An Alternative Method for Determining Penetration Limit Velocities Using Residual Velocity Data

U.S. ARMY COMBAT CAPABILITIES DEVELOPMENT COMMAND – ARMAMENTS CENTER

Kenneth Hohnecker, Christopher Drake, Michael Cataldi

US Army, DEVCOM, Picatinny Arsenal, NJ

Email: kenneth.h.hohnecker.civ@army.mil

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PRESENTATION OUTLINE

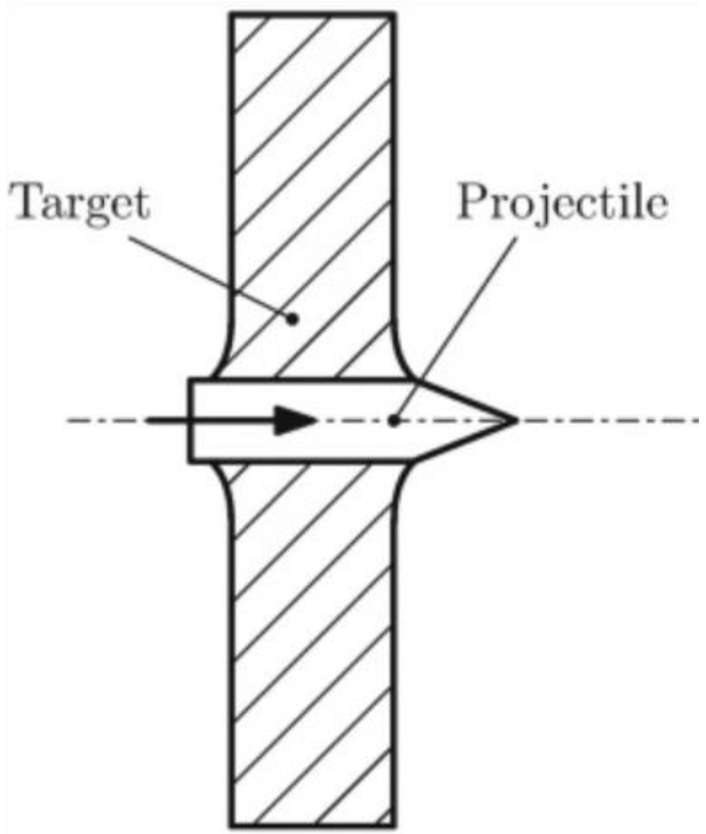
- Problem Statement
- Binary Data
- Current Methods
- Proposed Methodology
- Experimental Results
- Conclusion
- Future Work

PROBLEM STATEMENT

Determine the velocity at which a projectile will penetrate a target with some given probability (known as a *limit velocity (LV)*)

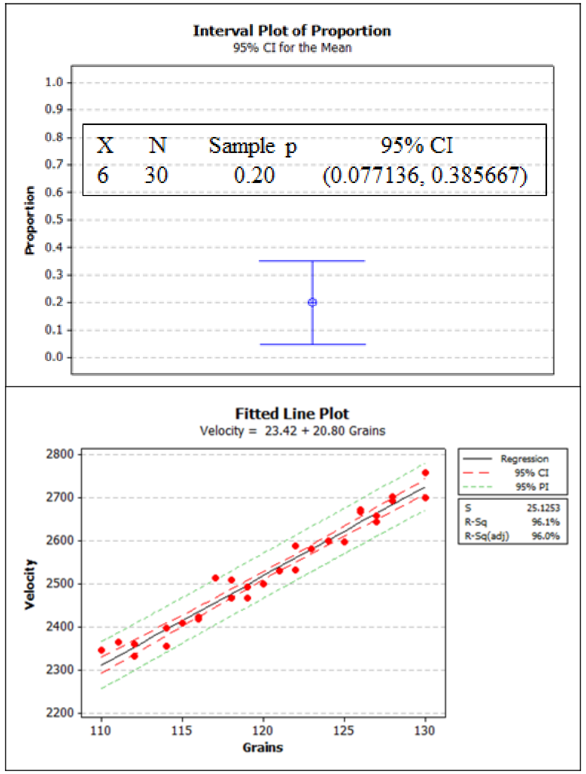
- *The* limit velocity is typically associated with the V50 (i.e., velocity at which a projectile penetrates 50% of the time) but other limit velocities may also be deemed important (e.g., V10, V90, etc.)

- LVs are commonly found in small arms requirement documents since they are a minimum requirement for terminal effectiveness behind a protective barrier



BINARY DATA

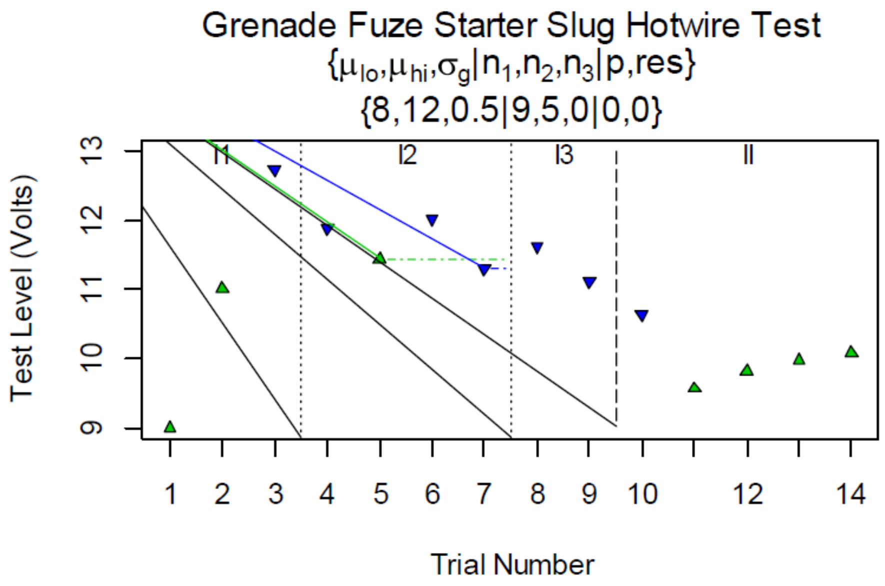
- **The issue with *binary* data:**
  - Information sparse (0/1, yes/no, go/no-go); *continuous* data contains more information (velocities, weights, etc.)
  - Requires many more samples than continuous data to acquire statistically meaningful results or build surrogate models
- **Are we leaving information on the table?**
  - Can we extract more information from the test?
  - Are there other things we can measure in parallel?



Is there any way we can avoid collecting only *binary* data?

CURRENT TESTING METHODS

- **3 Phase Optimal Design (3POD) [adaptive]**
  - The most efficient binary data gathering methodology we have today
  - [Phase 1] Finds go and no-go
  - [Phase 2] Establish overlap
  - [Phase 3] Strengthen overlap
  - Hone area of interest (V50, V90, etc.)



