practices. Those implementing the practices must develop the "how" — the implementation steps necessary to apply digital engineering in each enterprise. The Services should develop corresponding digital engineering implementation plans during 2018 to ensure the Department advances this timely and imperative effort.

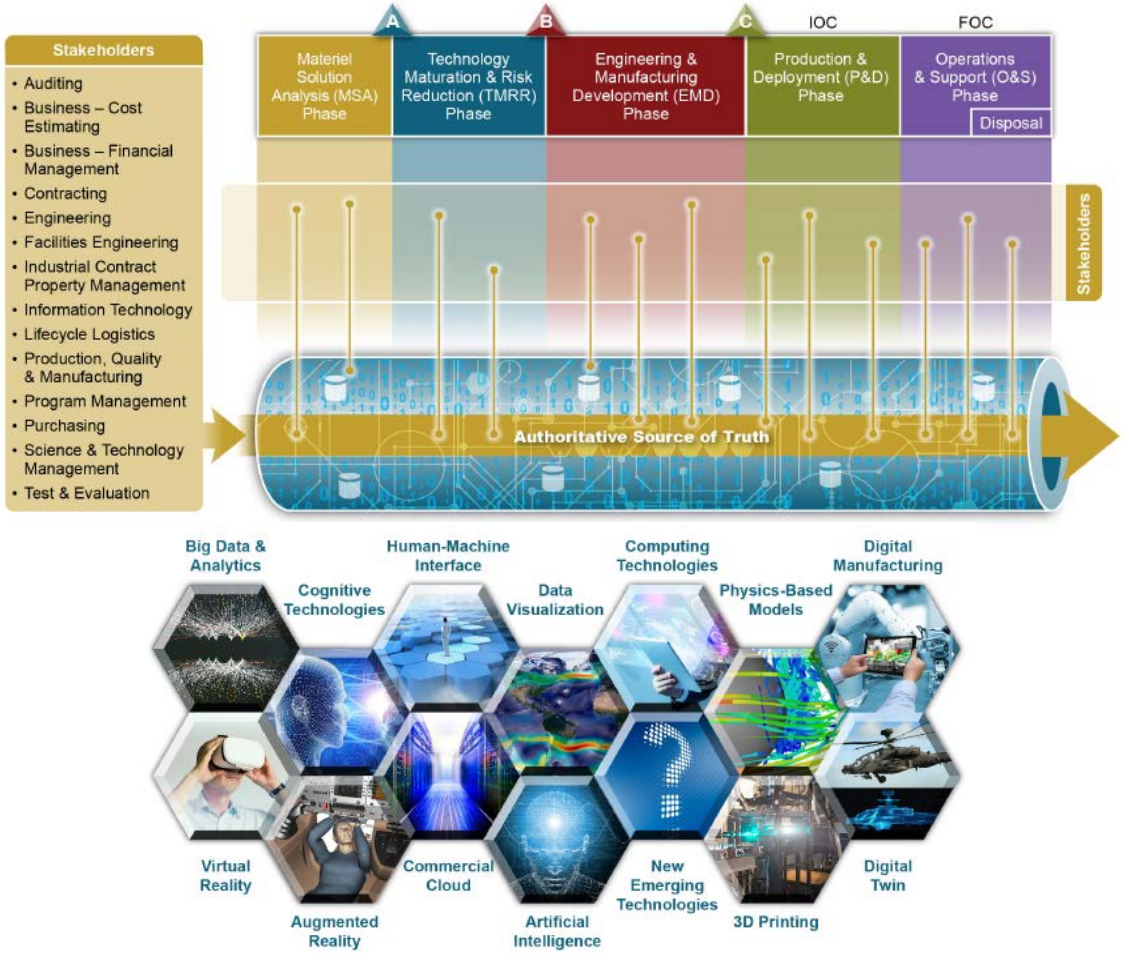
Michael D. Griffin
Under Secretary of Defense for Research and Engineering

The services develop the "how".

Refs:
U.S. DoD, Digital Engineering Strategy, Office of the ODASD(SE), 3030 Defense Pentagon 3C167, Washington, DC. Email: osd.atl.at-re.se@mail.mil. June 2018.

How Do We MBE?

- How do we find the right technologies to create an MBE structure?



Refs:

U.S. DoD, *Digital Engineering Strategy*, Office of the ODASD(SE), 3030 Defense Pentagon 3C167, Washington, DC. Email: osd.atl.at-re.se@mail.mil. June 2018. Distro A Public Release, Case #: 88ABW-2018-5163