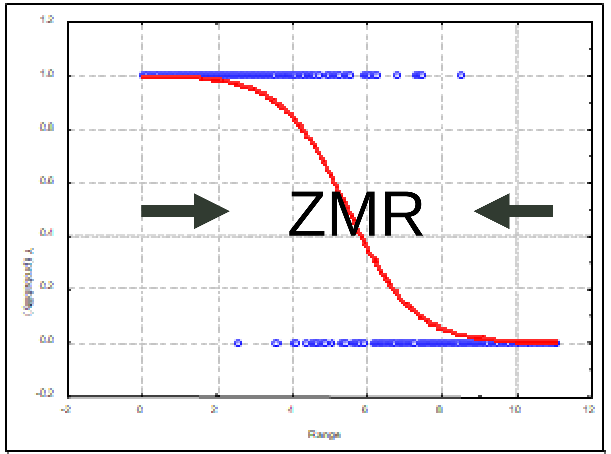
- **Langlie [adaptive]**
  - Common DoD method of sequentially gathering binary data developed in 1962

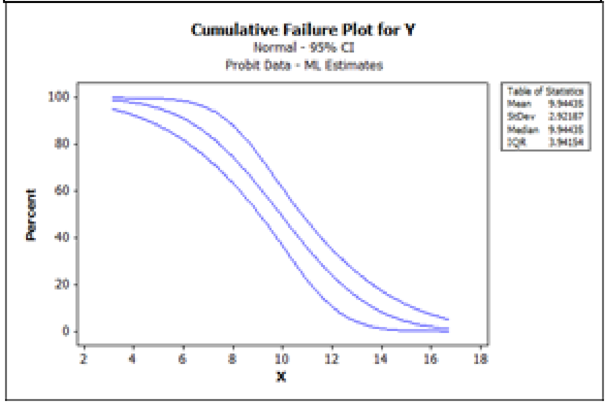
- **Brute force array [static]**
  - Build test assets at varying and equal spacing to cover area of interest (could potentially miss ZMR)

CURRENT MODELING METHODS

- **Binary logistic regression**
  - Looks at penetration / no penetration data only (Ex: logit, probit models)
  - Generates probabilistic model which requires many samples to obtain useful confidence bounds and predictions
  - Models may sometimes struggle at the far tails
  - Zone of Mixed Results (ZMR) historically necessary (although new methods exist to bypass this requirement using Firth bias-adjusted estimates)
- **Residual velocity**
  - Looks at residual velocity data
  - Ex: Jonas-Lambert
  - No proposed means of computing confidence intervals
  - Current methods do not work for limit velocities other than V50

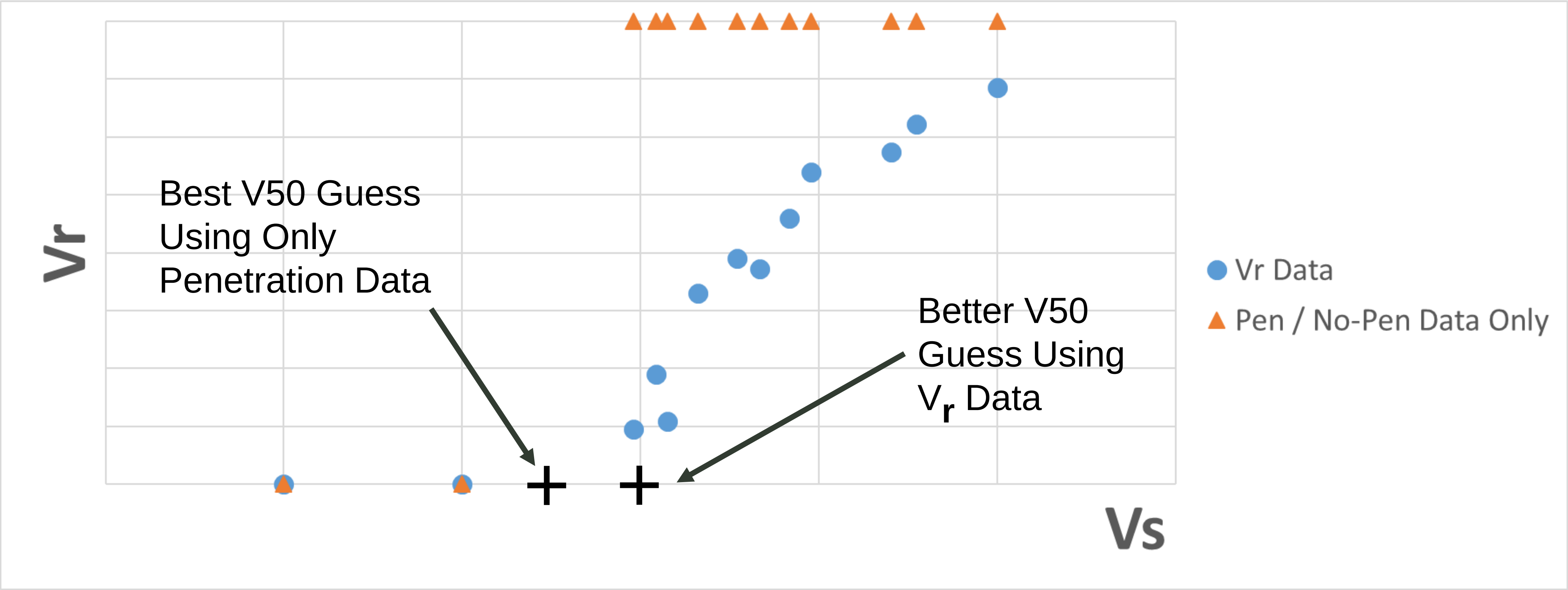
$$\pi(x) = \frac{e^{\beta_0 + \beta_1 x_1}}{1 + e^{\beta_0 + \beta_1 x_1}}$$





PROPOSED METHODOLOGY DISCUSSION

The proposed method uses projectile residual velocity (V_r) data in order to maximize "information" obtained from test



PROPOSED METHODOLOGY DISCUSSION (CONT.)

- **Need to model V_r as a function of striking velocity (V_s);** while there are existing models, the proposed V_r model was derived with the express purpose of being amenable to statistical analysis

- Current V_r models cannot accept non-penetrating test shots as data; **proposed method addresses this issue, thus eliminating "discarded" test shots**

PROPOSED METHODOLOGY DISCUSSION (CONT.)

- **Use standard non-dimensionalization techniques and application of first principles (i.e., Newton's Law)**
  - Assume instantaneous target resistance is a function of projectile velocity, contact area, and a "strength" parameter
- **Apply constraints**
  - No resistive force when projectile comes to rest; finite-time penetration event
- **Assume "average effective deformation" of projectile changes approximately linearly across small changes in impact velocity**
- **Define residual velocity as the amount of "unconsumed" velocity when the projectile reaches the back of the target**
  - This permits "negative" residual velocities if the projectile does not penetrate

PROPOSED METHODOLOGY DISCUSSION (CONT.)

- **Assume variation in residual velocity is dominated by projectile yaw, mass erosion, and the "strength" parameter; this results in the below distribution of V_r:**

$$\begin{cases} V_r \sim \operatorname{sign}(X)\,|X|^{\frac{1}{2-\alpha}} \\ X \sim v_s^{2-\alpha} - \dfrac{U^2}{v_\tau^{\alpha}\left[1 - c\left(\dfrac{v_s}{v_\tau} - 1\right)\right]} \end{cases}$$

v_s = striking velocity on the target
v_τ = limit velocity of interest (e.g., τ can be 50, 90, etc.)
α, c = empirical coefficients
U = random variable

- **The distribution of V_r is reduced to a 1D distribution**

- **It can be shown by the Central Limit Theorem implies that U is approximately log-normally distributed**

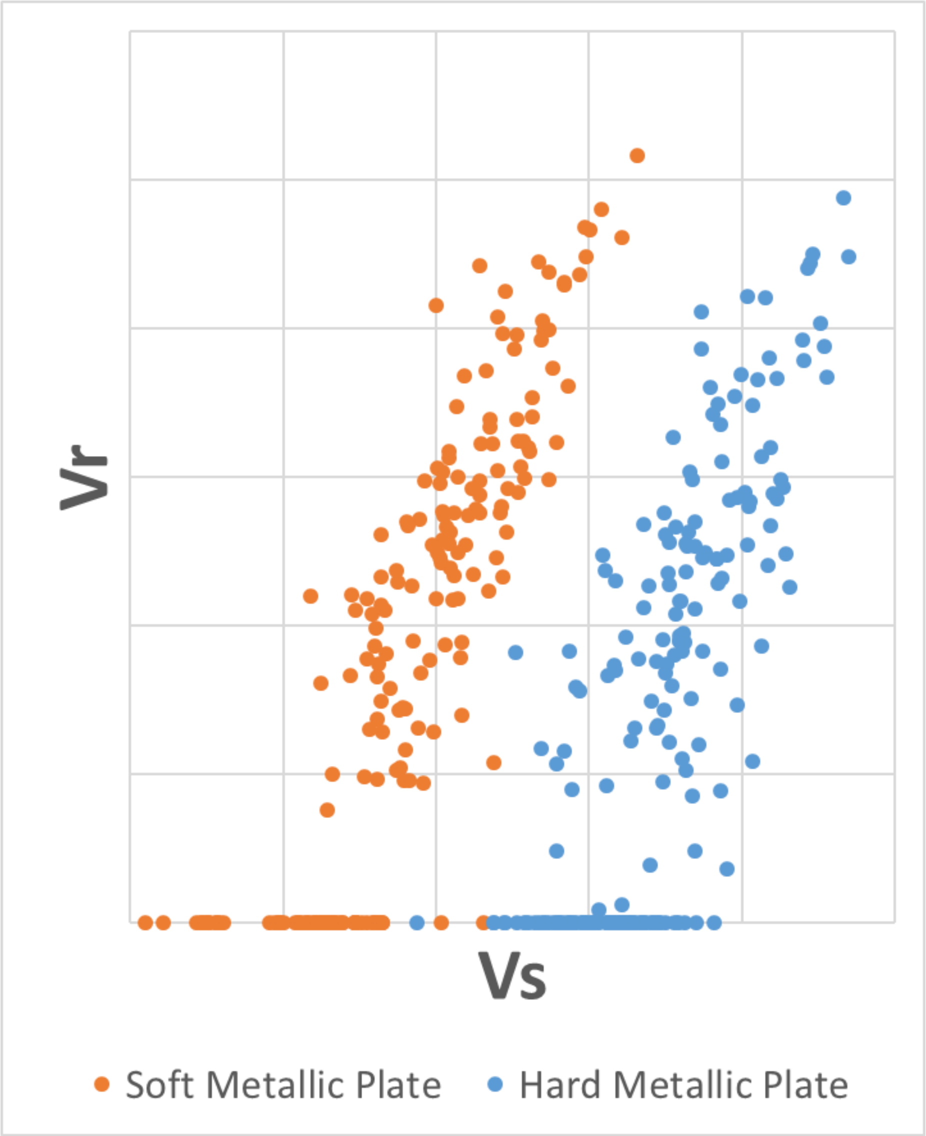
PROPOSED METHODOLOGY DISCUSSION (CONT.)

- **Use Maximum Likelihood Estimator (MLE) to approximate limit velocity**
- **For test shots that do not penetrate, use the probability of penetration in the likelihood function in place of the residual velocity distribution** (mixed continuous-discrete likelihood function)
- **Use parametric bootstrapping to determine confidence interval around estimated limit velocity**

EXPERIMENTAL RESULTS

- **Obtained sizable data set of a non-monolithic projectile against both hard and soft metallic target plates** (enough data to get "true" limit velocities)

- **Performed Monte Carlo simulations of typical plate penetration tests**
  - Residual velocities were randomly drawn from data set; V50, V90, and confidence intervals were calculated for each "test"
  - Residual velocities were sampled using two methods (Gaussian kernel, log-normal distribution of U)