How Do We MBE? -- "Cloudy" vs. Foundational Engineering

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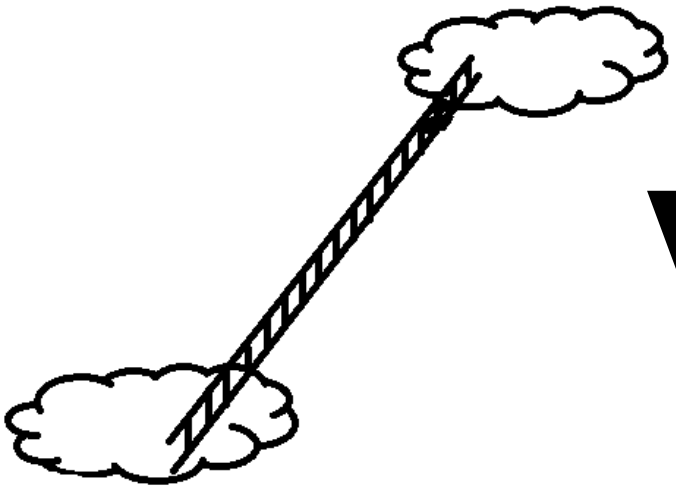
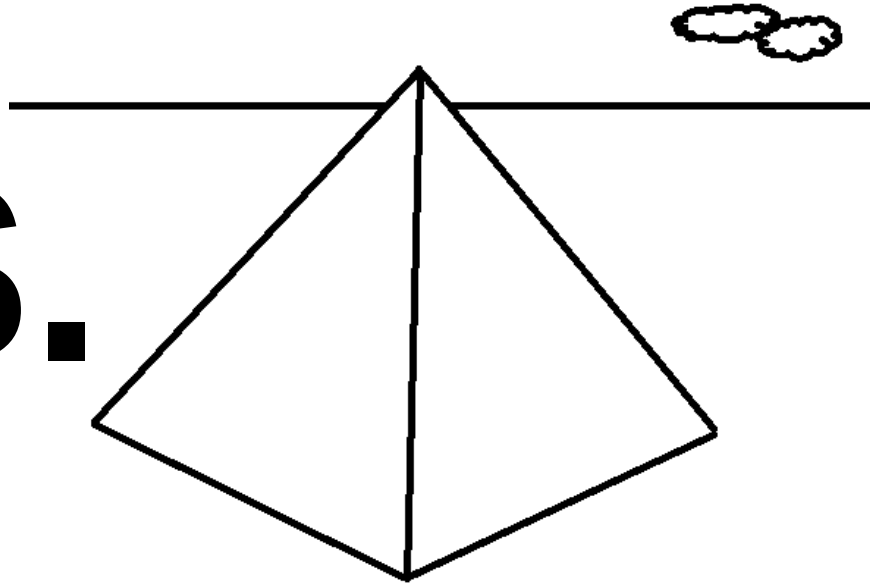
Uncertainty linked to uncertainty:
Collapses quickly



VS.

Well-established foundation:
Withstanding the tests of time



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ABCs of Axioms

- Axioms are the foundation of math
- Math is the foundation of science
- Science is the foundation of engineering

Model-based engineering NEEDS axioms!

"The laws of nature are but the mathematical thoughts of God."-- Euclid

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ABCs of Axioms

- an *axiom* is any mathematical statement that serves as a starting point from which other statements are logically derived
- *axiom* comes from the Greek word ἀξίωμα (*axíōma*), a verbal noun from the verb ἀξιόειν (*axioein*), meaning “to deem worthy”, but also “to require”
- *non-logical axioms* (e.g., $a + b = b + a$) are substantive assertions about the elements of the domain of a specific mathematical theory (such as arithmetic). “Axiom”, “postulate”, and “assumption” may be used interchangeably in this case.
- a *non-logical axiom* is *not* a self-evident truth, but rather a *formal logical expression* used in deduction to *build a mathematical theory*. To axiomatize a system of knowledge is to show that its claims can be derived from a small, well-understood set of sentences (the axioms).

Ref: https://en.wikipedia.org/wiki/Axiom

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Group Theory Requirements-- Are we Groupies?

- For some set and some operation:
  - **Four Requirements:**
    - **Closure/Totality (of the set w.r.t. the operation)**
      - even + even = $2k + 2m = 2*(k + m)$ ⇨ even ⇨ Closure
      - odd + odd = $(2k+1)+(2m+1)+=2*(k+m)+2$ ⇨ even ⇨ No Closure
    - **Associativity (property of the operation on the set)**
      - $(A \circ B) \circ C = A \circ (B \circ C)$
    - **Identity (w.r.t. the operation)**
      - A “Zero” or “Unit” in an open relational sense
    - **Invertibility (of an object w.r.t. the operation)**
      - An “inverse” in an open relational sense
  - And one more that can be useful for further restrictive abelian groups:
    - **Commutivity (property of the operation on the set)**

Ref: https://en.wikipedia.org/wiki/Axiom

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Testing Axiom Fit-for-Use

- Totality Axiom Issues:
  - Data aggregation can generate sensitivities
    - Record with: 9 digit integer + “SSN” + NAME ⇨ PII
    - Similar issues with:
      - ITAR, classified, HIPAA, PROPRIN, STINFO, etc.
  - So, closure / totality is not supported, unless the whole data archive is secured in the most secure “worst case” enclosure after aggregation considerations

Data aggregation of records increases sensitivity of the archive repository

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Testing Axiom Fit-for-Use

- Invertibility Axiom Issues:
  - What is the inverse of a CAD drawing? Or a CFD analysis? Or a part number? What is the operation on CAD drawings? Does it even have an identity?
    - Invertibility seems questionable