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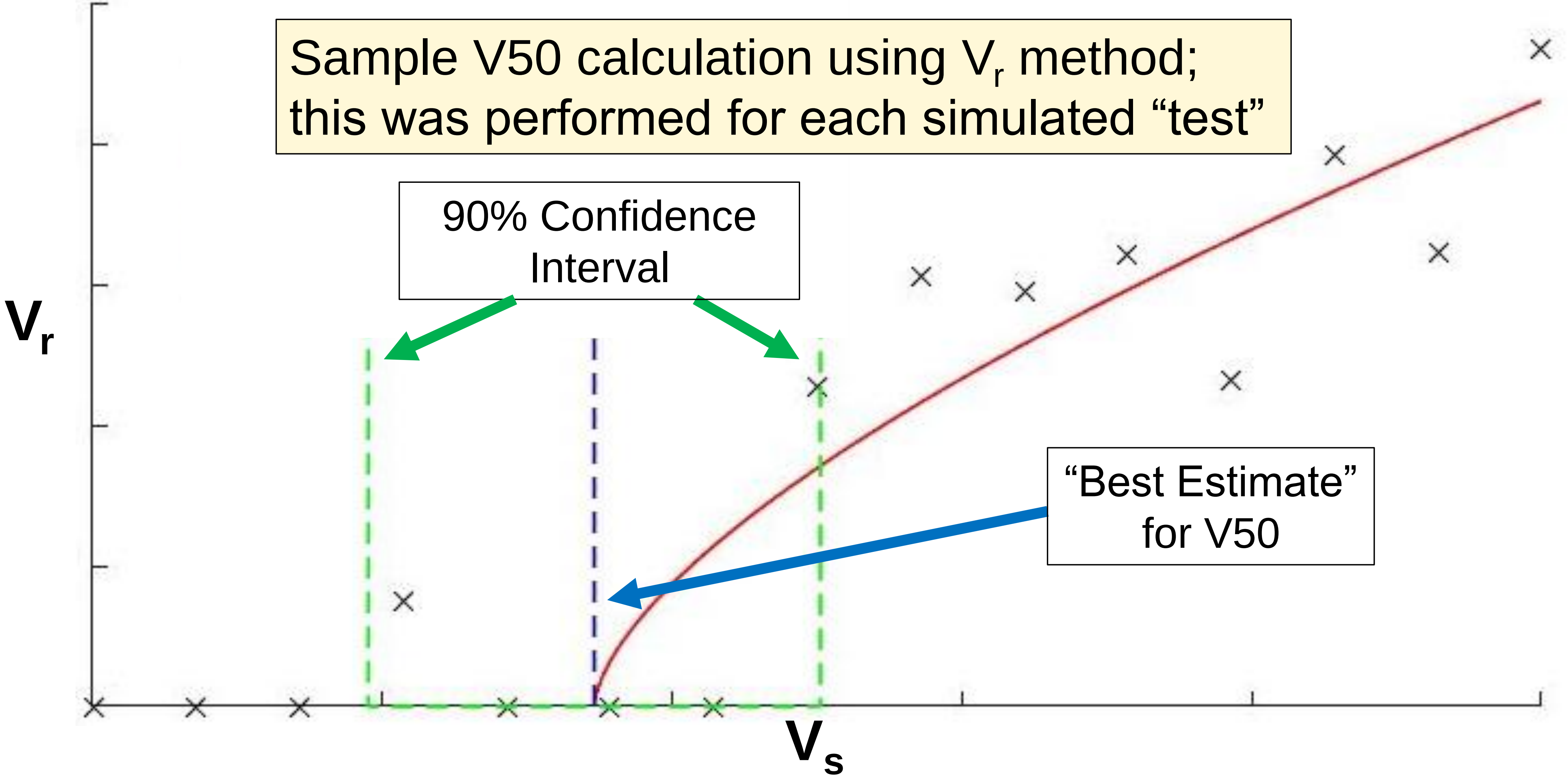
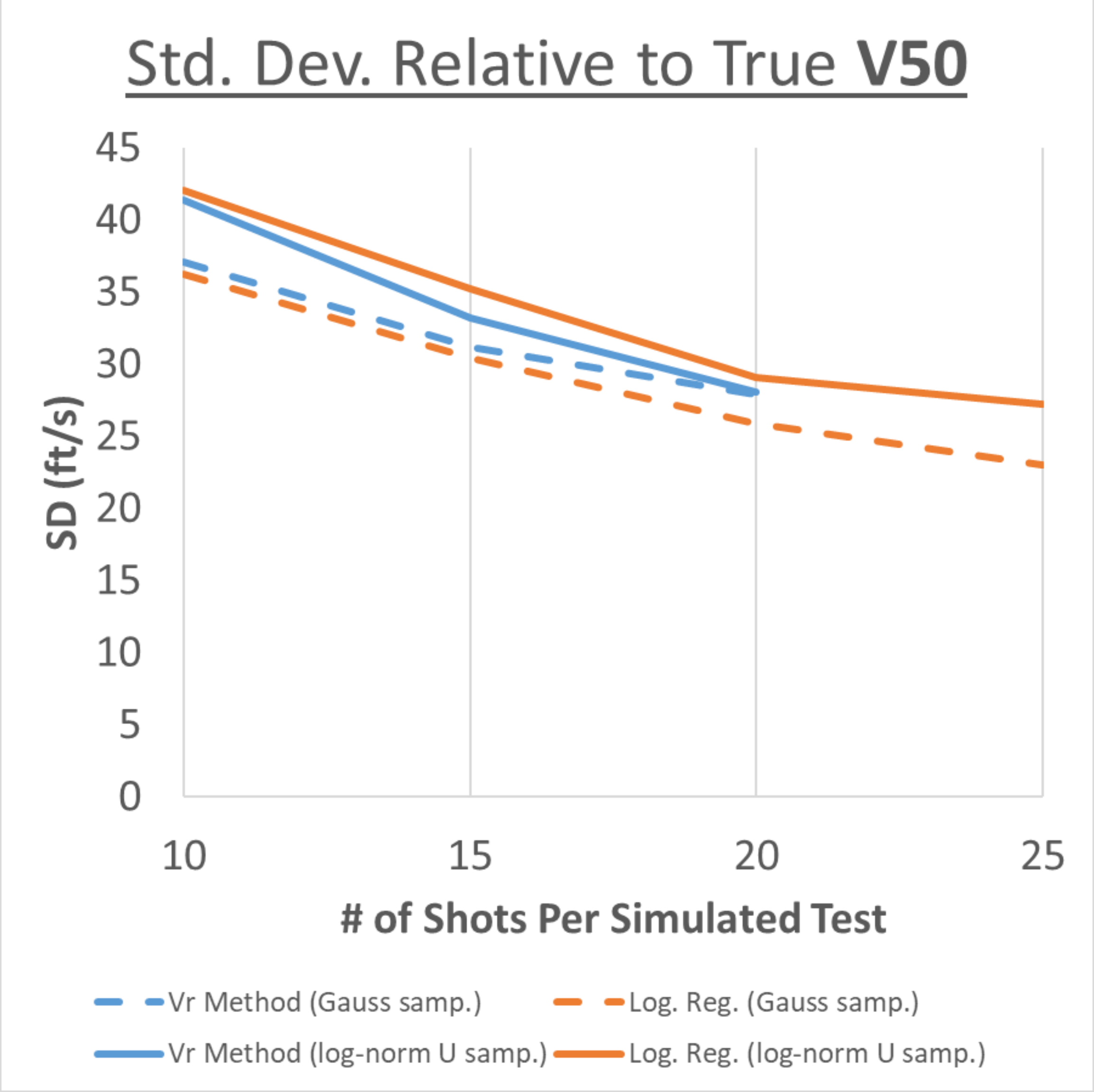
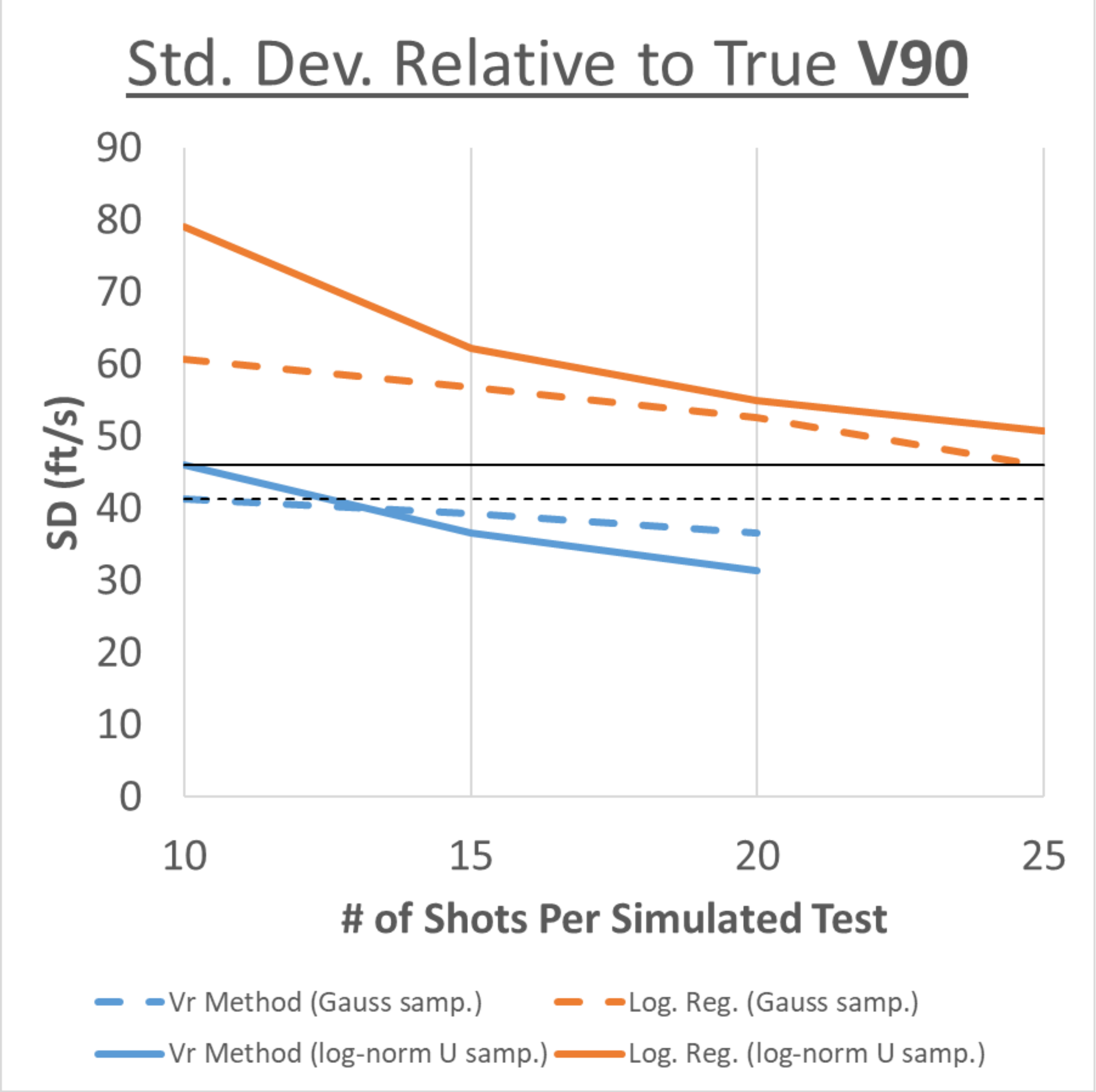


EXPERIMENTAL RESULTS (SOFT PLATE)





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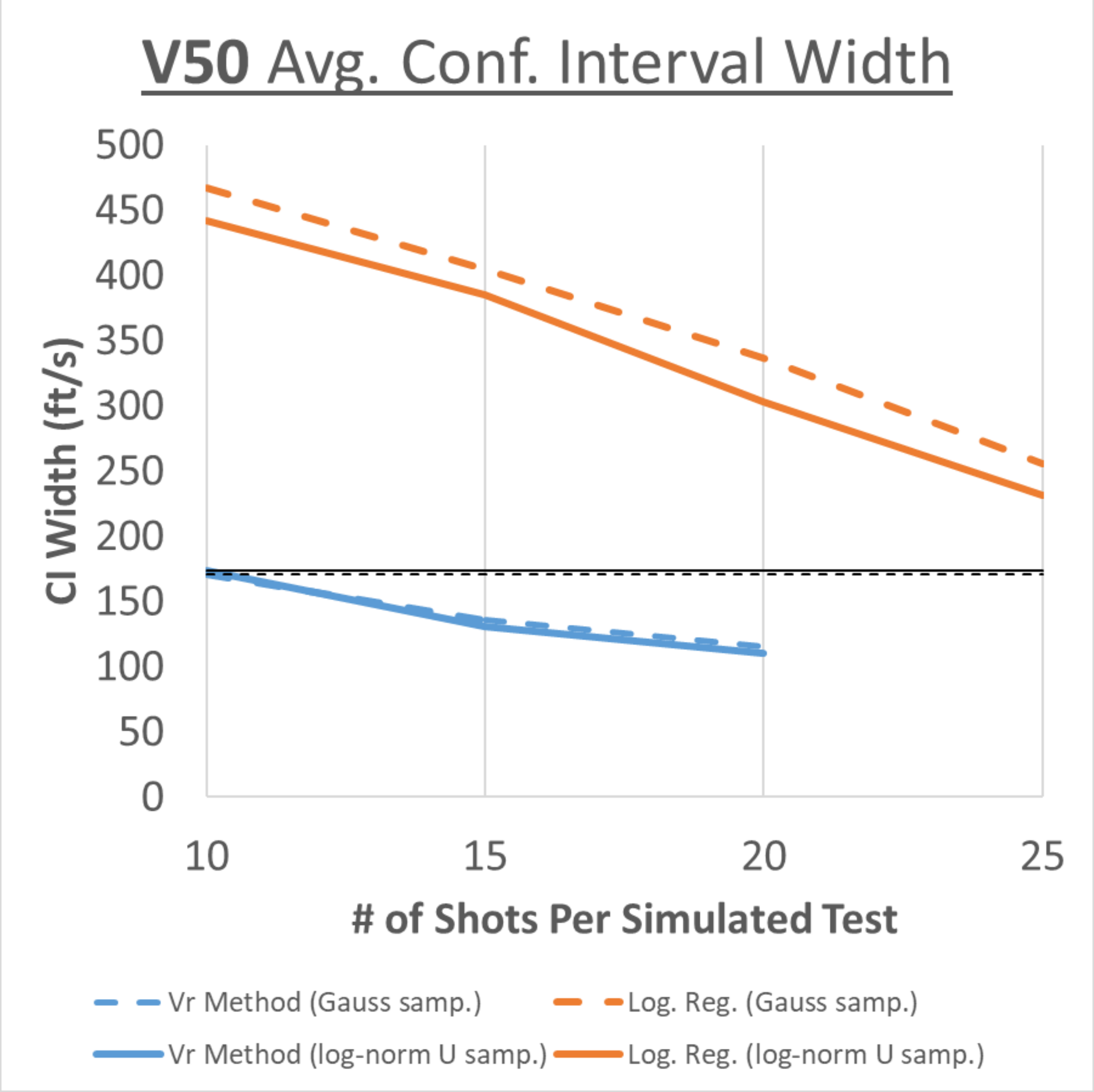