- Commutivity Axiom Issues:
  - A “door hinge” is not the same as a “hinge door”. A“key risk” is not the same as a “risk key”
    - Commutivity seems in question

Testing Axiom Fit-for-Use

- Associativity Axiom Acceptance:
  - Concatenation operation is useful for associativity of strings and of records
    - Concatenation (“a”,”bcd”) = Concatenation (“abc”, ”d”) = “abcd”
  - Associativity seems reasonable

- Identity Axiom Acceptance:
  - A null pointer or null string is useful for a “zero element” needed for identity
    - Concatenation (*a*,“”) = Concatenation (“”, *a*) = “” + *a* = *a* + “” = *a*
  - Identity seems reasonable

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Testing Axiom Fit-For-Use

Structure\Axiom	Totality	Associativity	Identity	Invertibility	Commutativity
Semigroupoid		Required			
Category		Required	Required		
Groupoid		Required	Required	Required	
Semigroup	Required	Required			
Monoid	Required	Required	Required		
Group	Required	Required	Required	Required	
Abelian Group	Required	Required	Required	Required	Required

We find ourselves in Category Theory.
We can consider using benefits of a Monoid algebra structure in very limited situations where sensitivity by aggregation is either over-controlled or structurally avoided.

Category Theory

- A category *C* consists of two entities or classes:
  - A class ob(*C*) whose elements are called objects
  - A class hom(*C*) whose elements are called morphisms or maps or *arrows*. Each morphism *f* as a source object *a* and a target object *b*.
    - A morphism is a structure-preserving map from one structure to another one of the same type.
      - Has to admit associative composition

Identical to a directed graph!

- Two required morphisms:
  - Identity morphism: ∀ object *X*, ∃ a morphism id_X: *X*→*X* called the identity morphism on *X* such that for every morphism f:A → B we have $id_B \circ f = f = f \circ id_A$.
  - Associativity morphism: $h \circ (g \circ f) = (h \circ g) \circ f$
    - *e.g. a B-52 object is a vehicle, a wheeled vehicle, and a flying vehicle after assigned inheritance attributes*

Ref: https://en.wikipedia.org/wiki/Morphism

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Empowering MBE by Using Category Theory Tools

- Having settled on axioms for MBE, let's call this common case "***Category Theoretic Model Based Engineering (CTMBE)***".
- Now what?
- Three Fruitful Activities:
  - Avoid misuse of tools not appropriate for CTMBE
  - Mathematically utilize our axiom base of Category Theory for "new innovations"
  - Use tools compatible with Category Theory

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Empowering MBE by Using Category Theory Tools

- Category Theory / Discrete Math has a Wealth of Tools:
  - Sorting and Ordering
    - Heap sorts
    - Quicksort
  - Data Structures
    - Stacks and queues
    - Hash tables
    - Binomial trees
    - Binary search trees
    - Binomial heaps
    - Fibonacci heaps
    - Spanning trees
  - Relations
    - Objects and pointers
    - Hash functions
    - Object oriented programming / Inheritance
  - Directed Graphs
    - Graph Theory
    - Clique analysis
    - Centrality
    - Min cut sets
    - Weighted graphs / network theory
      - Shortest path

Ref: Cormen, Thomas, et. Al. Introduction to Algorithms, 2nd Ed. MIT Press / McGraw-Hill Book Company, 2001.

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Empowering MBE by Using Category Theory Tools

- Pragmatic Examples:
  - Closure Assurance
    - What is the max theoretical information security state after aggregation of future record collection?
    - What IT solution architecture can handle such concerns?
  - Heap data structure → max priority queues
    - What is the highest priority Engineering Change Proposal pending in the weapon system authoritative source of truth stack?
  - Weighted networks → min cut sets
    - What are the minimum physical components or communication pathways which cause loss of primary system operations?

Ref: Cormen, Thomas, et. Al. Introduction to Algorithms, 2nd Ed. MIT Press / McGraw-Hill Book Company, 2001.

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Empowering MBE by Using Category Theory Tools

- For the CTMBE Tools Developer:
  - Design software data handling and menu tools to match the robust features of CTMBE
- For the MBE Tool Buyer:
  - Demand robust data structures, query features, and data analysis menu options that provide full value of CTMBE
- For a Program Chief Data Officer & Chief Analytics Officer:
  - Plan for a robust structure consistent with some of the best data management practices and data analysis techniques
- For the Future MBE Tool User:
  - Refine skills with database structures, advanced query language, object-oriented / inheritance methods, and network theory

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Summary

- Model-Based Engineering, like all science application, needs to leverage math and axioms to maximize tool development and application utility of MBE practice
- Category theory with its requirements of associativity and identity axioms fits well to MBE and is well known and developed in discrete math, data structures, sorting and ordering, directed graphs, and weighted networks.
- Monoid theory is also useful if we can either heavily restrict combinatorial spillage of security levels or create overly-secure enclaves of data stacks
- Applications are endless—go build CTMBE data repositories and the tools to manage them for benefits in structural efficiency, analytic tools, and insights!

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