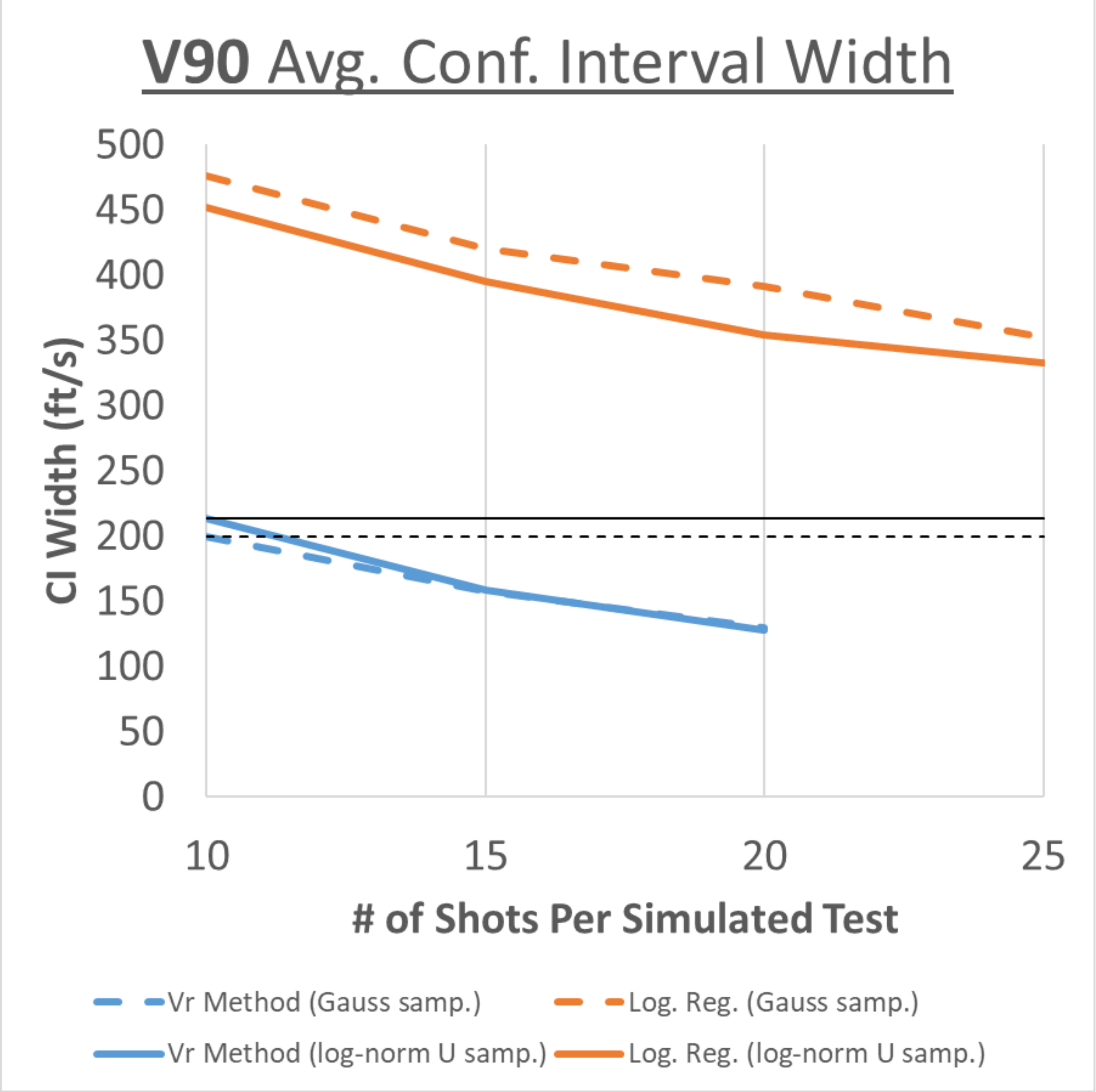
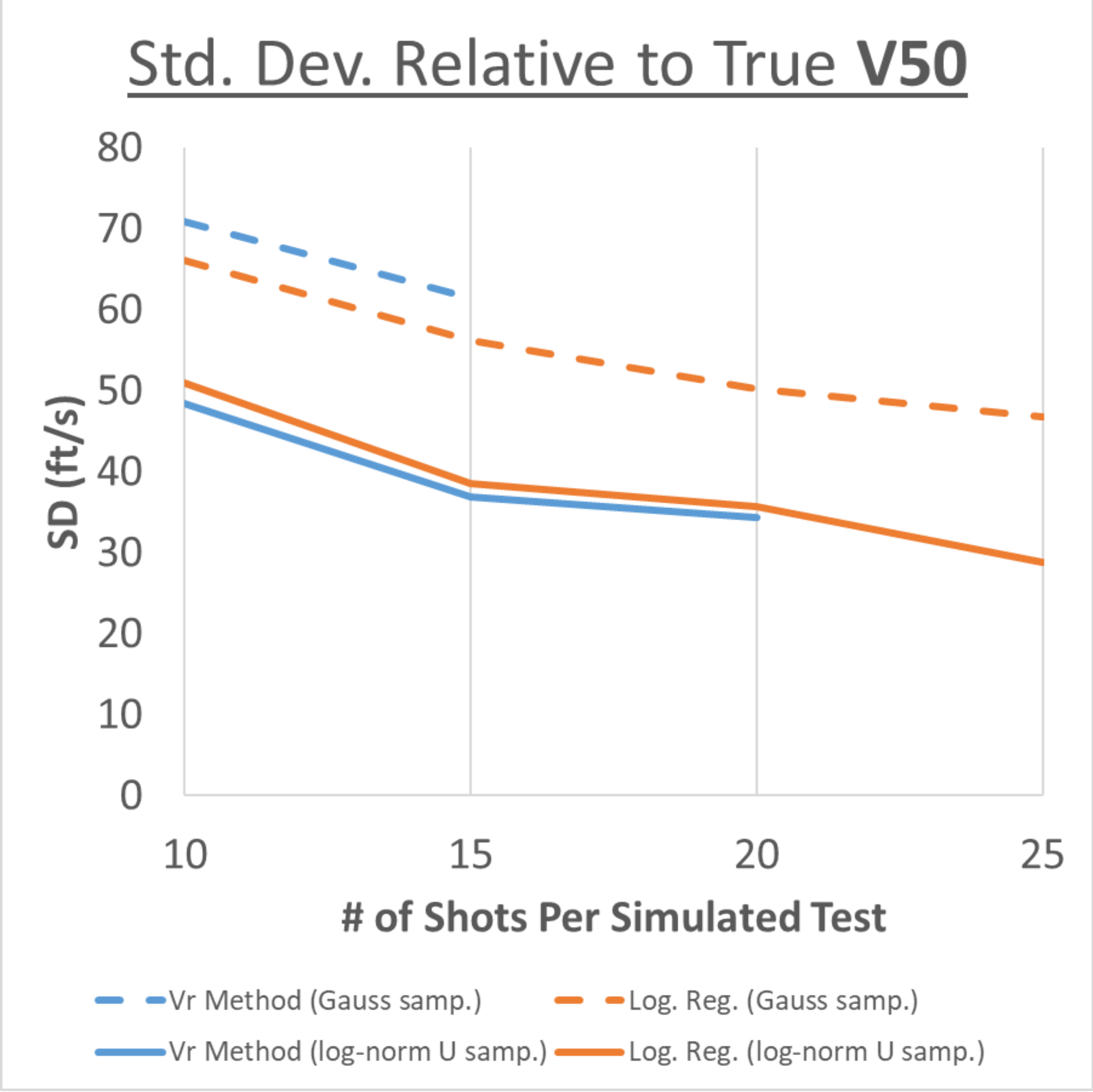
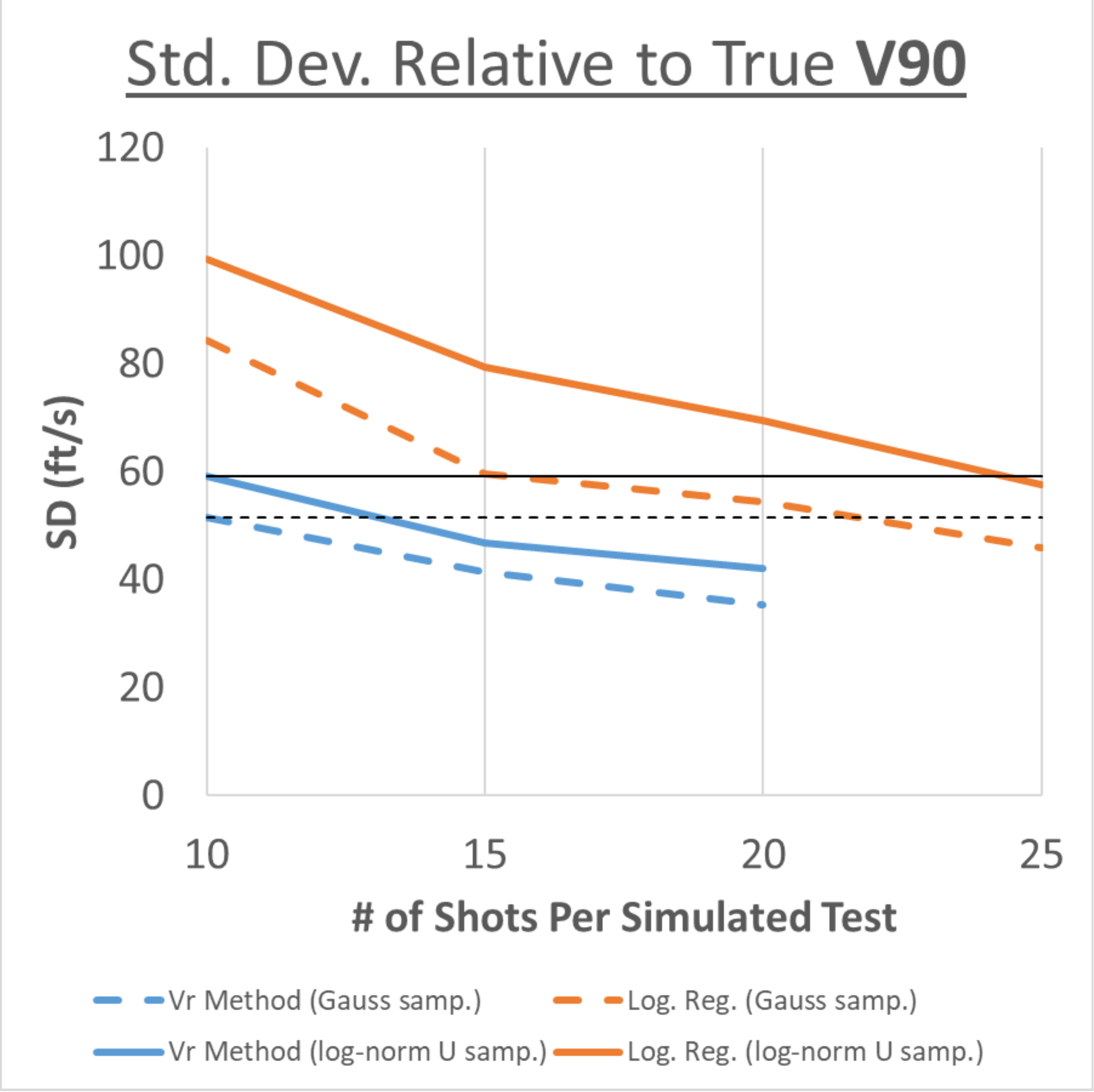




EXPERIMENTAL RESULTS (HARD PLATE)





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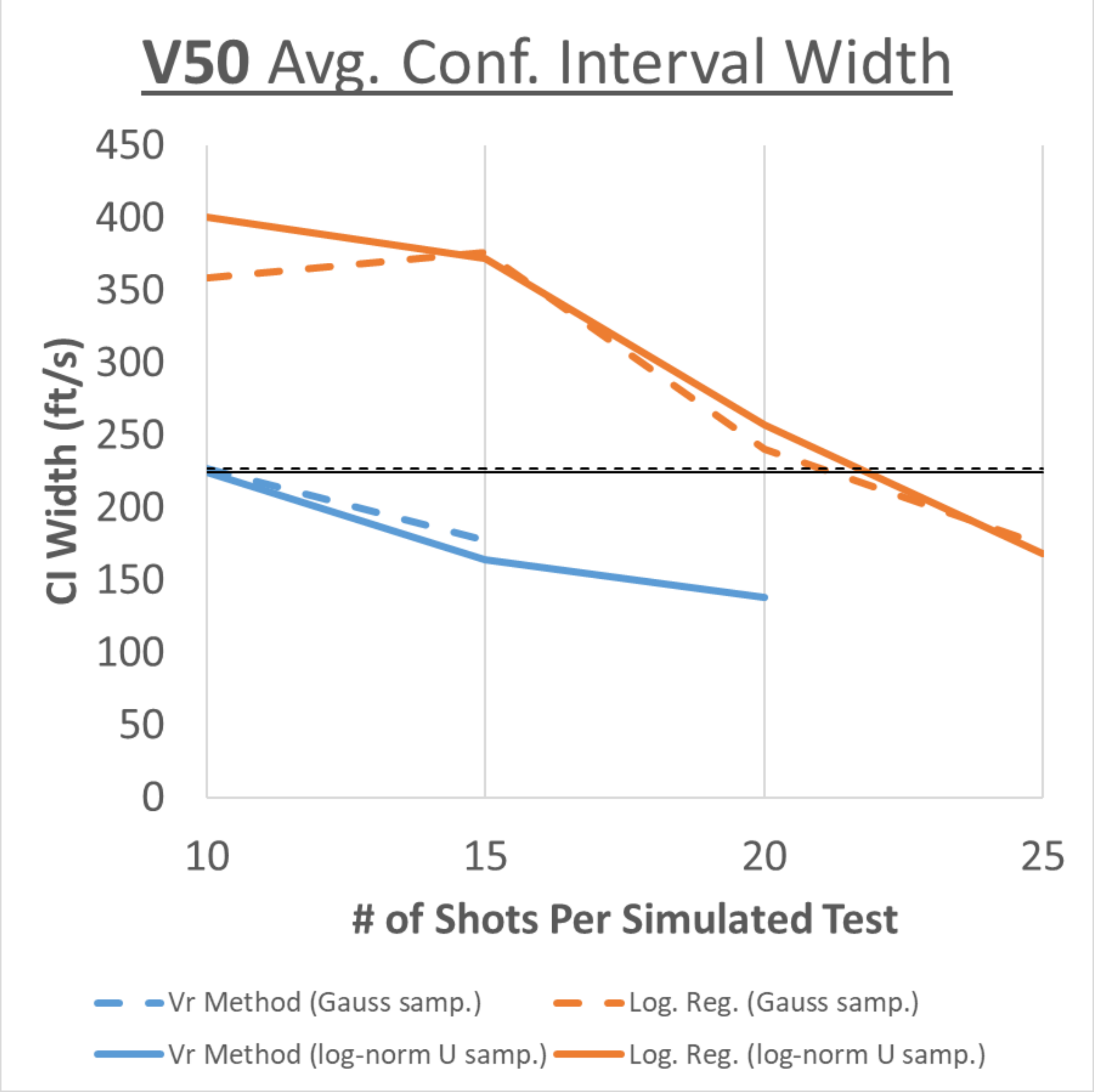
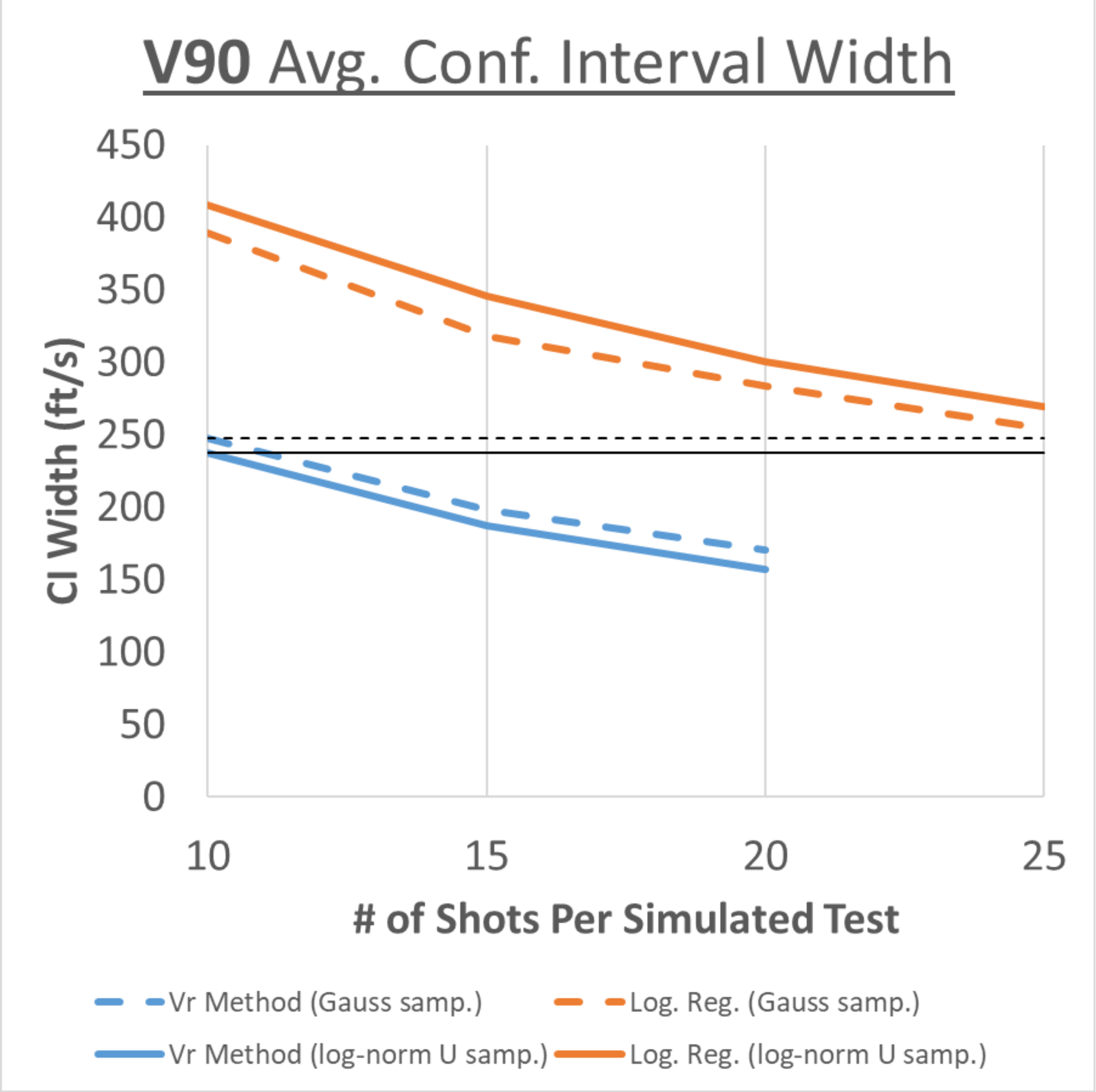




CONCLUSION

Initial investigation shows V_r method has potential to be on the order of ***2 to 3 times more efficient*** (in terms of shots required per penetration test) than current binary regression methods for small-sample penetration tests

FUTURE WORK

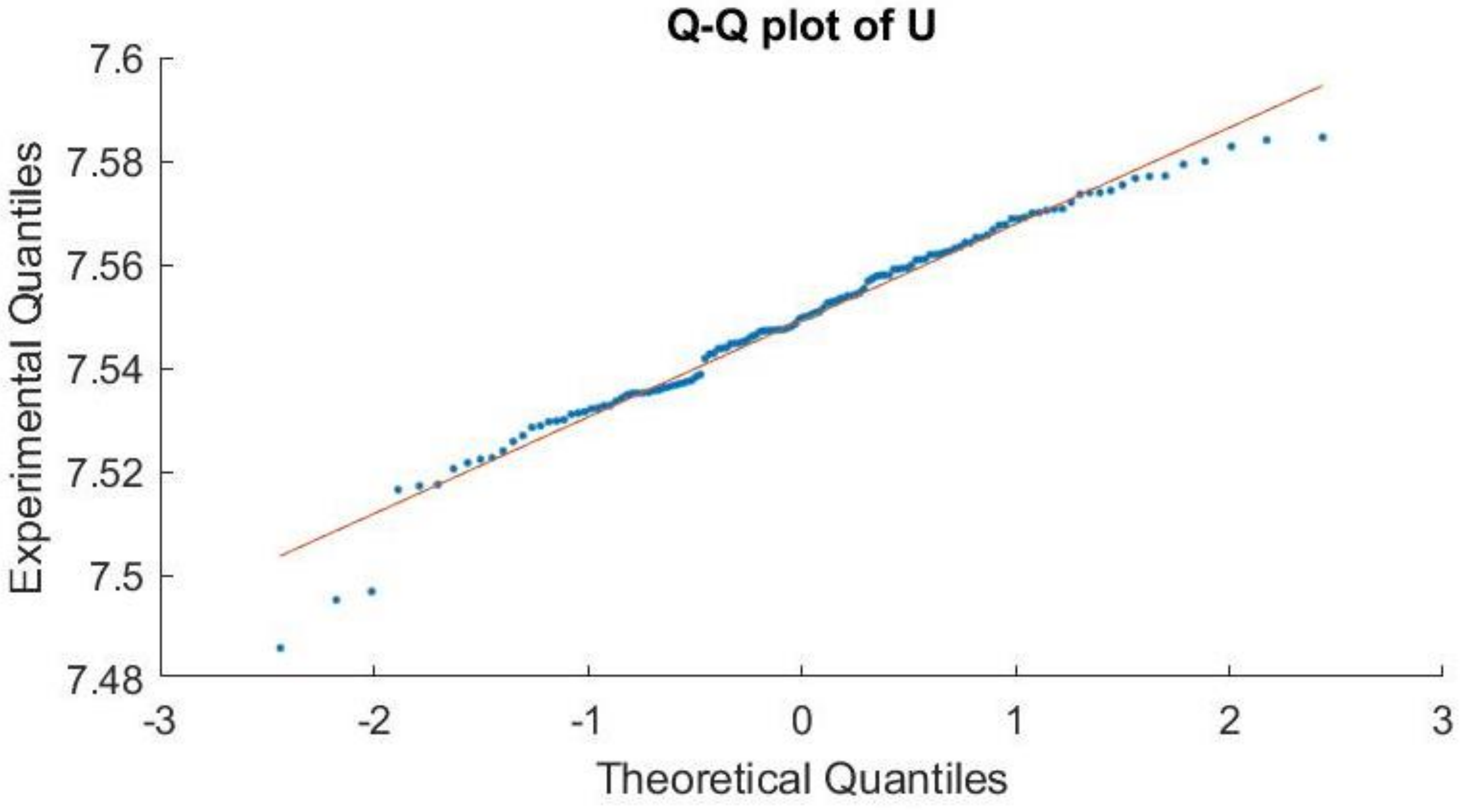
- Perform additional large-scale penetration tests across multiple calibers, projectiles, and target sets to test generality of V_r method
- Research and develop new V_r companion testing method (adaptive data gathering algorithm) to further optimize efficiency

Questions?

Back-ups

BACK-UPS

Soft Plate
V_r results



BACK-UPS

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